

[Time:3.00 Hrs]

[Marks:80]

Please check whether you have got the right question paper.

- N.B:**
1. All questions are compulsory.
 2. Figures to the right indicate full marks.
 3. Scientific calculator can be used.

- Q.1
- A) If $ca \equiv cb \pmod{n}$ then show that $a \equiv b \pmod{\frac{n}{d}}$, where $d = \gcd(c, n)$. 10
- B) Attempt any Two of the following:
- i) Define Derangement of finite objects. Let D_n denote the number of derangements of n objects. Show that $D_n = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}\right)$. 5
- ii) A building has 4 floors excluding the ground floor. 13 people get into the lift at the ground floor. Assuming that no body get into the lift from the other floors and that at least one person gets down at each floor, find the number of different possibilities so that the lift is emptied on the top floor. 5
- iii) Solve the linear Diophantine equations $172x + 20y = 1000$. 5
- Q.2
- A) If $S(n, k)$ denotes the Stirling number of Second kind, then show that $S(n, k) = \sum_{i=0}^k (-1)^i \binom{k}{i} C(k-i)^n$, $k \leq n$. 10
- B) Attempt any Two of the following:
- i) Fifteen children gathered 100 nuts. Prove that some pairs of children gathered the same number of nuts. 5
- ii) How many non-negative integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 67$? 5
- iii) In a class of 150 students, 70 have offered Mathematics, 80 have offered Physics and 90 have offered Chemistry. Of these, 40 students are Mathematics and Physics, 30 are for Mathematics and Chemistry and 50 are for Physics and Chemistry. If 20 students have neither of these subjects. Find the number of students have i) only Mathematics, ii) all three subjects. 5
- Q.3
- A) Show that $f_{m+n} = f_{m-1}f_n + f_m f_{n+1}$. Also deduce the value of f_{12} . 10
- B) Attempt any Two of the following:
- i) Find a recurrence relation and solve it for the following sequence. 5
0, 2, 6, 12, 20, 30, 42, ...
- ii) Determine the number of ways to select 4-letter combination from the set { A,B,C } if A can be included at most once, B at most twice, and C at most three times. 5
- iii) Find the coefficient of x^{16} in $(x^2 + x^3 + x^4 + \dots)^5$. Also find the general coefficient. i.e. the coefficient of x^r . 5
- Q.4
- A) Prove that "Let G be a finite group acting on a set X . Then $|Stab(x)| = \frac{|G|}{|Orb(x)|}$." 10
- B) Attempt any Two of the following:
- i) Suppose a necklace can be made from beads of three colors black, white, and red. How many different necklaces with n beads are there? 5
- ii) Find the number of benzene rings with Cl substituted in the place of H. 5
- iii) Find the number of 7-bead necklaces distinct under rotations using 3 black and 13 white beads. 5