AC – 28/03/2025

Item No. – 6.3 (N) (1ab) Sem. III & IV

As Per NEP 2020

University of Mumbai

Syllabus for Minor Vertical 2 (Scheme – I and III)			
Faculty of Science			
Board of Studies in Mathematics			
Second Year Programme in Minor (Mathematics)			
Semester		**III & IV**	
Title of Paper		**Sem. III**	**Total Credits 4**
Choose any one of the following:			
1	a) Calculus- III	III	2
	b) PM-3A Calculus III	III	2
OR			
2	a) Vector Spaces	III	2
	b) PM-3B Vector Spaces	III	2
OR			
3	a) Basic Mathematics in Real Life II	III	2
	b) PM-3C Basic Mathematics in Real Life II	III	2
Title of Paper		**Sem. IV**	**Total Credits 4**
Choose any one of the following:			
1	a) Calculus IV	IV	2
	b) PM-4A Calculus IV	IV	2
OR			
2	a) Linear Algebra	IV	2
	b) PM-4B Linear Algebra	IV	2
OR			
3	a) Basic Mathematics in Real Life III	IV	2
	b) PM-4C Basic Mathematics in Real Life III	IV	2
From the Academic Year			**2025-26**

Sem. - III

Syllabus
B.Sc. (Mathematics)
(Sem.- III)
Name of the Course: Calculus III (Minor I)

Sr. No	Heading	Particulars
1	**Description the course:** **Including but not limited to:**	Calculus finds extensive applications in diverse fields such as Physics, Chemistry, Biotechnology, Engineering, among others. This course aims to instill a deep understanding of Mathematical Analysis as it forms a rigorous foundation for Calculus. Learners will explore properties of Real Numbers, delve into concepts like Series and Riemann integration of functions. To provide practical context, the course incorporates applications of integration, offering students a broader perspective on the diverse uses of acquired knowledge.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course provides an introduction to advanced concepts in analysis with a strong emphasis on rigor. It aims to prepare students for more advanced courses in abstract analysis. The focus of the course is on developing formal proof skills, which not only deepens comprehension of the subject but also extends to broader applications in mathematics. **CO1:** Provide a solid understanding of fundamental principles and methods, equipping students with the skills to apply mathematical ideas and tools through modeling, solving, and interpretation. **CO2**: Illustrate the expansive nature of the subject by fostering the acquisition of essential mathematical tools for continued studies across various scientific fields. **CO3**: Foster students' comprehensive development by placing emphasis on problem-solving skills, nurturing creative talents, and enhancing communication abilities, all of which are vital for a range of employment opportunities. **CO4**: Ensure exposure to both global and local issues within the realm of Mathematical Sciences, allowing learners to explore diverse aspects of the discipline.	
8	**Course Outcomes (OC):**	

	After completion of the course, students will be able to **OC1** Understand and remember the concepts such as convergence/ divergence of series, Riemann Integration, beta-gamma functions and related results. **OC2**: Apply the formulae and concepts to solve the examples related to series, Riemann Integral, area between two curves etc. **OC3**: Analyse the convergence and divergence of series and integrability of given function. **OC4**: Justify/ check the integrability of function, absolute and conditional convergence of series. **OC5:** Construct counter examples related to absolutely convergent/ divergent series, non-integrable functions etc.
9	**Modules: -** **Module 1**: **Infinite Series (15 Lectures)**
	1. Infinite series in $\mathbb{R}$. Definition of convergence and divergence. Basic examples including geometric series. Elementary results such as if $\sum_{n=1}^{\infty} a_n$ is convergent then $a_n \to 0$ but converse is not true. Cauchy Criterion, Algebra of convergent series and related examples. 2. Tests for convergence: Comparison Test, Limit Comparison Test (without proof), Ratio Test (without proof), Root Test (without proof), Examples, p- series test. 3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence.
	Module 2: **Riemann Integration and Applications (15 Lectures)**
	1. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower Riemann sums for a bounded real valued function defined on a closed and bounded interval in $\mathbb{R}$. Definition of Riemann integral. 2. Criterion for Riemann integrability, Characterization of the Riemann integral as the limit of a sum. (without proof). Examples. 3. Algebra of Riemann integrable functions and basic results such as if (i) f:[a,b]⟶$\mathbb{R}$ is integrable, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (without proof) (ii) |f| is integrable and $\left\|\int_a^b f(x)dx\right\| \leq \int_a^b \|f\|(x)dx$ (iii) If $f(x) \geq 0$ for all x∈[a,b] then $\int_a^b f(x)dx \geq 0$ 4. Riemann integrability of a continuous function. Integrability of a bounded function whose set of discontinuities has only finitely many points (without proof). Riemann integrability of monotone functions. 5. First and Second Fundamental Theorems of Calculus. 6. Area between the two curves. Lengths of plane curves. Surface area of surfaces of revolution. 7. Gamma and Beta functions and their properties. Relationship between them (without proof).
10	**Recommended Reference Books:** 1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer. 2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970. 3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998. 4. T. Apostol; Calculus Vol. 2; John Wiley.
11	**Additional Reference Books** 1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014

2. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.
4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.
5. R.G. Bartle and D. R. Sherbert; Introduction to Real Analysis Second Ed.; John Wiley, New Yorm, 1992.
6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

Scheme of the Examination

The performance of the learners shall be evaluated in two parts.

- Internal Continuous Assessment of 20 marks.
- Semester End Examination of 30 marks.
- A separate head of passing is required for internal and semester-end examinations.

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13 **Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc.
(at least 3)

Sr. No.	Particulars	Marks
1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05
3	Seminar/ group presentation on any one topic related to the syllabus.	05

Paper pattern of the Test (Offline Mode with One hour duration):
Q1: Definitions/Fill in the blanks/ True or False with Justification. (04 Marks: 4 x 1).
Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)

14 **Format of Question Paper:**
The semester-end examination will be of 30 marks marks of one hour duration covering the entiresyllabus of the semester.

Note: Attempt any TWO questions out of THREE.			
Q.No.1	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.2	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.3	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: PM-3A Calculus III (Minor I)

Sr. No.	Heading	Particulars
1	**Description the course: Including but not limited to:**	Problem-solving is a fundamental aspect of any Mathematics course. While advanced courses often emphasize the theoretical nature of the subject, engaging in problem-solving reinforces concepts and enhances learners' ability to analyze existing problems and devise solutions. This activity not only motivates learners but also empowers them to formulate new results, propose conjectures, and develop innovative theories.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course emphases on problem solving and motivates to think on the basic concepts of Algebra and Analysis with rigour and prepares students to study further courses. **CO1.** To give sufficient knowledge of fundamental principles, methods and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting. **CO2.** To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. **CO3.** To enhance students' overall development, problem solving skills, creative talent, and power of communication. These are necessary for various kinds of employment. **CO4.** To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to **OC1**: Apply the formulae and concepts to solve the examples related to series, Riemann Integral, area between two curves etc. **OC2**: Analyze the convergence and divergence of series and integrability of given function. **OC3**: Justify/ check the integrability of function, absolute and conditional convergence of series. **OC4:** Construct counter examples related to absolutely convergent/ divergent series, non-integrable functions etc.	
9	**Modules: -** **Practical for Calculus III (30 Hours)**	

1.	Convergent and divergent series and algebra of convergent series.
2.	Comparison and limit comparison test.
3.	Ratio test and root test.
4.	Alternating Series and p-series test.
5.	Absolute and conditional convergence.
6.	Upper sum and lower sum.

7.	Riemann integral and its properties.
8.	Fundamental Theorems of Calculus.
9.	Area between two curves, lengths of plane curves and surface area of surfaces of revolution.
10.	Beta and Gamma functions.

10 Recommended Reference Books:

1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.
2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.
3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.
4. T. Apostol; Calculus Vol. 2; John Wiley.

11 Additional Reference Books

1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014
2. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.
4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.
5. R.G. Bartle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.
6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

<u>Scheme of the Examination</u>

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13 Continuous Evaluation through:

Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc.
(at least 3)

Sr. No.	Particulars	Marks
1	Objective question test	10
2	Overall performance	05
3	Viva	05

Paper pattern of the Test (Offline Mode):

Q1: (Attempt any 5 from 8) Multiple choice questions. (10 marks: 5 × 2)

Duration: 1Hrs

While setting question paper

	four MCQ on module 1 and four MCQ on module 2 both.	
14	**Format of Question Paper:**	

Scheme of examination:

At the end of the Semester III, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules.

Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2

1. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Name of the Course: Vector Spaces
(Minor II)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	This course covers fundamental concepts in linear algebra, concepts in mathematics with applications across various fields including physics, engineering, computer science, and economics. Learners will learn about subspaces, spanning sets, linear independence, and basis vectors, crucial concepts that form the building blocks of vector space theory.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course gives an introduction to vector spaces and system of linear equations and its solutions. Also, it deals with the basics of vector spaces, covering different examples and dealing with finite-dimensional vector spaces. CO1. To give sufficient knowledge of fundamental principles, methods, and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving, and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem-solving skills, creative talent, and power of communication are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Understand, remember the concepts and properties vector spaces. OC2: Apply the formulas and the concepts to solve examples subspaces, sum and intersection of subspaces. OC3: To analyse the properties of vector spaces, row space and column space of a matrix OC4: Justify or check a set to be a vector space. OC5: Construct counterexamples related to vector spaces and subspaces.	
9	**Modules: -** **Module 1: Vector spaces and subspaces (15 Hours)**	
	(a) Definition of a vector space over R. Examples such as: (i)Euclidean space R^n. (ii) The space of $m \times n$ matrices over R. (iii)The space of polynomials with real coefficients.	

	(b) Subspaces: definition, criterion for a nonempty subset to be a subspace of a vector space. Examples, including: (i) Lines in R^2, Lines and planes in R^3. (ii) The solutions of a homogeneous system of linear equations. (iii) The spaces of symmetric, skew-symmetric, upper triangular, lower triangular, and diagonal matrices. (iv) The space of polynomials with real coefficients of degree $\leq n$. (c) The sum, union and intersection of subspaces, direct sum of vector spaces. Cosets, Introduction to quotient space.
	Module 2: System of linear equations, Linear combination, Basis of vector spaces (15 Hours)
	(a) (i) Introduction to linear systems, Matrix representation of the system of homogeneous and non-homogeneous linear equations, row echelon form, Gauss Elimination. (ii) Linear combination of vectors. (iii) Linear span of a subset of a vector space. (iv) Linear dependence and independence of a set. (b) Basis of a vector space, Dimension of a vector space. The discussion of these concepts is for finitely generated vector spaces only. (c) (i) Row space, column space of a $m \times n$ matrix over R and row rank, column rank of a matrix. (ii) Equivalence of row rank and column rank, computing the rank of a matrix by row reduction.
10	**Text Books** 1. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd edition, Pearson. 2. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition). 3. Serge Lang, Introduction to Linear Algebra, Springer. 4. S Kumaresan, Linear Algebra: A Geometric Approach, PHI Learning.
11	**Reference Books** 1. Sheldon Axler, Linear Algebra done right, Springer. 2. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers. 3. David W. Lewis, Matrix theory.
	<u>Scheme of the Examination</u>
	The performance of the learners shall be evaluated in two parts. ● Internal Continuous Assessment of 20 marks. ● Semester examination of 30 marks. ● A separate head of passing is required for internal and semester-end examinations.

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**
13	**Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc. (at least 3)	

Sr. No.	Particulars	Marks
1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
2	Project on any one topic	05

	related to the syllabus or a quiz (offline/online) on one of the modules.		
3	Seminar/group presentation on any one topic related to the syllabus.	05	

Paper pattern of the Test (Offline Mode with One-Hour Duration):

Q1: Definitions/Fill in the blanks/ True or False with Justification. (04 Marks: 4 x 1).

Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)

14 Format of Question Paper:

The semester-end examination will be of 30 marks of one-hour duration, covering the entire syllabus of the semester.

Note: Attempt any TWO questions out of THREE.			
Q.No.1	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.2	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.3	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: PM-3B Vector Spaces (Minor II)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	This course covers fundamental concepts in linear algebra, concepts in mathematics with applications across various fields including physics, engineering, computer science, and economics. Learners will learn about subspaces, spanning sets, linear independence, and basis vectors, crucial concepts that form the building blocks of vector space theory.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course gives an introduction to vector spaces and system of linear equations and its soutions. Also, it deals with the basics of vector spaces, covering different examples and dealing with finite-dimensional vector spaces. CO1. To give sufficient knowledge of fundamental principles, methods, and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving, and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem-solving skills, creative talent, and the power of communication are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Apply the formulas and the concepts to solve examples related to vector spaces. OC2: To analyse and test the property of vector subspaces on sets. OC3: To check linear independence, and dependence of vectors. OC4: Construct basis and counter-examples related to vector spaces and subspaces.	
9	**Modules: -** **Module 1: Vector spaces and subspaces (30 Hours)**	

1.	Vector spaces-I (Examples)
2.	Vector spaces-II (To check which of the given sets are vector spaces)
3.	Subspaces of Euclidean space
4.	Subspaces of Polynomial space
5.	Subspaces of Matrix space
6.	Sum of subspaces
7.	Intersection of subspaces
8.	Union of subspaces

9.	Direct sum of subspaces
10.	Cosets, Quotient spaces

Module 2: System of linear equations, Linear combination, Basis of vector spaces (30 Hours)

1.	System of linear equations
2.	Linear combination of vectors
3.	Linear span of vectors in vector spaces
4.	Linear dependence.
5.	Linear independence.
6.	Standard Basis of vector spaces.
7.	Basis of vector spaces
8.	Dimension of vector spaces.
9.	Row rank and column rank of the matrix.
10.	Computing rank of matrix by row reduction.

10 Text Books

1. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd edition, Pearson.
2. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition.
3. Serge Lang, Introduction to Linear Algebra, Springer.
4. S Kumaresan, Linear Algebra: A Geometric Approach, PHI Learning.

11 Reference Books

1. Sheldon Axler, Linear Algebra done right, Springer.
2. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.
3. David W. Lewis, Matrix theory.

Scheme of the Examination

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13 Continuous Evaluation through:

Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc.
(at least 3)

Sr. No.	Particulars	Marks
1	Objective question test	10
2	Overall performance	05
3	Viva	05

Paper pattern of the Test (Offline Mode):

Q1: (Attempt any 5 from 8) Multiple-choice questions. (10 marks: 5 × 2)

	Duration: 1Hrs **While setting the question paper, four MCQs on module 1 and four MCQs on module 2 both.**	
14	**Format of Question Paper:** **Scheme of examination:**	

At the end of Semester III, Practical examinations of three hours duration and 30 marks shall be conducted based on both modules.

Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple-choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2,
Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Name of the Course: Basic Mathematics in Real Life-II (Minor III)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	To demonstrate the importance of mathematics in real life by considering interdisciplinary applications of basic concepts in real life.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course is a balanced mixture of the basic concepts of mathematics, which highlights the crucial role of mathematics in other sciences. In this course, students from various science streams will be able to see mathematics being applied in their area of interest and learn CO1: To develop methods for polynomial interpolation. CO2: To identify row echelon form and row reduced echelon form for matrices. CO3: To associate mathematical notation for solving real-life problems like those related to forest management. CO4: To recognize the role of geometry (in particular Platonic solids) in science streams.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Understand and remember the polynomials, polynomial graphs, Fibonacci sequence, platonic solids and Matrices OC2: Apply the formulas and concepts to solve problems and examples related to quadratic equations recurrence relations, matrices and platonic solids OC3: Examine and investigate the growth matrices, echelon form of matrices, and Kirchhoff's Law using matrices. OC4: Justify and check the inverse of matrices and applications of the golden ratio. OC5: Design and construct circuits using Kirchhoff's Laws	
9	**Modules: -** **Module 1: Polynomials and interpolation (15 Hours)**	
	1. Solving a quadratic equation: conditions for repeated roots, discriminant 2. Plotting a polynomial graph for degree two when roots are real. Example: (monic; not monic) 3. Relation between roots and coefficients of a polynomial (degree two), plotting polynomial of degree three (some specific examples). 4. Fibonacci numbers: motivation and recurrence relation. Simple examples of recurrence. 5. Sunflower and Golden ratio. 6. Polynomial interpolation: Statement of the problem and motivation; calculation in degree two. 7. Matrices (not necessarily square) and multiplication formula for two matrices under suitable conditions; examples. 8. Forest management—I: introduction to growth matrix.	

	9. Forest management II: Statements and notations for optimal sustainable yield. 10. Forest management III : Computation of solution 11. Row echelon and Row reduced echelon form: Definition and computation in 2 by 2 matrices. 12. Row echelon and row reduced echelon form: computation in 3 by 3 matrices. 13. Definition of the inverse of a matrix, Elementary matrices and calculation of inverse in particular examples (size 2,3) 14. Polynomial interpolation in degree three: simple examples. 15. Vander monde matrix and computation of its determinant (by stating properties of determinant).
	Module 2: Applications of linear systems and introduction to platonic solids (15 Hours)
	1. Kirchoff's laws recall and setting up notation. 2. Kirchoff's laws and determination of current in a circuit (setting up a linear system of equations). 3. Kirchoff's laws and explicit examples. 4. Cofactor, Adjoint of a Matrix: Definitions. 5. Computation of cofactor and adjoint of two-by-two matrices. 6. Computation of cofactor and adjoint of three-by-three matrices. 7. Formula stating the relation between a matrix, adjoint, and inverse. 8. Computation of adjoint and inverse for higher-size matrices. 9. Relation between invertibility and uniqueness of solution to a linear system of equations (only statement) and examples. 10. Counting edges, faces, and vertices in planar and non-coplanar figures. Statement of Euler's formula. 11. Platonic solids: introducing five platonic solids with names, verifying Euler's formula. 12. Proof of the existence of only five platonic solids. 13. Duals of platonic solids, the existence of molecules in the shape of platonic solids, and the impossibility of certain crystal shapes. 14. George Mendel and his experiment and introduction to The hardy-Weinberg principle in population genetics 15. Punnett square and associated binomial expansions.
10	**Text Books:** 1. Hermann Weyl, Symmetry, Princeton University Press, 1952. 2. Elementary Linear Algebra Application Version, H. Anton, C. Rorres, Wiley, Tenth Edition.
11	**Reference Books:** 1. Contemporary Abstract Algebra, J. A. Gallian, Narosa publishing house. 2. Tipler, Paul (2004). Physics for Scientists and Engineers: Electricity, Magnetism, Light, and Elementary Modern Physics (5th ed.). W. H. Freeman.
	<u>Scheme of the Examination</u>
	The performance of the learners shall be evaluated in two parts. ● Internal Continuous Assessment of 20 marks. ● Semester-end examination of 30 marks. ● A separate head of passing is required for internal and semester-end examinations.

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13	**Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc. (at least 3)	

Sr. No.	Particulars	Marks
1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05
3	Seminar/group presentation on any one topic related to the syllabus.	05

Paper pattern of the Test (Offline Mode with One Hour Duration):

Q1: Definitions/Fill in the blanks/ True or False with Justification.
(04 Marks: 4 x 1).

Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)

14 Format of Question Paper:

The semester-end examination will be of 30 marks of one hour duration, covering the entire syllabus of the semester.

Note: Attempt any TWO questions out of THREE.			
Q.No.1	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.2	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.3	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: PM-3C Basic Mathematics in Real Life-II (Minor III)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	To demonstrate the importance of mathematics in real life by considering interdisciplinary applications of basic concepts in real life.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course is a balanced mixture of the basic concepts of mathematics, which highlights the crucial role of mathematics in other sciences. In this course, students from various science streams will be able to see mathematics being applied in their area of interest and learn CO1: To develop methods for polynomial interpolation. CO2: To identify row echelon form and row reduced echelon form for matrices. CO3: To associate mathematical notation for solving real-life problems like those related to forest management. CO4: To recognise the role of geometry (in particular Platonic solids) in science streams.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Apply the formulas and concepts to solve interpolation problems, problems, and examples related to polynomials up to degree three, equations of recurrence relations. OC2: analyse the solutions of the system of linear equations and graphs OC3: Check the relation between non-planar graphs and Euler's formula and verify the Hardy-Weinberg principle. OC4: Construct the recurrence relations and design circuits based on Kirchhoff's Law	
9	**Modules: -** **Module 1: Practicals for Polynomials and Interpolation (30 Hours)**	

1.	Computing roots for quadratic equations.
2.	Plotting polynomials of degree at most three.
3.	Setting up Recurrence Relations.
4.	Polynomial Interpolation with Examples.
5.	Multiplication of matrices of arbitrary sizes.
6.	Computation of Row-Echelon Form in 2 by 2 and 3 by 3 matrices.
7.	Computation of Row-Reduced-Echelon Form in 2 by 2 and 3 by 3 matrices.
8.	Calculating inverses of 2 by 2 matrices and 3 by 3 matrices.
9.	Polynomial Interpolation in Degree Three.
10.	Vandermonde Determinant and Invertibility.

	Module 2: Practicals for Applications of linear systems and introduction to Platoonic solids (30 Hours)

1.	Kirchhoff's Law in Computing Current in given examples.
2.	Computing cofactors of 2 by 2 and 3 by 3 matrices.
3.	Computing adjoint and inverse of 2 by 2 matrices.
4.	Computing the adjoint and inverse of 3 by 3 matrices.
5.	Determining solutions to linear systems of equations using matrices.
6.	Examples in n by n matrices to System of Linear Equations.
7.	Euler's Formula via Examples.
8.	Planar Figures and Graphs, Definition and Examples.
9.	Non-Planar Figures and Relation to Euler's Formula.
10.	Problems based on the Hardy-Weinberg principle.

10 **Text Books:**

1. Artin, Algebra, Pearson, Second Edition.
2. Elementary Linear Algebra Application Version, H. Anton, C. Rorres, Wiley, Tenth Edition.

11 **Reference Books:**

1. Contemporary Abstract Algebra, J. A. Gallian, Narosa publishing house.
2. Tipler, Paul (2004). Physics for Scientists and Engineers: Electricity, Magnetism, Light, and Elementary Modern Physics (5th ed.). W. H. Freeman.

Scheme of the Examination

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13 **Continuous Evaluation through:**
Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc.
(at least 3)

Sr. No.	Particulars	Marks
1	Objective question test	10
2	Overall performance	05
3	Viva	05

Paper pattern of the Test (Offline Mode):

Q1: (Attempt any 5 from 8) Multiple-choice questions. (10 marks: 5 × 2)

Duration: 1Hrs
While setting the question paper, four MCQs on module 1 and four MCQs on module 2

	both.	

14 **Format of Question Paper:**

Scheme of examination:

At the end of the Semester III, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules.
Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple-choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2,
Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Sem. – IV

Name of the Course: Calculus IV (Minor I)

Sr. No.	Heading	Particulars
1	**Description the course:** **Including but not limited to:**	Calculus finds extensive applications in diverse fields such as Physics, Chemistry, Biotechnology, Engineering, and more. This course seeks to provide learners with a comprehensive understanding of Multivariable Calculus, building upon a rigorous foundation laid by Mathematical Analysis. Through the exploration of various properties of derivatives of scalar fields and vector fields. Students will gain valuable insights into the analytical aspects of Multivariable Calculus. To enhance practical understanding, the course incorporates real-world applications of differentiation in multiple dimensions, allowing learners to grasp the diverse uses of the acquired knowledge.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course aims to equip students with a comprehensive understanding of functions of several variables and the principles of differentiation for scalar and vector fields in multivariable calculus. **CO1**: To develop the understanding of vectors in $\mathbb{R}^n$ focusing on $\mathbb{R}^2$ and $\mathbb{R}^3$ and acquire proficiency in working with real-valued functions of several variables. **CO2**: To demonstrate competence in analyzing neighbourhoods in $\mathbb{R}^n$ and applying concepts of limits and continuity to scalar fields. **CO3**: To define and compute partial and directional derivatives of scalar fields, focusing on $\mathbb{R}^2$ and $\mathbb{R}^3$, and understand the Mean Value Theorem for scalar fields. **CO4**: To explore the basic properties of differentiability, such as continuity at a point, existence of partial derivatives, and differentiability when partial derivatives exist and are continuous. **CO5**: To utilize concept of differentiation for practical applications, including the understanding of tangent planes and maxima-minima. **CO6**: To understand higher-order partial derivatives and their applications, including the Mixed Partial Derivatives Theorem, Taylor's Theorem for twice continuously differentiable functions, the Method of Lagrange Multipliers and the Second Derivative Test for functions of two variables.	
8	**Course Outcomes (OC):** After completion of the course, students will be able **OC1**: understand and remember the concepts such as Euclidean spaces, norm, inner product, limit, continuity, derivatives of scalar fields etc. **OC2**: apply first and second derivative tests to find extreme values of scalar fields. **OC3**: verify the relationship between Differentiability and Continuity, directional derivative and continuity etc.	

	OC4: check differentiability and continuity of scalar and vector fields. **OC5**: create counter examples related to continuity and differentiability, directional derivative and continuity, partial derivatives and total derivative etc.
9	**Modules: -** **Module 1**: **Functions of Several Variables (15 Lectures)**
	1 Review of vectors in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and basic notions such as addition and scalar multiplication, inner product, length (norm) and distance between two points. 2 Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples. 3 Sequence in $\mathbb{R}^n$ [with emphsis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and their limits. Neighbourhoods in $\mathbb{R}^n$. Limits and continuity of scalar fields. Sequential characterizations (without proof), Composition of continuous functions. Algebra of limits and continuity (Results with proofs). Iterated and simultaneous limits of scalar fields. Limits and continuity of vector fields. Algebra of limits and continuity of vector fields. (without proofs). 4 Partial derivatives, directional derivatives and gradient of scalar fields (with emphasis on $\mathbb{R}^2$ and $\mathbf{R}^3$). Existence of directional derivative implies continuity. Mean Value Theorem for scalar fields. 5 Differentiability of scalar fields (in terms of linear transformation). Concept of total derivative and its uniqueness, basic results such as (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability and (iii) differentiability when the partial derivatives exist and are continuous.
	Module 2: Applications of Differentiability (15 Lectures)
	1 Relation between total derivative and gradient of a function. Chain rule (without proof). Geometric properties of gradient. Tangent planes. 2 Euler's Theorem, Higher order partial derivatives. Mixed Partial Derivatives Theorem (n=2). 3 Taylor's Theorem for twice continuously differentiable functions (without proof). 4 The maximum and minimum rate of change of scalar fields. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables (statement only). Examples. Method of Lagrange Multipliers.
10	**Recommended Reference Books:** 1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley. 2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer. 3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc. 4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer. 5. D. Somasundaram and B. Choudhary; A First Course in Mathematical Analysis, Narosa New Delhi, 1996. 6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.
11	**Additional Reference Books** 1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-

Wesley, 1998.

2. Howard Anton; Calculus- A new Horizon, (Sixth Edition); John Wiley and Sons Inc, 1999.

3. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.

4. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

<u>Scheme of the Examination</u>

The performance of the learners shall be evaluated in two parts.

- Internal Continuous Assessment of 20 marks.
- Semester End Examination of 30 marks.
- A separate head of passing is required for internal and semester-end examinations.

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**

13 **Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc.
(at least 3)

Sr. No.	Particulars	Marks
1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05
3	Seminar/ group presentation on any one topic related to the syllabus.	05

Paper pattern of the Test (Offline Mode with One hour duration):

Q1: Definitions/Fill in the blanks/ True or False with Justification.
(04 Marks: 4 x 1).

Q2: Attempt any 2 from 3 descriptive questions. (06 marks: 2 × 3)

14 **Format of Question Paper:**

The semester-end examination will be of 30 marks of one hour duration covering the entiresyllabus of the semester.

Note: Attempt any TWO questions out of THREE.			
Q.No.1	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Q.No.2	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.3	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: PM-4A Calculus IV (Minor I)

Sr. No.	Heading	Particulars
1	**Description the course: Including but not limited to:**	Problem solving forms one of the basic aspects of any course in Mathematics. Higher courses in Mathematics focus mainly on the theoretical nature of the subject, nevertheless, the problem- solving activity strengthens the concepts and helps the learners develop their ability to think over the existing problems in the subject, and also to create and crack new problems! This way a learner is not just motivated, but elevated also, to formulate new results, suggest new postulates (usually known as conjectures), and design new theories.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course introduces basic concepts of Calculus, Linear Algebra and differential equation with rigour and prepares students to study further courses. **CO1**. To give sufficient knowledge of fundamental principles, methods, and a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving and interpreting. **CO2**. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. **CO3**. To enhance students' overall development, problem solving skills, creative talent, and power of communication, which are necessary for various kinds of employment. **CO4**. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able **OC1**: apply first and second derivative tests to find extreme values of scalar fields. **OC2**: verify the relationship between Differentiability and Continuity, directional derivative and continuity etc. **OC3**: check differentiability and continuity of scalar and vector fields. **OC4**: create counter examples related to continuity and differentiability, directional derivative and continuity, partial derivatives and total derivative etc.	
9	**Modules: -** **Practical for Calculus IV (30 Hours)**	

1.	Limits and continuity of scalar fields, using "definition and otherwise", iterated limits.
2.	Directional derivatives, partial derivatives and mean value theorem of scalar fields.
3.	Differentiability of scalar field and Total derivative.
4.	Gradient, level sets and tangent planes.

	5.	Chain rule, higher order partial derivatives and mixed partial derivatives of scalar fields.	
	6.	Maximum and minimum rate of change of scalar fields. Finding Hessian/Jacobian matrix.	
	7.	Taylor's Theorem.	
	8.	Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.	
	9.	Wronskian and linear independence of solutions.	
	10.	Higher order homogeneous linear differential equations with constant coefficients.	

10 Text Books

1. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.
2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.
3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.
4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.
5. D. Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa New Delhi, 1996.
6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.

11 Reference Books

1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-Wesley, 1998.
2. Howard Anton; Calculus- A new Horizon, (Sixth Edition); John Wiley and Sons Inc, 1999.
3. S L Gupta and Nisha Rani; Principles of Real Analysis; Vikas Publishing house PVT LTD.
4. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.
5. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

Scheme of the Examination

12	**Internal Continuous Assessment: 40%** **Semester End Examination: 60%**	**Semester End Examination: 60%**
13	**Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc. (at least 3)	

Sr. No.	Particulars	Marks
1	Objective question test	10
2	Overall performance	05
3	Viva	05

	Paper pattern of the Test (Offline Mode): Q1: (Attempt any 5 from 8) Multiple choice questions. (10 marks: 5 × 2) **Duration: 1Hrs** **While setting question paper four MCQ on module 1 and four MCQ on module 2 both.**	
14	**Format of Question Paper:**	

Scheme of examination:

At the end of the Semester IV, Practical examinations of three hours duration and 30 marks shall be conducted based on both the modules.

Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2

1. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Name of the Course: Linear Algebra (Minor II)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	The system of linear equations arises naturally in other science courses. This course provides a sound knowledge of Matrix theory, starting with the basic requirement of its study in the form of solving a system of equations. Also, it advances the discussion of vector space dealing with linear transformations, developing its connection with matrix theory.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course gives an introduction to linear transformations with an understanding of its geometry. Also, it covers a brief amount of matrix theory. CO1. To give sufficient knowledge of fundamental principles and methods, as well as a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving, and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem-solving skills, creative talent and power of communication are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Remember and understand the concept of a system of linear equations, matrices and linear transformations OC2: Calculate and find the solutions to a system of equations using various methods, finding the inverse of matrices OC3: Analyse the linear transformation and its various aspects OC4: Verify the Rank Nullity theorem OC5: Construct counterexamples related to linear transformations	
9	**Modules: -** **Module 1: Matrices (15 Hours)**	
	(a) Systems of homogeneous and non-homogeneous linear equations, Simple examples of finding solutions of such systems. Geometric and algebraic understanding of the solutions. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with a number of unknowns greater than the number of equations has infinitely many solutions. (b) Equivalence of statements (in which A denotes an $n \times n$ matrix) such	

as the following.(without proof)

(i) The system $A\boldsymbol{x} = \boldsymbol{b}$ of non-homogeneous linear equations has a unique solution.

(ii) The system $A\boldsymbol{x} = \boldsymbol{0}$ of homogeneous linear equations has no nontrivial solution.

(iii) A is invertible.

(iv) $\det A \neq 0$.

(v) $\text{rank}(A) = n$.

Examples.

(c) Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) Invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using the Gauss elimination method.

Module 2: Linear Transformation (15 Hours)

(a) Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \rightarrow W$; where V, W are vector spaces over R and V is a finite-dimensional vector space is completely determined by its action on an ordered basis of V.

(b) Null space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (without proof).

(c) Matrix associated with linear transformation of $V \rightarrow W$ where V and W are finite dimensional vector spaces over R. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.

(d) Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

10 Text Books:

1. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd edition, Pearson.
2. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition).
3. Serge Lang, Introduction to Linear Algebra, Springer.
4. S Kumaresan, Linear Algebra: A Geometric Approach, PHI Learning.
5. Sheldon Axler, Linear Algebra done right, Springer.
6. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.
7. David W. Lewis, Matrix theory.

11 Reference Books

1. Matrix Analysis and its Applications, Carl D. Mayor, SIAM publications.
2. Linear Algebra, Kenneth Hoffman & Ray Kunze, Prentice-Hall Inc.

Scheme of the Examination

The performance of the learners shall be evaluated in two parts.

- Internal Continuous Assessment of 20 marks.
- Semester-end examination of 30 marks.
- A separate head of passing is required for internal and semester-end examinations.

<table>
<tr><td>12</td><td>Internal Continuous Assessment: 40%</td><td>Semester End Examination: 60%</td></tr>
<tr><td>13</td><td>Continuous Evaluation through:

Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc.
(at least 3)

<table>
<tr><td>Sr. No.</td><td>Particulars</td><td>Marks</td></tr>
<tr><td>1</td><td>A class test of 10 marks is to be conducted during each semester in an Offline mode.</td><td>10</td></tr>
<tr><td>2</td><td>Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.</td><td>05</td></tr>
<tr><td>3</td><td>Seminar/group presentation on any one topic related to the syllabus.</td><td>05</td></tr>
</table>

Paper pattern of the test (offline mode with one-hour duration):
Q1: Definitions/Fill in the blanks/ True or False with Justification.
(04 Marks: 4 x 1).
Q2: Attempt any two of the three descriptive questions. (06 marks: 2 × 3)</td><td></td></tr>
<tr><td>14</td><td colspan="2">Format of Question Paper:
The semester-end examination will be of 30 marks of one-hour duration, covering the entire syllabus of the semester.

<table>
<tr><td colspan="4">Note: Attempt any TWO questions out of THREE.</td></tr>
<tr><td>Q.No.1</td><td>Module 1 and 2</td><td>Attempt any THREE out of FOUR.
(Each question of 5 marks)
(a) Question based on OC1
(b) Question based on OC2
(c) Question based on OC3
(d) Question based on OC4/OC5</td><td>15 Marks</td></tr>
<tr><td>Q.No.2</td><td>Module 1 and 2</td><td>Attempt any THREE out of FOUR.
(Each question of 5 marks)
(a) Question based on OC1
(b) Question based on OC2
(c) Question based on OC3
(d) Question based on OC4/OC5</td><td>15 Marks</td></tr>
<tr><td>Q.No.3</td><td>Module 1 and 2</td><td>Attempt any THREE out of FOUR.
(Each question of 5 marks)</td><td>15 Marks</td></tr>
</table></td></tr>
</table>

				(a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5		

Name of the Course: PM-4B Linear Algebra (Minor II)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	The system of linear equations arises naturally in other science courses. This course provides a sound knowledge of Matrix theory, starting with the basic requirement of its study in the form of solving a system of equations. Also, it advances the discussion of vector space dealing with linear transformations, developing its connection with matrix theory.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course deepens the understanding of system of linear equations with an understanding of the geometry of its solutions. Also, it covers a brief amount of matrix theory and the theory of linear transformations. CO1. To give sufficient knowledge of fundamental principles and methods, as well as a clear perception of numerous powers of mathematical ideas and tools and the skills to use them by modelling, solving, and interpreting. CO2. To reflect the broad nature of the subject and develop mathematical tools for continuing further study in various fields of sciences. CO3. To enhance students' overall development, problem-solving skills, creative talent, and the power of communication are necessary for various kinds of employment. CO4. To give adequate exposure to global and local concerns that would help learners explore many aspects of Mathematical Sciences.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Calculate solutions to a system of equations using various methods, and the inverse of matrices OC2: Analyse the null space and rank space related to the linear transformation. OC3: Verify the Rank Nullity theorem OC4: Construct and design counterexamples related to linear transformations	
9	**Modules: -** **Module 1: Practical for Matrices (30 Hours)**	

1.	System of linear equations with examples.
2.	The geometry of solutions of a system of linear equations.
3.	Matrix representation of a system of linear equations.
4.	Algebra of solutions of a system of homogenous linear equations.
5.	Elementary row operations on matrices.
6.	Determinants and rank of a matrix.
7.	Solving a system of linear equations using determinants.
8.	Elementary matrices and their relations with elementary operations on matrices.

9.	Invertibility of elementary matrices.
10.	Computing the inverse of a matrix by the Gauss elimination method.

Module 2: Practical for Linear Transformation (30 Hours)

1.	Linear transformations and their elementary properties.
2.	Composite of linear transformations.
3.	Determining linear transformation by knowing its action on basis vectors.
4.	Null space and image space of linear transformation.
5.	Rank and nullity of a linear transformation with verification of rank-nullity theorem.
6.	Computing matrix associated with a linear transformation.
7.	Matrix associated with a composite of two linear transformations.
8.	Effect on change of basis on linear transformation.
9.	Similar matrices.
10.	Equivalence of rank of a matrix with the associated linear transformation.

10 Text Books:

1. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd edition, Pearson.
2. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition.
3. Serge Lang, Introduction to Linear Algebra, Springer.
4. S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.
5. Sheldon Axler, Linear Algebra done right, Springer.
6. Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.
7. David W. Lewis, Matrix theory.

11 Reference Books

1. Matrix Analysis and its Applications, Carl D. Mayor, SIAM publications.
2. Linear Algebra, Kenneth Hoffman & Ray Kunze, Prentice-Hall Inc.

<u>Scheme of the Examination</u>

12	**Internal Continuous Assessment: 40%**	**Semester End Examination: 60%**
13	**Continuous Evaluation through:** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments etc. (at least 3)	

Sr. No.	Particulars	Marks
1	Objective question test	10
2	Overall performance	05
3	Viva	05

Paper pattern of the Test (Offline Mode):

Q1: (Attempt any 5 from 8) Multiple-

	choice questions. (10 marks: 5 × 2) **Duration: 1Hrs** **While setting the question paper, four MCQs on module 1 and four MCQs on module 2.**	
14	**Format of Question Paper:** **Scheme of examination:** At the end of Semester IV, Practical examinations of three hours duration and 30 marks shall be conducted based on both modules. Paper pattern: The question paper shall have two questions.	

Q. No. 1	Five out of Eight multiple-choice questions (four from module 1 and four from module 2) (OC1 to OC3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2
1. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Name of the Course: Basic Mathematics in Real Life-III (Minor IV)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	To underline the importance of concepts in mathematics that have physical interpretation, especially in other sciences like physics and chemistry.
2	**Vertical:**	Minor
3	**Type:**	Theory
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course emphasizes the concepts of mathematics applicable, especially in physics and chemistry. In this course, students from various science streams will be introduced to fundamental concepts from mathematics relevant to daily life and learn CO1: To identify eigenvalues and eigenvectors. CO2: To develop insight into the cross-product of vectors. CO3: To associate counting with real-life puzzles and games to make it easy to understand. CO4: recognizes symmetries in existing objects.	
8	**Course Outcomes (OC):** OC1: Remember and understand the concept of inner product spaces OC2: Calculate and find the inner products, moments, Eigenvalues Eigenvectors and orthogonal vectors OC3: Analyse the Cauchy-Schwarz inequality and the symmetries of triangles OC4: Verify the Cauchy-Schwarz inequality. OC5: Construct counterexamples related to inner product spaces	
9	**Modules: -** **Module 1: Linear dependence, independence and inner product spaces (15 Hours)**	
	1. Linear dependence and independence in two and three dimensions over reals, definitions, and simple examples. 2. Further examples of dependence and independence. 3. Linear dependence and independence of vectors and relation to determinants. 4. Cross products of vectors in R^3 basic properties like the angle between two vectors. 5. Cross product on R^3 and Jacobi identity; characterization of cross product is zero. 6. Use of cross product to calculate moment about a point: definition and formula 7. Moment of a force about a point: examples. 8. Problems involving angle bars and estimates on the magnitude of the moment about a point. 9. Inner product spaces definitions, examples, and properties. 10. Arithmetic mean—geometric mean inequality: statement, proof (for two numbers only) and applications (for two or more).	

	11. Cauchy-Schwarz inequality (real numbers) statement and proof, norm of a vector. 12. Problems based on Cauchy-Schwarz inequality, like finding the maximum possible value of a dot product. 13. Inner product spaces with complex coefficients. 14. Proof of Cauchy-Schwarz inequality for complex numbers. 15. Hermitian and unitary matrices and their examples.
	Module 2: Eigenvalues and eigenvector orthonormalization and symmetry (15 Hours)
	1. Definition of eigenvector and eigenvalue. 2. Examples of eigenvector and eigenvalue in 2 by 2 matrices. 3. Examples of eigenvectors and eigenvalues in 3 by three matrices. 4. Gram-Schmidt orthogonalization process: formula. 5. Gram-Schmidt orthogonalization process with examples. 6. Playing cards and counting permutations (ordered arrangements) 7. Further problems on cards. 8. Set game: introduction, counting: Calculation of total number of cards, calculation of the number of sets, calculation of cards with certain properties. 9. Permutations of an equilateral triangle. 10. Writing composition tables for symmetries (group) of equilateral triangles. 11. Permutation on four symbols. 12. Rule for composition of above permutations. 13. Symmetries of the square. 14. Writing a composition table for symmetries of a square. 15. Introduction to quaternions and their composition table.
10	**Text Books:** 1. Halliday and Resnick's Principles of Physics, Wiley, Eleventh Edition. 2. Hoffman and Kunze, Linear Algebra, Second Edition, Pearson.
11	**Reference Books:** 1. Shaeffer, R.E. *Elementary Structures for Architects and Builders.* 2. Elementary Linear Algebra Application Version, H. Anton, C. Rorres, Wiley & Sons.
	<u>Scheme of the Examination</u>
	The performance of the learners shall be evaluated in two parts. ● Internal Continuous Assessment of 20 marks. ● Semester-end examination of 30 marks. ● A separate head of passing is required for internal and semester-end examinations.
12	**Internal Continuous Assessment: 40%** | **Semester End Examination: 60%**

13	**Continuous Evaluation through** Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc. (at least 3)	

Sr. No.	Particulars	Marks
1	A class test of 10 marks is to be conducted during each semester in an Offline mode.	10
2	Project on any one topic related to the syllabus or a quiz (offline/online) on one of the modules.	05
3	Seminar/ group presentation on any one topic related to the syllabus.	05

Paper pattern of the test (offline mode with one-hour duration):

Q1: Definitions/Fill in the blanks/
True or False with Justification.
(04 Marks: 4 x 1).
Q2: Attempt two of 3 descriptive questions. (06 marks: 2 × 3)

14	**Format of Question Paper:** The semester-end examination will be of 30 marks for one hour, duration covering the entire syllabus of the semester.

Note: Attempt any TWO questions out of THREE.			
Q.No.1	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.2	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks
Q.No.3	Module 1 and 2	Attempt any **THREE** out of **FOUR**. (Each question of 5 marks) (a) Question based on OC1 (b) Question based on OC2 (c) Question based on OC3 (d) Question based on OC4/OC5	15 Marks

Name of the Course: PM-4C Basic Mathematics in Real Life-III (Minor IV)

Sr. No.	Heading	Particulars
1	**Description of the course: Including but not limited to:**	To underline the importance of concepts in mathematics that have physical interpretation, especially in other sciences like physics and chemistry.
2	**Vertical:**	Minor
3	**Type:**	Practical
4	**Credits:**	2 credits (1 credit = 15 Hours for Theory or 30 Hours of Practical work in a semester)
5	**Hours Allotted:**	30 Hours
6	**Marks Allotted:**	50 Marks
7	**Course Objectives (CO):** This course emphasizes the concepts of mathematics applicable, especially in physics and chemistry. In this course, students from various science streams will be introduced to fundamental concepts from mathematics relevant to daily life and learn CO1: To identify eigenvalues and eigenvectors. CO2: To develop insight into the cross-product of vectors. CO3: To associate counting with real-life puzzles and games to make it easy to understand. CO4: recognizes symmetries in existing objects.	
8	**Course Outcomes (OC):** After completion of the course, students will be able to OC1: Calculate and find the cross product, inner product, Norm, moments of force, Eigenvalues Eigenvectors and orthogonal vectors OC2: Analyse the Vector Spaces, Cauchy-Schwarz inequality and the symmetries of triangles OC3: Verify the Cauchy-Schwarz inequality and permutations as a function. OC4: Construct counterexamples related to inner product spaces.	
9	**Modules: -** **Module 1: Practical for linear dependence, independence and inner product spaces (30 Hours)**	

1.	Checking Linear dependence and independence.
2.	Definition of Linear Span of Vectors and Examples of Finding the Span.
3.	Calculations based on cross product.
4.	Examples based on a moment about a point.
5.	Further computations based on cross-product.
6.	Verification of inner product via examples for real vector spaces.
7.	Norms in inner product spaces, examples and properties.
8.	Definition of vector space over real and complex numbers, Examples.
9.	Inner product spaces over complex numbers, Definition and Examples.
10.	Eigenvalues and Eigenvectors with basic calculations.

Module 2: Practical for eigenvalues and eigenvector orthonormalisation and symmetry (30 Hours)

1.	Eigenvectors and Linear Independence.
2.	Orthogonalization Formula and Examples.
3.	Gram-Schmidt process with examples only of the rom real plane.
4.	Gram-Schmidt process with examples in three dimensions.
5.	Problems based on permutations and their composition, Examples.
6.	Problems based on the formula for permutations with possible constraints.
7.	Problems based on the set game.
8.	Composition of two permutations and further properties.
9.	Symmetries of rectangles and pentagons and other figures.
10.	Symmetries of letters of the alphabet of Indian languages and English.

<table>
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2. Hoffman and Kunze, Linear Algebra, Second Edition, Pearson.</td></tr>
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Quizzes, Class Tests, presentations, projects, role play, creative writing, assignments, etc.
(at least 3)

<table><tr><td>Sr. No.</td><td>Particulars</td><td>Marks</td></tr><tr><td>1</td><td>Objective question test</td><td>10</td></tr><tr><td>2</td><td>Overall performance</td><td>05</td></tr><tr><td>3</td><td>Viva</td><td>05</td></tr></table>
Paper pattern of the Test (Offline Mode):
Q1: (Attempt any 5 from 8) Multiple-choice questions. (10 marks: 5 × 2)

Duration: 1Hrs
While setting the question paper, four MCQs in module 1 and MCQs in module 2.</td><td></td></tr>
<tr><td>14</td><td colspan="2">Format of Question Paper:
Scheme of examination:

At the end of Semester IV, Practical examinations of three hours duration</td></tr>
</table>

	and 30 marks shall be conducted based on the modules. Paper pattern: The question paper shall have two questions.

Q. No. 1	Five out of Eight multiple-choice questions (four from module 1 and four from module 2) (OC1, OC2 and OC 3)	Marks (3 × 5 = 15 Marks)
Q. No.2	Attempt any Two out of Four (two from module 1 and two from module 2). (OC3 and OC4)	(5 × 2 = 10 Marks)

Marks for Journals:

For both Module 1 and Module 2
2. Journal: 5 marks (2.5 marks for each module 1 & module 2)

The students are required to perform 75% of the Practical for the journal to be duly certified. The students are required to present a duly certified journal for appearing at the practical examination, failing which they will not be allowed to appear for the examination.

Sd/-
Sign of the BOS Chairman
Prof. B.S. Desale.
BOS in Mathematics

Sd/-
Sign of the Offg. Associate Dean
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