

M.Sc (maths) Part - II (Sem. IV) 80.20 dt 10/01/25

[Time: 3:00 Hrs.]

[Marks : 80]

- N.B: 1. Please check whether you have got the right question paper.
 2. Figures to the right indicate full marks.
 3. Scientific calculator can be used.

- Q.1**
- a) State and Prove Parseval's Identity of periodic function. 10
 - b) Attempt any two of the following 10
 - i) Expand the Fourier series for the periodic function 5

$$f(x) = -\pi, -\pi < x < 0$$

$$= x, 0 < x < \pi$$
 - ii) Find the Fourier series of $f(x) = x^2$ in $(0, 2\pi)$. 5
 - iii) Find a cosine series of period 2π to represent $\sin x$ in $0 \leq x \leq \pi$. 5
- Q.2**
- a) State and prove Dirichlet Theorem. 10
 - b) Attempt any two of the following 10
 - i) $\frac{1}{2\pi} \int_{-\pi}^{\pi} D_N(\theta) d\theta = 1$ where, D_N N^{th} Dirichlet kernel. 5
 - ii) The N^{th} Dirichlet Kernel is given by $D_N(\theta) = \sum_{n=-N}^N e^{in\theta} = \frac{\sin(N+\frac{1}{2})\theta}{\sin\frac{1}{2}\theta}$ 5
 - iii) Suppose that f is periodic and integrable then n^{th} partial sum of Fourier series expansion of f is given by $S_N(f)(x) = (D_N * f)(x) = (f * D_N)(x)$. 5
- Q.3**
- a) Define Fejer's Cesaro's mean of Fourier series. The N^{th} is Fejer's kernel 10

$$\text{is given by } F_N(x) = \frac{1}{N} \frac{\sin^2(\frac{Nx}{2})}{\sin^2(\frac{x}{2})}.$$
 - b) Attempt any two of the following 10
 - i) Prove that $\frac{1}{2\pi} \int_{-\pi}^{\pi} P_r(\theta) d\theta = 1$ where $P_r(\theta)$ is the poisson kernel. 5
 - ii) The Abel Mean can be written as convolution of periodic integrable function f and the Poisson kernel $P_r(\theta)$ as $A_r f(\theta) = (f * P_r)(\theta)$ 5
 - iii) State and Prove Uniqueness of Fourier series. 5
- Q.4**
- a) State and Prove the Cauchy-schwarz inequality for Hilbert space. 10
 - b) Attempt any two of the following 10
 - i) Define Hilbert space. State and Prove Properties of Hilbert space. 5
 - ii) Prove Bessels's inequality for $L^2[-\pi, \pi]$ 5
 - iii) Explain Laplace operator and Harmonic functions. 5
