**Paper / Subject Code: 82136 / Mathematics. : Topology (Rev 2021)**

M.Sc(Maths) Part I (Sem-II) 80:20 dt 09/01/25

**Time: 3 Hours** **Max. Marks: 80**

**Note: 1) All Questions are compulsory**

**2) Figures to the right indicate full marks**

Q.1 (a) Define a topology on a set and a basis for a topology. Find a basis for each of the standard and discrete topologies on the set of real numbers. Show that the topology generated by a basis equals the collection of all unions of elements of basis. (10)

(b) Attempt any two of the following. (10)

(i) Define subspace topology. Let $Y$ be a subspace of $X$. Prove that a set is closed in the subspace $Y$ if and only if it equals the intersection of a closed set of $X$ with $Y$.

(ii) Define closure and limit points of a set. Let $A$ be a subset of the topological space $X$, $A'$ be the set of all limit points of $A$ and $\bar{A}$ be the closure of $A$, then show that $\bar{A} = A \cup A'$.

(iii) Define continuous function and homeomorphism between two topological spaces. Construct an example of a continuous function that is not a homeomorphism.

Q.2 (a) Define a connected space. If the sets $C$ and $D$ form a separation of $X$, then show that a connected space $Y$ of $X$ lies entirely within $C$ or $D$. Further, show that the union of a collection of connected spaces that have a point in common is connected. (10)

(b) Attempt any two of the following. (10)

(i) Show that a continuous image of a connected space is connected.

(ii) Define a path-connected space. Prove that every path connected space is connected. Is the converse true?

(iii) Define first countable and second countable spaces. Show that every second countable space is first countable. Give an example to show that a first-countable space need not be second-countable.

Q.3 (a) What is finite intersection property? Let $X$ be a topological space. Prove that $X$ is compact if and only if for every collection of closed sets $C$ in $X$ having the finite intersection property, the intersection $\bigcap_{c \in C} c$ of all the elements of $C$ is nonempty. (10)

(b) Attempt any two of the following. (10)

(i) Define a compact space. Let $\mathbb{R}$ be the set of real numbers with standard topology. Prove that $\mathbb{R}$ is not compact.

(ii) Prove that a bijective continuous map from a compact space to a Hausdorff space is a homeomorphism.

(iii) Prove that closed subspace of a compact space is compact.

Q.4 (a) Assume one-point sets are closed. Define a regular space. Prove that, (10)

- A regular space is Hausdorff. Give an example of a space that is Hausdorff but not regular.
- Is the set of real numbers with lower limit topology a regular space? Justify your answer.

(b) Attempt any two of the following. (10)

(i) Show that every compact Hausdorff space is normal.

(ii) State Urysohn Lemma. Give a direct proof of Urysohn's lemma for the metric spaces.

(iii) Define a metrizable space. Show that a metrizable space is normal.