Paper / Subject Code: 82142 / Mathematics. : Partial Differential Equations (Rev 2021)

[Time:3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Scientific calculator can be used.**

Q.1 a) If $\phi(x, y_1 u) = C_1$ *and* $\psi(x, y, u) = C_2$ are two first order integrals of the Ordinary Partial Differential Equations $\frac{dx}{p(x,y,u)} = \frac{dy}{q(x,y,u)} = \frac{du}{r(x,y,u)}$ and $\emptyset^2 + \psi^2 \neq 0$ then the general solution of the partial differential equation $p(x, y, u)u_x + q(x, y, u)u_y = r(x, y, u)$ is given by $f(\phi(x, y, u), \psi(x, y, u)) = 0$ **10**

b) Attempt **any Two** of the following: **10**

i) Solve the Partial Differential Equation $(y + z)z_x - (z + x)z_y = x - y$ **5**

ii) Find the general solution of the Non-Linear PDE $u_x + uu_y = 0$ **5**

iii) Solve the Initial Value Problem for the Partial Differential Equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ with the Initial Condition $u(x, 0) = e^{-x}$ **5**

Q.2 a) Reduce the Tricomi Equation, $u_{xx} + xu_{yy} = 0$ to its normal form. **10**

b) Attempt **any Two** of the following: **10**

i) Reduce the following Partial Differential Equation to Normal Form $u_{xx} + x^2u_{yy} = 0, x \in \mathbb{R}, x \neq 0$ **5**

ii) Given the function $u(x, y) = 3x^2 + 2y^2 - 12x - 8y + 10$ determine the points of Extremum. **5**

iii) Classify the following Partial differential Equation **5**

$x^2u_{xx} + 4u_{yy} + 2xyu_{xy} = 0$

Q.3 a) i) Define Green's Function **10**

ii) Prove: If $G(x, x_0)$ is a Green's Function then the solution to the Dirichlet Problem is given by the formula

$$u(x_0) = \iint_{\partial D} u(x) \frac{\partial G(x, x_0)}{\partial n} dS$$

b) Attempt **any Two** of the following: **10**

i) A vector field F is given by $F = \sin y i + x(1 + \cos y) j$. Evaluate the line integral $\int_C F \cdot dr$ where C is the circular path given by $x^2 + y^2 = a^2$. **5**

ii) Let C be the circle $x^2 + y^2 = 4$, oriented counterclockwise. Use Green's Theorem to evaluate $\oint_C (\cos x^2 - y^3)\, dx + x^3\, dy$. **5**

iii) Define Neumann Function and state its properties. **5**

Q.4 a) Derive the One-Dimensional Heat Equation **10**

b) Attempt **any Two** of the following: **10**

i) Solve $\frac{\partial u}{\partial x} = \frac{2\partial u}{\partial t} + u$ using the method of Separation of variables where $u(x, 0) = 6e^{-3x}$. **5**

ii) Solve using method of Separation of Variables $u_{xx} - 2u_x + u_y = 0$ **5**

iii) Give the One-Dimensional Wave equation also state the assumptions made in its derivation. **5**

****