Paper / Subject Code: 82133 / Mathematics. : Algebra II (Rev 2021)

M.Sc (Maths) Sem-II 07/01/2025

[Time: 3 Hours] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q.1 **a)** State and prove first isomorphism theorem for groups. **10**

b) Attempt **any Two** of the following: **10**

i) Prove that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if $\gcd(m, n) = 1$. **5**

ii) Let G be a group. When can we say H is a normal subgroup of G. Further prove that set of all left cosets is a group. **5**

iii) Let G_1, G_2 be two groups. Let $f: G_1 \to G_2$ be an onto homomorphism. Prove that (I) If G_1 is abelian then G_2 is abelian
(II) If G_1 is cyclic then G_2 is cyclic. **5**

Q.2 **a)** Define a group action. Further check that a map $(g, s) \to gsg^{-1}$, is it a group G acting on itself? **10**

b) Attempt **any Two** of the following: **10**

i) State the class equation. Hence prove that p −group has a nontrivial center. **5**

ii) Prove that every group of order 15 is cyclic. **5**

iii) State second Sylow theorem. Hence prove that unique p −sylow subgroups are always normal. **5**

Q.3 **a)** Prove that R/A is integral domain if and only if A is prime ideal of R. **10**

b) Attempt **any Two** of the following: **10**

i) Prove that characteristic of a field is either 0 or prime. **5**

ii) Prove that ideal $< x >$ is prime ideal but not maximal ideal in $\mathbb{Z}[x]$. **5**

iii) Define prime ideal of a ring R. Hence prove that $2\mathbb{Z}$ is prime ideal in $\mathbb{Z}[x]$. **5**

Q.4 **a)** If F is field, then prove that $F[x]$ is principal ideal domain. **10**

b) Attempt **any Two** of the following: **10**

i) Define irreducible element in a ring. Check whether $1+\sqrt{-3}$ is irreducible in $\mathbb{Z}[\sqrt{-3}]$. **5**

ii) Prove that $\mathbb{Z}[x]$ is not a PID. **5**

iii) Check the irreducibility of a polynomial $f(x) = 21x^3 - 7x^2 + 4x + 1$ over $\mathbb{Q}$. **5**
