[Time:3.00 Hrs] **[Marks:80]**

Please check whether you have got the right question paper.

N.B: **1. All questions are compulsory.**
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q.1 A) Derive the least square approximation using orthogonal polynomial 10

B) Attempt any Two of the following:

i) Find the least square polynomial approximation of degree two for the following data: 5

X	0	1	2	3	4
y	-4	-1	4	11	20

ii) Express the polynomial $1 + x - x^2 + x^3$ as a sum of Chebyshev polynomial. 5

iii) Compute the 4 points inverse DFT of sequences $\left(\frac{1}{2}, i - \frac{1}{2}, \frac{1}{2}, -i - \frac{1}{2}\right)$ 5

Q.2 A) Derive the general formula of Picard's method to solve differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$. Also solve the differential equation $\frac{dy}{dx} = xe^y$ with initial condition $y(0) = 0$ by Picard's method and find $y(0.2)$. 10

B) Attempt any Two of the following:

i) Solve the difference equation $y_{n+2} + y_{n+1} + y_n = n^2$ 5

ii) Given differential equation $\frac{dy}{dx} = 1 + y^2$ where $y(0) = 0$. Estimate $y(0.8)$ using the Milne Simpson predication correction method taking $h = 0.2$. 5

iii) Given that $\frac{dy}{dx} = log(x + y)$ with $y(1) = 2$ take $h = 0.2$ find $y(1.6)$ Using the Euler's Modified method. 5

Q.3 A) Derive the Boole's rule error formula in interval [a, b]. 10

B) Attempt any Two of the following:

i) Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Simpson's 1/3 rule taking $h = 0.25$. Compare the error with the exact value. 5

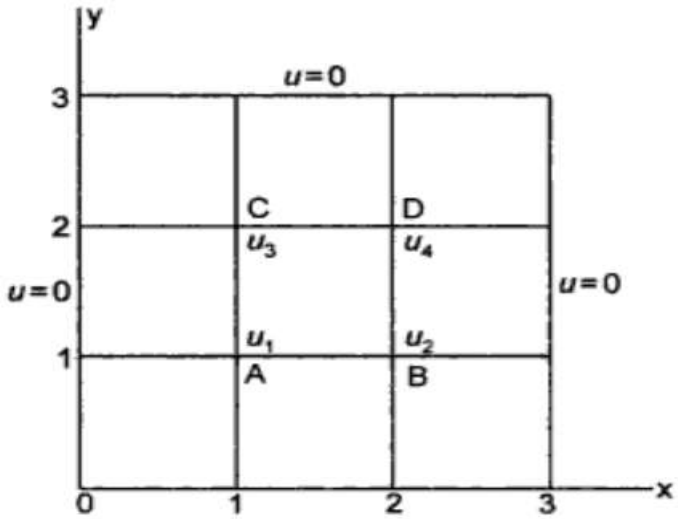
ii) Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(1+x)^4}\,dx$ by the Gaussian quadrature three-point formula. 5

iii) Using trapezoidal rule, evaluate $\int_1^2 \int_1^2 \frac{dx\,dy}{xy}$ taking four sub-intervals. 5

Q.4 A) What is Bender – Schmidt recurrence equation? Derive the Bender – Schmidt recurrence equation formula. 10

B) Attempt any Two of the following:

i) Solve the passion equation $u_{xx} + u_{yy} = -10(x^2 + y^2 + 10)$ in the domain of figure given below. 5



ii) Show that the equation $U_{xx} + 2xU_{xy} + (1 - y^2)U_{yy} = 0$ is elliptical for x and y in the region $x^2 + y^2 < 1$, outside for region. 5

iii) Using Crank-Nicholson's method solve $U_{xx} = 16U_t,\ 0 < x < 1, t > 0$ given $U(x,0) = 0, U(0,t) = 0\ U(1,t) = 50t$. Compute U for two steps in t-direction taking $h = \frac{1}{4}$. 5