Time: 2 ½ Hours **Marks: 80**

Note : Figures to the right indicate full marks.

Q1) A] State and Proof Separability of Hilbert Space. (10)

Q.1] B] Attempt any two. (10)

i) Let $\{e_j\}$ be a orthonormal set in Hilbert Space H. If x be any vector in Hilbert Space H than $x - \sum \langle x,e_i \rangle e_j \perp e_j$ for all j.

ii) State and prove Baire Theorem for general space.

iii) Define a sequence of $\{f_n\}$ on R by $f_n(x)= \sin(n + x)$. Proof that the family $\{f_n/n \text{ in } N\}$ is equicontinous.

Q.2] A] Let l^∞ be consists all bounded sequence in K. Show that l^∞ is vector space over K with respect to addition and scalar multiplication for x in l^∞ $||x||=\sup\{|x_n| : n \text{ in } N\}$. Show that $||\cdot||_{l^\infty}$ is norm space. (10)

Q.2] B] Attempt any two. (10)

i) Show that $L^p(x)$ is vector space over K and $||\cdot||_{L^P}$ is norm function on $L^p(x)$.

ii) Show that $(B, ||\cdot||_{sup})$ is normed linear space under $||f||= \sup\{ |f(x)|, x \text{ in } X\}$.

iii) Let N be a non –zero normed linear space and prove that N is banach space $\leftrightarrow$ $\{ x : ||x||=1\}$ is complete .

Q.3] A] State and prove Riezs –Representation theorem for Hilbert space. (10)

Q.3] B] Attempt any two. (10)

i) Let X be an n-dimensional vector space and $E =\{ e_1,e_2,\ldots,e_n\}$ is basis for X ,then
$F = \{ f_1,f_2,\ldots,f_3\}$ $f_k(e_j)=\partial_{jk} =0$, if $j\neq k = 1$, if j=k
Is basis for algebraic dual X* of X and dim X*=dim x =n .

ii) Show that l^∞ is not separable.

iii) Prove that $|\, ||x|| - ||y||\, | \leq ||x-y||$

Q.4] A] A bounded linear operator T from a banach space X onto a banach space Y has the property that the image $T(B_0)$ of the open unit ball $B_0=B(0,1) \subset X$ contain an open ball about 0 belong to Y .

Q.4] B] Attempt any two. (10)

i) Let X be norm space and let $x_0 \neq 0$ be any element of X then their exits a bounded linear functional $\bar{f}$ on X such that $\|\bar{f}\| = 1, \bar{f}(x_0) = \|x_0\|$.

ii) Let B be banach space and N a normal linear space .If $\{T_1\}$ is non –empty set of continuous linear transformation of of B in N_1 with property that $\{T_i(x)\}$ is bounded subset of N for each x in B $\{\|T_i\|\}$ is bounded set of number.

iii) State and prove closed graph theorem.

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