[Time:3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. The figures to the right indicate full marks.
3. A scientific calculator can be used.

Q.1 **a)** Let $f_n: X \subset \mathbb{C} \to \mathbb{C}$ be a function on X such that $|f_n\ x| \leq M_n$ for every $z \in X$ and suppose that $\sum_{n=1}^{\infty} M_n$ is convergent series of real numbers then prove that $\sum_{n=1}^{\infty} f_n$ is uniformly convergent **10**

b) Attempt **any Two** of the following: **10**

i) Prove that log z is not continuous on the negative real axis. **5**

ii) Find the domain of the region of convergence of the following power series $\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot \ldots \cdot (2n-1)}{n!}\left(\frac{(1-z)}{z}\right)^n$. **5**

iii) Given a series $\sum_{n=1}^{\infty} z^n(1-z)$. Prove that the series uniformly converges to the sum z for $|z| \leq \frac{1}{2}$ **5**

Q.2 **a)** Let f be analytic in a simply connected domain G, $G \subset \mathbb{C}$. If x is a closed rectifiable curve in G and $\alpha \in G / \{x\}$, then $\frac{1}{2\pi i}\int_x \frac{f(z)}{z-\alpha} dz = f(\alpha)\eta(x;\alpha)$. **10**

b) Attempt **any Two** of the following: **10**

i) If O is the origin, L is the point $z = 3$, M is the point $z = 3 + i$, evaluate $\int z^2 dz$ along the path OLM. **5**

ii) Prove that the function $f(z) = ze^z - z$ has a zero of order 2 at the origin. **5**

iii) Give the power series expansion for $f(z) = sin\ z$ around z = $\frac{\pi}{4}$. **5**

Q.3 **a)** State and prove Liouville's theorem. **10**

b) Attempt **any Two** of the following: **10**

i) Let $f(z) = e^z$ and $T = \overline{B}(2+3i, 1)$. Find a point in T at which $|f|$ attains its maximum value. **5**

ii) Define the following: i)Singularity ii) Essential singularities iii) Removable singularities. **5**

iii) Find Laurent Series expansions of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$. **5**

Q.4 **a)** State and prove Rouche's Theorem. **10**

b) Attempt **any Two** of the following: **10**

i) Using contour integration evaluate $\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx$. **5**

ii) Prove that the circle $|z-2| = 3$ is mapped onto a circle $\left|w + \frac{2}{5}\right| = \frac{9}{25}$ under the transformation $w = \frac{1}{z}$. **5**

iii) Use the Argument principle to evaluate $\int_x \frac{f'(z)}{f(z)}$ where $f(z) = \frac{z-1}{z^2(z-2)(z-3)}$ and x is the circle $|z| = \pi$ **5**

****