[Time:3.00 Hrs] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Scientific calculator can be used.**

Q.1 A) State and prove that division algorithm in $\mathbb{Z}$. 10

B) Attempt any Two of the following:

i) If x is an odd integer, not divisible by 3, prove that $x^2 \equiv 1\ (mod\ 24)$. 5

ii) A building has 4 floors excluding the ground floor. 13 people get into the lift at the ground floor. Assuming that no body get into the lift from the other floors and that at least one person get down at each floor, find the number of different possibilities so that the lift is emptied on the top floor. 5

iii) The sum of two positive integers is 100, if one divided by 7 the remainder is 1 and if the other is divided by 9 the remainder is 7. Find the number using the Diophantine equation. 5

Q.2 A) Define Derangement of finite objects. Let D_n denote the number of derangements of n objects. Show that $D_n = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots \ldots \ldots \ldots . + (-1)^n \frac{1}{n!}\right)$. Also find number of derangement on 4 symbol. 10

B) Attempt any Two of the following:

i) Prove that in any group of 6 people, there are 3 mutual friends or 3 mutual stangers. 5

ii) Prove that $S(n,2) = 2^{(n-1)} - 1$. 5

iii) In a class of 150 students, 70 have offered Mathematics, 80 have offered Physics and 90 have offered Chemistry. Of these, 40 students are Mathematics and Physics, 30 are for Mathematics and Chemistry and 50 are for Physics and Chemistry. If 20 students have neither of these subjects. Find the number of students have i) only Mathematics, ii) all three subjects. 5

Q.3 A) Prove that "If the characteristic roots $x_1, x_2, \dots \dots, x_r$ of an linear homogeneous recurrence relation with constant coefficients are distinct, then every solution of the linear homogeneous recurrence relation with constant coefficients is a linear combination of $x_1{}^n, x_2{}^n, \dots \dots, x_r{}^n$. Moreover, the solution is unique if *r* consecutive initial conditions are given." 10

B) Attempt any Two of the following:

i) Find a recurrence relation and solve it for the following sequence. 5
0, 2, 6, 12, 20, 30, 42,……

ii) Find the number of ways to collect 15 rupees from 20 distinct people if each of the 19 people can give a rupee (or nothing) and the twentieth person can give a 1-rupee or 5-rupees (or nothing). 5

iii) Show that $f_{m+n} = f_{m-1}f_n + f_m f_{n+1}$. 5

Q.4 A) Prove that "Let G be a finite group acting on a set X. Then $|Stab\ (x)| = \frac{|G|}{|Orb\ (x)|}$." 10

B) Attempt any Two of the following:

i) Prove that "Orbits under the action of the group *G* partition the set *X*" 5

ii) Consider an $n \times n$ chessboard, $n \geq 2$, where every square is either colored by blue (B) or red (R). How many different colourings are there, if different means that one cannot obtain a colouring from another by either a rotation or a reflection? 5

iii) A stick is painted with equal sized cylindrical bands. Each band can be painted black or white. If the stick is un-oriented as when spun in the air, how many different 2-coloring of the stick are possible if the stick has i) 2 band? ii) 3 bands? 5

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