M.Sc. (Maths) (Sem-II) July-2023

Probability Theory (Rev 2021)

[Time: 3:00 Hrs.] **[Marks : 80]**

N.B: 1. Please check whether you have got the right question paper.
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q.1	a)	Define Probability. State and prove the below elementary theorems: (i) Probability of Complimentary events P (A^C) = 1- p (A). (ii) For any events A and B ; P($A^C \cap$ B) = p(B) - P(A ∩ B)	10
	b)	Attempt any two of the following	10
	i)	What is the probability that in a random arrangement of alphabets of word "REGULATIONS" (i) All vowels are together? (ii)No two vowels are together?	5
	ii)	If P(A) = 1/5, P(B) = 2/3, P(A ∩ B) = 1/15 Find (i) P(A ∪ B) (ii) P($A^C \cap$ B) (iii) P(A $\cap$ B^C), (iv) P($A^C \cap B^C$) (v) P($A^C \cup B^C$)	5
	iii)	Three light bulbs are chosen at random from 12 bulbs of which 5 are defective. Find the probability that (i) All the defective (ii) One is defective (iii) two is defective.	5
Q.2	a)	State and Prove Bayes Theorem In a bolt factory machines A, B, C manufacture 20%, 30%, 50% of the total of their output 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. What is the probability that is manufactured by machines A, B, C?	10
	b)	Attempt any two of the following	10
	i)	Define conditional probability. State multiplication theorem on probability for two events.	5
	ii)	An urn contains 5 white and 7 black balls.3 balls are drawn in succession. What is the probability that all are white'? If ball are drawn (i) with replacement (ii) Without replacement.	5
	iii)	Three light bulbs are chosen at random from 12 bulbs of which 5 are defective. Find the probability that (i) All the defective (ii) One is defective (iii) two is defective.	5
Q.3	a)	The Probability that a man hitting a target is 1/3. (a) If he fires 6 times, what is the probability of hitting (i) at the most 5 times (ii) at least 5 times (iii) exactly once If he fires so that the probability of his hitting the target at least once is greater than ¾, find n? By binomial distribution method Function?	10

b) Attempt any two of the following 10

i) Let X be a discrete random variable with the following probability distribution. Find (i) P(X<1) (ii) p(x ≥ 3) (iii) p(1<x<4) (iv) p(2≤x≤3) 5

X	0	1	2	3	3
P(X=x)	0.1	0.2	0.3	0.15	0.25

ii) Explain about Normal distribution. If X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that (i) $26 \leq x \leq 40$ (ii) $x \geq 45$. 5

iii) Explain Continuous random variable and its probability density function. 5

Q.4 a) State and Prove weak law of large numbers. 10

b) Attempt any two of the following 10

i) Given that the mean heights of students in a class is 158 cms with standard deviation of 20 cms. Find how many students heights lie between 150 cms and 170 cms if there are 100 students in the class. 5

ii) Out of 800 families with 5 children each how many would you expect to have (i) 3 boys (ii) either 2 or 3 boys 5

iii) State and Prove Cauchy Schwarz's inequality for any two r.v.s X ,Y, $E[XY]^2 \leq E[X^2] E[Y^2]$. 5

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