

**N.B: 1. Please check whether you have got the right question paper.**

**2. Figures to the right indicate full marks.**

**3. Scientific calculator can be used.**

- Q.1 a) State and Prove Riemann-Lebesgue lemma 10  
 b) Attempt any two of the following 10  
 i) Expand the Fourier series for the periodic function 5  

$$f(x) = -\pi, -\pi < x < 0$$

$$= x, 0 < x < \pi$$
  
 ii) Find the Fourier expansion of  $f(x) = x^2, -\pi \leq x \leq \pi$ . Check the limit and decide it is even or odd function. 5  
 iii) Find a cosine series of period  $2\pi$  to represent  $\sin x$  in  $0 \leq x \leq \pi$  5
- Q.2 a)  $\int_{-\pi}^{\pi} |D_N| dx \geq c \log N$  as  $N \rightarrow \infty$  where  $C$  is any constant and  $D_N(x)$  is  $N^{\text{th}}$  Dirichlet Kernel. 10  
 b) Attempt any two of the following 10  
 i) If  $f$  is  $2\pi$  period and piece wise smooth on  $\mathbb{R}$  then show that 5  

$$\lim_{N \rightarrow \infty} S_N f(\theta) = \frac{1}{2} [f(\theta^-) + f(\theta^+)].$$
 for every  $\theta$  where  $f$  is continuous.  
 ii) The  $N^{\text{th}}$  Dirichlet Kernel is given by  $D_N(\theta) = \sum_{n=-N}^N e^{in\theta} = \frac{\sin(N+\frac{1}{2})\theta}{\sin\frac{1}{2}\theta}$  5  
 iii) Suppose that  $f$  is periodic and integrable then  $n^{\text{th}}$  partial sum of Fourier series expansion of  $f$  is given by  $S_N(f)(x) = (D_N * f)(x) = (f * D_N)(x)$  5
- Q.3 a) Define Fejer's Cesaro's mean of Fourier series. The  $N^{\text{th}}$  is Fejer's kernel 10  
 is given by  $F_N(x) = \frac{1}{N} \frac{\sin^2(\frac{Nx}{2})}{\sin^2(\frac{x}{2})}$ .  
 b) Attempt any two of the following 10  
 i) For  $\delta > 0$ ,  $\int_{\delta \leq |\theta| \leq \pi} |P_r(\theta)| d\theta \rightarrow 0$  as  $r \rightarrow 1$  5  
 ii) The Abel Mean can be written as convolution of periodic integrable function  $f$  and the Poisson kernel  $P_r(\theta)$  as  $A_r f(\theta) = (f * P_r)(\theta)$  5  
 iii) The Poisson kernel  $P_r(\theta) \geq 0$ . 5
- Q.4 a) State and Prove solution of DIRICHLET problem for UNIT DISC 10  
 b) Attempt any two of the following 10  
 i) Define Hilbert space. State and Prove Properties of Hilbert space. 5  
 ii) State and prove Pythagorean theorem. 5  
 iii) Explain Laplace operator and Harmonic functions. 5

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