M.Sc. (Maths) (Sem-II) July-2023

Mathematics. : Partial Differential Equations (Rev 2021)

[Time: 3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. **All questions are compulsory.**
2. **Figures to the right indicate full marks.**
3. **Scientific calculator can be used.**

Q.1 a) If $\phi(x,y,u) = c_1,\ \varphi(x,y,u) = c_2$ be two independent first integrals of $\frac{dy}{dx} = \frac{b(x,y)}{a(x,y)}$ and $\phi_u^2 + \varphi_u^2 \neq 0$ then prove that general solution of $\frac{dy}{dx} = \frac{b(x,y)}{a(x,y)}$ is given by $h(\phi(x,y,u),\ \varphi(x,y,u) = 0)$ **10**

b) Attempt **any Two** of the following: **10**

i) Find the characteristic equation of the following PDE $2xy\, u_x - (x^2 + y^2)\, u_y = 0$ **5**

ii) find the general solution of $\frac{y^2 u}{x} u_x + xu\, u_y = y^2$ **5**

iii) find the solution of the Cauchy problem. **5**

$(y + 2ux)u_x - (x + 2uy)\, u_y = \frac{1}{2}(x^2 - y^2)$ at $u = 0, x - y = 0$

Q.2 a) State and prove Poisson's integral formula in 2D. **10**

b) Attempt **any Two** of the following: **10**

i) Check whether the equation $u_{xx} + u_{yy} = u_{zz}$is hyperbola, parabola, or ellipse. **5**

ii) Write a canonical/normal form of $\frac{\partial^2 z}{\partial^2 x} - \frac{\partial^2 z}{\partial^2 y} = 0$ **5**

iii) Find the characteristics of $y^2 r - x^2 t = 0$ **5**

Q.3 **a)** Let $f(x+iy) = u + iv$ represent the mapping of $D + \partial D$ onto the unit circle in u,v place where $f(x+iy)$ is a simple analytic function of the complex variable $x + iy$, then the Green function for D is given by, **10**

$G(a_1, a_2; x, y) = -\frac{1}{2\pi}\, Re \log\left[\frac{f(a_1+ia_2)-f(x+iy)}{\overline{f(a_1+\iota a_2)}f(x+iy)-1}\right]$

b) Attempt **any Two** of the following: **10**

i) Prove that the BVP $\Delta_m u = d(x)\ in\ D,\ \ and\ u = f(x) on\ \partial D.\ d(x) \in C^0\ in\ D + \partial D\ \ f(x) \in C^0\ on\ \partial D$, has at most one solution $u(x) \in C^0\ in\ D + \partial D\ and \in C^2\ in\ D.$ **5**

ii) Consider a sphere with center at origin and radius 'a' apply the divergence theorem to the sphere and show that $\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(r)$. where $\delta(r)$ is a Dirac delta function. **5**

iii) Find the solution of BVP $\Delta_2 u = f, with\ boundary\ conditions\ u = h\ on\ \partial D.$ **5**

Q.4 **a)** If $u(x,t)$ satisfy the diffusion equation, in the strip $0 < t \le c$, and if $\lim(x,t) \to (x^0, 0)$ $u(x,t) = 0\ for\ all\ x^0$ and if $|u(x,t)| \le Me^{Ax^2}, then\ u(x,t)$ is identically zero in the strip. **10**

b) Attempt **any Two** of the following: **10**

i) Solve the BVP $4u_{tt} = u_{xx}, u(x,0) = 0\ \ u_t(x,0) = 8\sin 2x\ , (x,t) \in R \times (0,\infty).$ **5**

ii) Solve the BVP $u_{xx} = \frac{1}{k}u_t$ satisfying the conditions, $u(0,t) = 0 = u(l,t)$ **5**

iii) Find the general solution for the heat equation $u_{xx} = u_t,\ \ u(x,0) = \sin\left(\frac{\pi x}{l}\right) + 3\sin\left(\frac{5\pi x}{l}\right)$ **5**

$u(0,t) = 0 = u(l,t)$.

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