M.Sc.(Maths) (Sem-II) July-2023

Mathematics : Algebra II (Rev 2021))

[Time 3:00Hrs] **[Marks: 80]**

Please Check whether you have got the right question paper

N.B 1. All questions are compulsory
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q-1	**a)**	If a group G is the internal direct product of subgroups $H_1, \cdots H_n$ then prove that $G = H_1 \oplus H_2 \; ... \oplus H_n$	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Prove that A_n is a normal subgroup of S_n. Prove that $O(A_n) = \frac{1}{2}\, O(S_n)$.	**5**
	ii)	List down 4 non-isomorphic abelian groups of order 360.	**5**
	iii)	Let $\phi: \mu(30) \rightarrow \; \mu(30)$be a group homomorphism and $\ker(\,\phi) = \{\overline{1}, \overline{11}\,\}$. If $\phi(7) = 7,$ then find all elements of $\mu(30)$ that are mapped to $\overline{7}$.	**5**
Q-2	**a)**	Let G be a group acting on a set S. Then, the relation $\backsim$ on S defined by for $a, b \in S, a \backsim b \; iff \; a = gb$ for some $g \in G$ is an equivalence relation. Prove that for each $a \; \in S$, the number of elements in the equivalence containing a is $[G: G_a]$, the index of the stabilizer of a.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Prove that for a prime $'p'$, a group of order p^2 is abelian.	**5**
	ii)	Show that a Sylow -p- subgroup of a group which is normal is unique.	**5**
	iii)	Let $\lvert G \rvert = \mathbb{Z}_p$ where p is an odd prime. Then prove that $G \approx \mathbb{Z}_{\{2p\}}$ or $G = D_p$	**5**
Q-3	**a)**	Prove that in a ring every Maximal ideal is prime ideal.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Let R=Z[i] and $I = < 2 - i >$. Find the number of elements in $\frac{R}{I}$.	**5**
	ii)	Let R be commutative ring of prime characteristic P. Define $\phi: R \rightarrow R$ as $\phi(x) = x^p$. Show that ϕ is ring homomorphism.	**5**
	iii)	Prove that every field contains Z_p or Q.	**5**
Q-4	**a)**	Prove that in a Principal ideal domain every irreducible is a prime.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Let R Principal ideal domain. Then for any $a, b \in R$, greatest common divisor of a and b exists.	**5**
	ii)	Prove that the product of two primitive polynomials is primitive.	**5**
	iii)	Prove that $\mathbb{Z}[x]$ is not a PID.	**5**

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