UNIVERSITY OF MUMBAI

Syllabus

for M. Sc./ M. A. Semester I & II (CBCS)
Program: M. Sc./ M. A.

Course: Mathematics

with effect from the academic year 2020-2021

Syllabus M.Sc./M.A. Part I (Sem. I & II)

Choice Based Credit System (CBCS)

Sr. No.	Subject code	Units	Subject	Credits	L/W
Semester I					
Algebra I					
01	PSMT/PAMT 101	Unit I	Dual Spaces	05	04
		Unit II	Determinants and Characteristics Polynomial		
		Unit III	Triangulation of Matrices		
		Unit IV	Bilinear Forms		
Analysis I					
02	PSMT/PAMT 102	Unit I	Euclidean Space	05	04
		Unit II	Differentiable Functions		
		Unit III	Inverse Function Theorem and Implicit Function Theorem		
		Unit IV	Riemann Integration		
Complex Analysis					
03	PSMT/PAMT 103	Unit I	Holomorphic Functions	05	04
		Unit II	Contour Integration and Cauchy-Goursat theorem		
		Unit III	Holomorphic Functions and Their Properties		
		Unit IV	Residue Calculus and Mobius Transformation		
Ordinary Differential Equations					
04	PSMT/PAMT 104	Unit I	Existence and Uniquness of Solutions	05	04
		Unit II	Linear Equations with Constant Coefficients		
		Unit III	Linear Equations with		

			Variable Coefficients		
		Unit IV	Strum_Liouville Problem and Qualitative Properties of Solutions		
Discrete Mathematics					
05	PSMT/PAMT 105	Unit I	Number Theory	04	04
		Unit II	Advanced Counting		
		Unit III	Recurrence Relations		
		Unit IV	Polyas Theory of Counting		
Semester II					
Algebra II					
01	PSMT/PAMT 201	Unit I	Groups and Group Homomorphisms	05	04
		Unit II	Group Acting on Sets and Sylow Theorems		
		Unit III	Rings and Fields		
		Unit IV	Divisibility in Integral Domains		
Topology					
02	PSMT/PAMT 202	Unit I	Topology and Topological Spaces	05	04
		Unit II	Connected Topological Spaces		
		Unit III	Compact Topological Spaces		
		Unit IV	Metrizable Spaces and Tychonoff Theorem		
Analysis II					
03	PSMT/PAMT 203	Unit I	Measures and Measurable Sets	05	04
		Unit II	Measurable functions and their Integration		
		Unit III	Convergence Theorems on Measure space		
		Unit IV	Space of Integrable functions		

Partial Differential Equations					
04	PSMT/PAMT 204	Unit I	First Order Partial Differential Equations	05	04
		Unit II	Second Order Partial Differential Equations		
		Unit III	Green’s Functions and Integral Representations		
		Unit IV	The Diffusion Equation and Parabolic Differential Equations		
Probability Theory					
05	PSMT/PAMT 205	Unit I	Probability Basics	04	04
		Unit II	Probability Measure		
		Unit III	Random Variables		
		Unit IV	Limit Theorems		

Teaching Pattern for Semester I and II
1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

Semester-I

PSMT101 /PAMT101 ALGEBRA I

Course Outcomes:

1. Students will be able to understand the notion of dual space and double dual, Annihilator of a subspace and its application to counting the dimension of a finite dimensional vector space, Basics of determinants, applications to solving system of equations, Nilpotent operators, invariant subspaces and its applications, Bilinear forms and spectral theorem with examples of spectral resolution and Symmetric bilinear form and Sylvesters law.

2. Students will be able to understand the applicability of the above concepts in different courses of pure and applied mathematics and hence in other disciplins of science and technology.

Unit I. Dual spaces (15 Lectures)
Para 1 and 2 of Unit I are to be reviewed without proof (no question be asked).

1. Vector spaces over a field, linear independence, basis for finite dimensional and infinite dimensional vector spaces and dimension.

2. Kernel and image, rank and nullity of a linear transformation, rank-nullity theorem (for finite dimensional vector spaces), relationship of linear transformations with matrices, invertible linear transformations. The following are equivalent for a linear map $T : V \to V$ of a finite dimensional vector space V :

   (a) T is an isomorphism.
   (b) $\ker T = \{0\}$.
   (c) $Im(T) = V$.

3. Linear functionals, dual spaces of a vector space, dual basis (for finite dimensional vector spaces), annihilator W^o in the dual space V^* of a subspace W of a vector space V and dimension formula, a k-dimensional subspace of an n-dimensional vector space is intersection of $n-k$ many hyperspaces. Double dual V^{**} of a Vector space V and canonical embedding of V into V^{**}. V^{**} is isomorphic to V when V is of finite dimension. (ref:[1] Hoffman K. and Kunze R.).

4. Transpose T^t of a linear transformation T. For finite dimensional vector spaces: rank (T^t) =rank T, range(T^t) is the annihilator of kernel (T), matrix representing T^t. (ref:[1] Hoffman K and Kunze R)

Unit II. Determinants & Characteristic Polynomial (15 Lectures)
Rank of a matrix. Matrix of a linear transformation, change of basis, similar matrices. Determinants as alternating n-forms, existence and uniqueness, Laplace expansion of determinant, determinants of products and transposes, adjoint of a matrics. determinants and invertible linear transformations, determinant of a linear transformation. Solution of system of linear equations using Cramer's rule. Eigen values and Eigen vectors of a linear transformation, Annihilating polynomial, Characteristic polynomial, minimal polynomial, Cayley-Hamilton theorem. (Reference for Unit II: [1] Hoffman K and Kunze R, Linear Algebra).

Unit III. Triangulation of matrices (15 Lectures)
Triangulable and diagonalizable linear operators, invariant subspaces and simple matrix representation (for finite dimension). (ref: [5] N.S. Gopalkrishnan & [3] Serge Lang) Nilpotent linear transformations on finite dimensional vector spaces, index of a Nilpotent linear transformation. Linear independence of $\{u, N, \ldots, N^{k-1}u\}$ where N is a nilpotent linear transformation of index $k \geq 2$ of a vector space V and $u \in V$ with $Nu \neq 0$. (Ref: [2] I.N.Herstein).

For a nilpotent linear transformation N of a finite dimensional vector space V and for any subspace W of V which is invariant under N, there exists a subspace V_1 of V such that $V = W \oplus V_1$. (Ref:[2] I.N.Herstein).

Computations of Minimum polynomials and Jordan Canonical Forms for 3×3-matrices through examples. (Ref:[6] Morris W. Hirsch and Stephen Smale).

Unit IV. Bilinear forms (15 Lectures)
Para 1 of Unit IV is to be reviewed without proof (no question be asked).

1. Inner product spaces, orthonormal basis, Gram-Schmidt process.

2. Adjoint of a linear operator on an inner product space, unitary operators, self adjoint operators, normal operators. (ref:[1] Hoffman K and Kunze R). Spectral theorem for a normal operator on a finite dimensional complex inner product space. (ref:[4] Michael Artin, Ch. 8). Spectral resolution (examples only). (ref:[1] Hoffman K and Kunze R, sec 9.5).

3. Bilinear form, rank of a bilinear form, non-degenerate bilinear form and equivalent statements. (ref:[1] Hoffman K and Kunze R).

4. Symmetric bilinear forms, orthogonal basis and Sylvester's Law, signature of a Symmetric bilinear form. (ref:[4] Michael Artin).

Recommended Text Books

1. Hoffman K and Kunze R: Linear Algebra, Prentice-Hall India.

2. I.N.Herstein: Topics in Algebra, Wiley-India.

3. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.

4. Michael Artin: Algebra, Prentice-Hall India.

5. N.S. Gopalkrishnan: University Algebra, New Age International, third edition, 2015.

6. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

PSMT102 / PAMT102 ANALYSIS I

Course Outcomes

1. This course is the foundation course of mathematics, especially mathematical analysis.

2. Student will be able to grasp approximation of a differentiable function localized at a point.

3. Inverse function theorem helps to achieve homeomorphism locally at a point whereas implicit function theorem justifies the graph of certain functions. Indirectly or directly Unit III talks about value of a function in the neighbourhood of a known element.

4. In Unit IV, student will be able to understand the concept of Riemann integration.

Unit I. Euclidean space $\mathbb{R}^n$ (15 Lectures)
Euclidean space $\mathbb{R}^n$: inner product $\langle x, y\rangle = \sum_{j=1}^{n} x_j y_j$ of $x = (x_1, .., x_n), y = (y_1, .., y_n) \in R^n$ and properties, norm $||x|| = \sqrt{\sum_{j=1}^{n} x_j^2}$ of $x = (x_1, .., x_n) \in R^n$, Cauchy-Schwarz inequality, properties of the norm function $||x||$ on $\mathbb{R}^n$. (Ref. W. Rudin or M. Spivak).

Standard topology on $\mathbb{R}^n$: open subsets of $\mathbb{R}^n$, closed subsets of $\mathbb{R}^n$, interior A^o and boundary ∂A of a subset A of $\mathbb{R}^n$. (ref. M. Spivak)

Operator norm $||T||$ of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ ($||T|| = sup\{||T(v)|| : v \in \mathbb{R}^n \& ||v|| \leq 1\}$) and its properties such as: For all linear maps $S, T : \mathbb{R}^n \to \mathbb{R}^m$ and $R : \mathbb{R}^m \to \mathbb{R}^k$

1. $||S + T|| \leq ||S|| + ||T||$,

2. $||RoS|| \leq ||R||||S||$,and

3. $||cT|| = |c|||T||(c \in \mathbb{R})$.

(Ref. C. C. Pugh or A. Browder)

Compactness: Open cover of a subset of $\mathbb{R}^n$, Compact subsets of $\mathbb{R}^n$ (A subset K of $\mathbb{R}^n$ is compact if every open cover of K contains a finite subover), Heine-Borel theorem (statement only), the Cartesian product of two compact subsets of $\mathbb{R}^n$ is compact (statement only),every closed and bounded subset of $\mathbb{R}^n$ is compact. Bolzano-Weierstrass theorem: Any bounded sequence in $\mathbb{R}^n$ has a converging subsequence.

Brief review of following three topics:

1. Functions and Continuity Notation: $A \subset \mathbb{R}^n$ arbitary non-empty set. A function $f : A \to \mathbb{R}^m$ and its component functions, continuity of a function($\in, \delta$ definition). A function $f : A \to \mathbb{R}^m$ is continuous if and only if for every open subset $V \subset \mathbb{R}^m$ there is an open subset U of $\mathbb{R}^n$ such that $f^{-1}(V) = A \cap U$.

2. Continuity and compactness:Let $K \subset \mathbb{R}^n$ be a compact subset and $f : K \to \mathbb{R}^m$ be any continuous function. Then f is uniformly continuous, and $f(K)$ is a compact subset of $\mathbb{R}^m$.

3. Continuity and connectedness:Connected subsets of $\mathbb{R}$ are intervals. If $f : E \to \mathbb{R}$ is continuous where $E \subset \mathbb{R}^n$ and E is connected, then $f(E) \subset \mathbb{R}$ is connected.

Unit II. Differentiable functions (15 Lectures)
Differentiable functions on $\mathbb{R}^n$, the total derivative $(Df)_p$ of a differentiable function $f : U \to \mathbb{R}^m$ at $p \in U$ where U is open in $\mathbb{R}^n$,uniqueness of total derivative, differentiability implies continuity.(ref:[1] C.C.Pugh or[2] A.Browder)

Chain rule. Applications of chain rule such as:

1. Let γ be a differentiable curve in an open subset U of $\mathbb{R}^n$. Let $f : U \to \mathbb{R}$ be a differentiable function and let $g(t) = f(\gamma(t))$. Then $g'(t) = \langle (\bigtriangledown f)(\gamma(t)), \gamma'(t) \rangle$.
2. Computation of total derivatives of real valued functions such as
   (a) the determinant function $\det(X)$, $X \in M_n(\mathbb{R})$,
   (b) the Euclidean inner product function $\langle x, y \rangle$, $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$.

(ref. M. Spivak, W. Rudin)

Results on total derivative:

1. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a constant function,then $(Df)_p = 0 \ \forall p \in \mathbb{R}^n$.
2. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map, then $(Df)_p = f \ \forall p \in \mathbb{R}^n$.
3. A function $f = (f_1, f_2, ..f_m) : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $p \in \mathbb{R}^n$ if and only if each f_j is differentiable at $p \in \mathbb{R}^n$, and $(Df)_p = ((Df_1)_p, (Df_2), p, .., (Df_m)_p)$. (ref. M. Spivak).

Partial derivatives, directional derivative $(D_u f)(p)$ of a function f at p in the direction of the unit vector, Jacobian matrix, Jacobian determinant. Results such as :

1. If the total derivative of a map $f = (f_1, .., f_m) : U \to \mathbb{R}^m$($U$ open subset of $\mathbb{R}^n$) exists at $p \in U$, then all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ exists at p.
2. If all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ of a map $f = (f_1, .., f_m) : U \to \mathbb{R}^m$($U$ open subset of $\mathbb{R}^n$) exist and are continuous on U, then f is differentiable.(ref. W. Rudin)

Derivatives of higher order, C^k-functions, C^∞-functions.(ref. T. Apostol)

Unit III. Inverse function theorem and Implicit function theorem (15 Lectures)
Theorem (Mean Value Inequality): Suppose $f : U \to \mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$ and there is a real number such that $||f(Df)_x|| \leq M \; \forall x \in U$. If the segment $[p, q]$ is contained in U, then $||f(q) - f(p)|| \leq M||q - p||$. (ref. C. C. Pugh or A. Browder).

Mean Value Theorem: Let $f : U \to \mathbb{R}^m$ is a differentiable on an open subset U of $\mathbb{R}^n$. Let $p, q \in U$ such that the segment $[p, q]$ is contained in U. Then for every vector $\mathbf{v} \in \mathbb{R}^n$ there is a point $x \in [p, q]$ such that $\langle \mathbf{v}, f(q) - f(p) \rangle = \langle \mathbf{v}, (Df)_x(q - p) \rangle$. (ref:T. Apostol) If $f : U \to \mathbb{R}^m$ is differentiable on a connected open subset U of R^n and $(Df)_x = 0 \; \forall x \in U$, then f is a constant map.

Taylor expansion for a real valued C^m-function defined on an open subset of R^n, stationary points(critical points), maxima, minima, saddle points, second derivative test for extrema at a stationary point of a real valued C^2-function defined on an open subset of $\mathbb{R}^n$. Lagrange's method of undetermined multipliers.
(ref. T. Apostol)

Contraction mapping theorem. Inverse function theorem, Implicit function theorem.(ref. A. Browder)

Unit IV. Riemann Integration(15 Lectures)
Riemann Integration over a rectangle in $\mathbb{R}^n$, Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgues Theorem(statement only), Fubini's Theorem and applications. (Reference for Unit IV: M. Spivak, Calculus on Manifolds).

Recommended Text Books

1. C. C. Pugh, Mathematical Analysis, Springer UTM.
2. A. Browder, Mathematical Analysis an Introduction, Springer.
3. T. Apostol, Mathematical Analysis, Narosa.
4. W. Rudin, Principals of Mathematical Analysis, McGraw-Hill India.
5. M. Spivak, Calculus on Manifolds, Harper-Collins Publishers.

PSMT103 / PAMT103 COMPLEX ANALYSIS

Course Outcomes

1. In this course the students will learn about series of functions and power series. The concept of radius of convergence will be introduced and calculated.
2. This course gives insight of complex integration which is different from integration of real valued functions. In particular, Cauchy integral formula will be proved.
3. The students will learn that if a function is once (complex) differentiable then it is infinitely many times differentiable. This will be a sharp contrast with the theorems of real analysis.
4. The various properties of Möbius transformations that have a wide variety of applications along with major theorems of theoretical interest like Cauchy-Goursat theorem, Morera's theorem, Rouché's theorem and Casorati-Weierstrass theorem will be studied.

Unit I. Holomorphic functions (15 Lectures)
Note: A complex differentiable function defined on an open subset of $\mathbb{C}$ is called a holomorphic function.

Review: Complex numbers, Geometry of the complex plane, Weierstrass's M-test and its aplication to uniform convergence, Ratio and root test for convergence of series of complex numbers. (no questions to be asked).

Stereographic projection, Sequence and series of complex numbers, Sequence and series of functions in $\mathbb{C}$, Complex differential functions, Chain rule for holomorphic function.

Power series of complex numbers, Radius of convergence of power series, Cauchy-Hadamard formula for radius of convergence of power series. Abel's theorem: let $\sum_{n\geq 0} a_n(z-z_0)^n$ be a power series of radius of convergence $R > 0$. Then the function $f(z)$ defined by $f(z) = \sum a_n(z-z_0)^n$ is holomorphic on the open ball $|z-z_0| < R$ and $f'(z) = \sum_{n\geq 1} na_n(z-z_0)^{n-1}$ for all $|z-z_0| < R$. Trigonometric functions, Applications of Abel's theorem to trigonometric functions.

Applications of the chain rule to define the logarithm as the inverse of exponential, branches of logarithm, principle branch $l(z)$ of the logarithm and its derivative on $\mathbb{C} \setminus \{z \in \mathbb{C} \mid \mathrm{Re}(z) \leq 0, \mathrm{Im}(z) = 0\}$.

Unit II. Contour integration, Cauchy-Goursat theorem (15 Lectures)
Contour integration, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Cauchy's theorem(general domain), Cauchy integral formula, Cauchy's estimates, The index(winding number) of a closed curve, Primitives. Existence of primitives, Morera's theorem. Power series representation of holomorphic function (Taylor's theorem).

Unit III. Properties of Holomorphic functions (15 Lectures)
Entire functions, Liouville's theorem. Fundamental theorem of algebra. Zeros of holomorphic functions, Identity theorem. Counting zeros; Open Mapping Theorem, Maximum modulus theorem, Schwarz's lemma. Automorphisms of unit disc.

Isolated singularities: removable singularities and Removable singularity theorem, poles and essential singularities. Laurent Series development. Casorati-Weierstrass's theorem.

Unit IV. Residue calculus and Mobius transformation (15 Lectures)
Residue Theorem and evaluation of standard types of integrals by the residue calculus method. Argument principle. Rouché's theorem. Conformal mapping, Mobius Transformation.

Recommended Text Books

1. J.B. Conway, Functions of one Complex variable, Springer.
2. A.R. Shastri: An introduction to complex analysis, Macmillan.
3. Serge Lang: Complex Analysis. Springer.
4. L.V. Ahlfors:Complex analysis, McGraw Hill.
5. R. Remmert: Theory of complex functions, Springer.
6. J.W. Brown and R.V. Churchill:Complex variables and Applications, McGraw-Hill.

PSMT104 / PAMT104 ORDINARY DIFFERENTIAL EQUATIONS

Course Outcomes

1. Through this course students are expected to understand the basic concepts of existence and uniqueness of solutions of Ordinary Differential Equations (ODEs).
2. In case of nonlinear ODEs, students will learn how to construct the sequence of approximate solutions converges to the exact solution if exact solution is not possible.
3. Students will be able to understand the qualitative features of solutions.
4. Students will be able to identify Sturm Liouville problems and to understand the special functions like Legendre's polynomials and Bessel's function.
5. Students will be to understand the applicability of the above concepts in different disciplins of Techonolgy.

Unit I. Existeence and Uniqueness of Solutions (15 Lectures)
Existence and Uniqueness of solutions to initial value problem of rst order ODE- both autonomous, non autonomous, ϵ-approximate solutions, Ascoli lemma, Cauchy-Peano existence theorem, Lipschitz condition, Picard's method of successive approximations, Picard-Lindelof theorem, System of Differential equations. Reduction of n-th order differential equations.
[Reference Unit-I of Theory of Ordinary Differential Equations; Earl A. Coddington and Norman Levinson, Tata McGraw Hill, India.]

Unit II. Linear Equations with constant coefficients (15 Lectures)
The second order homogeneous equations, Initial value problem for second order equations, Uniqueness theorem, linear dependance and independence of solutions, Wronskian,a formula for the Wronskian, The second order non-homogeneous equations, The homogeneous equations of order n, Initial value problem for n^{th} order equations, The non-homogeneous equations of order n, Algebra of constant coefficient operators.
[Reference Unit-II of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]

Unit III. Linear Equations with variable coefficients (15 Lectures)
Initial value problem for the homogeneous equation of order n,, Existence and Uniqueness theorem, solution of the homogeneous equations, Wronskian and linear independance, reduction of the order of a homogeneous equation, the non-homogeneous equations of order n,.
[Reference III of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]

Unit IV. Sturm-Liouville Problem & Qualitative Properties of Solutions (15 Lectures)

Eigenvalue problem, Eigenvalues and Eigenfunctions, the vibrating string problem, Sturm Liouville problems, homogeneous and non-homogeneous boundary conditions, orthogonality property of eigenfunctions, Existence of Eigenvalues and Eigenfunctions, Sturm Seperation theorem, Sturm comparison theorem. Power series solution of second order linear equations, ordinary points, singular points, regular singular points, existence of solution of homogeneous second order linear equation, solution of Legendre's equation, Legendre's polynomials, Rodrigues' formula, orthogonality conditions, Bessel differential equation, Bessel functions, Properties of Bessel function, orthogonality of Bessel functions.

[Reference Unit IV (24, 25), Unit V (Review), Unit-VII (40, 43, Appendix A) and Unit VIII (44, 45, 46, 47) : G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India]

Recommended Text Books

1. Earl A., Coddington and Norman Levinson, Theory of Ordinary Differential Equations; Tata McGraw Hill, India.

2. Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.

3. G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India

4. Hurewicz W., Lectures on ordinary differential equations, M.I.T. Press.

5. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

PSMT105/PAMT105 DISCRETE MATHEMATICS

Course Outcomes

1. Students will solve Linear Diophantine equations, cubic equation by Cardanos Method, Quadratic Congruence equation. Students will learn the multiplicativity of function τ, σ and ϕ.

2. Students will be able to understand the proof of Erdos- Szekers theorem on monotone sub-sequences of a sequence with n^2+1 terms and the applicability of Forbidden Positions.

3. Stundent will learn the Fibonacci sequence, the Linear homogeneous recurrence relations with constant coefficient, Ordinary and Exponential generating Functions, exponential generating function for bell numbers, the applications of generating Functions to counting and use of generating functions for solving recurrence relations.

4. Stundents will be able to understand Polyas Theory of counting, Orbit stabilizer theorem, Burnside Lemma and its applications, Applications of Polya's Formula.

Unit I. Number theory (15 Lectures)
Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Quadratic residues, Arithmetic functions σ, τ, ϕ and their multiplicative property. Advanced counting: Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Selections with Repetitions.

Unit II. Advanced counting (15 Lectures)
Stirling numbers of second and first kind. Pigeon-hole principle, generalized pigeonhole principle and its applications, Erdos-Szekers theorem on monotone subsequences, A theorem of Ramsey. Inclusion Exclusion Principle and its applications. Derangement. Permutations with Forbidden Positions, Restricted Positions and Rook Polynomials.

Unit III. Recurrence Relations (15 Lectures)
The Fibonacci sequence, Linear homogeneous and Non-homogeneous recurrence relations. Proof of the solution Linear homogeneous recurrence relations with constant coefficient in case of distinct roots and statement of the theorem giving a general solution (in case of repeated roots), Iteration and Induction. Ordinary generating Functions, Exponential Generating Functions, algebraic manipulations with power series, generating functions for counting combinations with and without repetitions, applications to counting, use of generating functions for solving homogeneous and non-homogeneous recurrence relations.

Unit IV. Polyas Theory of counting (15 Lectures)
Equivalence relations and orbits under a permutation group action. Orbit stabiliser theorem, Burnside Lemma and its applications, Cycle index, Polyas Formula, Applications of Polyas Formula.

Recommended Text Books

1. D. M. Burton, Introduction to Number Theory, McGraw-Hill.
2. Nadkarni and Telang, Introduction to Number Theory
3. V. Krishnamurthy:Combinatorics: Theory and applications, Affiliated East-West Press.
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. A. Tucker: Applied Combinatorics, John Wiley & Sons.
6. Norman L. Biggs: Discrete Mathematics, Oxford University Press.
7. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
8. Sharad S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

Semster-II
PSMT201 / PAMT201 ALGEBRA II

Course Outcomes

1. Students will learn Dihedral groups, Matrix groups, Automorphism group, Inner automorphisms, Structure theorem for finite abelain groups via examples.

2. Students will be able to understand group actions and orbit-stabilizer formula; Sylow theorems and applications to classification of groups of small order.

3. Students will be able to earn knowledge of prime avoidance theorem, Chinese remainder theorem, and specialized rings like Euclidean domains, principal ideal domains, unique factorization domains, their inclusions and counter examples.

Unit I. Groups and Group Homomorphisms (15 Lectures)
Review: Groups, subgroups, normal subgroups, center $Z(G)$ of a group. The kernel of a homomorphism is a normal subgroup. Cyclic groups. Lagrange's theorem. The product set $HK = \{hk/h \in H \;\&\; \in K\}$ of two subgroups of a group G: Examples of groups such as Permutation groups, Dihedral groups, Matrix groups, U_n-the group of units of $\mathbb{Z}_n$ (no questions be asked).

Quotient groups. First Isomorphism Theorem and the following two applications (reference: Algebra by Michael Artin)

1. Let $\mathbb{C}^*$ be the multiplicative group of non-zero complex numbers and $\mathbb{R} > 0$ be the multiplicative group of positive real numbers. Then the quotient group $\mathcal{C}^*/U$ is isomorphic to $\mathbb{R} > 0$:

2. The quotient group $GL_n(\mathbb{R})/SLn(\mathbb{R})$ is isomorphic to the multiplicative group of non-zero real numbers $\mathbb{R}^*$:

Second and third isomorphism theorems for groups, applications.

Product of groups. The group $\mathbb{Z}_m \times \mathbb{Z}_m$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if $\gcd(m, n) = 1$. Internal direct product (A group G is an internal direct product of two normal subgroups H, K if $G = HK$ and every $g \in G$ can be written as $g = hk$ where $h \in H; k \in K$ in a unique way). If H, K are two finite subgroups of a group, then $|HK| = \frac{|H|\,|K|}{|H \cap K|}$. If H, K are two normal subgroups of a group G such that $H \cap K = \{e\}$ and $HK = G$, then G is internal direct product of H and K. If a group G is an internal direct product of two normal subgroups H and K then G is isomorphic to $H \times K$. (Reference: Algebra by Michael Artin) Inner automorphisms, Automorphisms of a group. If G is a group, then $A(G)$; the set of all automorphisms of G, is a group under composition. If G is a finite cyclic group of order r; then $A(G)$ is isomorphic to U_r; the groups of all units of $\mathbb{Z}_r$ under multiplication modulo r. For the infinite cyclic group Z; $A(Z)$ is isomorphic to $\mathbb{Z}_2$. Inner automorphisms of a group. (Reference: Topics in Algebra by I.N.Herstein). Structure theorem of Abelian groups (statement only) and applications (Reference: A first Course in Abstract Algebra by J. B. Fraleigh).

Unit II. Groups acting on sets and Sylow theorems
Center of a group, centralizer or normalizer $N(a)$ of an element $a \in G$; conjugacy class $C(a)$ of a in G: In finite group G; $|C(a)| = o(G)/o(N(a)$ and $o(G) = \sum o(G)/o(N(a)$ where the summation is over one element in each conjugacy class, applications such as:

1. If G is a group of order p^n where p is a prime number, then $Z(G) \neq \{e\}$:
2. Any group of order p^2; where p is a prime number, is Abelian. (Reference: Topics in Algebra by I. N. Herstein).

Groups acting on sets, Class equation, Cauchy's theorem: If p is a positive prime number and $p|o(G)$ where G is finite group, then G has an element of order p. (Reference: Topics in Algebra by I. N. Herstein). p-groups, Sylow theorems and applications:

1. There are exactly two isomorphism classes of groups of order 6:
2. Any group of order 15 is cyclic

(Reference for Sylow's theorems and applications: Algebra by Michael Artin).

Unit III. Rings and Fields (15 lectures)
Review: Rings (with unity), ideals, quotient rings, prime ideals, maximal ideals, ring homomorphisms, characteristic of a ring, first and second Isomorphism theorems for rings, correspondence theorem for rings (If $f : R \to R'$ is a surjective ring homomorphism, then there is a $1-1$ correspondence between the ideals of R containing the ker f and the ideals of R'). Integral domains, construction of the quotient field of an integral domain. (no questions be asked).

For a commutative ring R with unity:

1. An ideal M of R is a maximal ideal if and only if the quotient ring R/M is a field.
2. An ideal N of R is a prime ideal if and only if the quotient ring R/M is an integral domain.
3. Every maximal ideal is a prime ideal.
4. Every proper ideal is contained in a maximal ideal.
5. If an ideal I is contained in union of prime ideals $P_i, P_2, \cdots, P_n$, then I is contained in some P_i.

6. If a prime ideal P contains an intersection of ideals $I_i, I_2, \cdots, I_n$, then P contains some ideal I_j.

Rings of fractions, inverse and direct imges of ideals, Comaximal ideals, Chinese Remainder Theorem in rings and its applications to congruences.

Definition of field, characteristic of a field, sub field of a field. A field contains a sub field isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$:

Polynomial ring $F[X]$ over a field, irreducible polynomials over a field. Prime ideals, and maximal ideals of a Polynomial ring $F[X]$ over a field F: A non-constant polynomial $p(X)$ is irreducible in a polynomial ring $F[X]$ over a field F if and only if the ideal $(p(X))$ is a maximal ideal of $F[X]$: Unique Factorization Theorem for polynomials over a field (statement only).

Unit IV. Divisibility in integral domains (15 lectures)
Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma, $Z[X]$ is a UFD, irreducibility criterion, Eisenstein's criterion, Euclidean domains. $Z[\sqrt{-5}]$ is not a UFD.
Reference for Unit IV: Michael Artin: Algebra, Prentice-Hall India.

Recommended Text Books

1. Michael Artin: Algebra, Prentice-Hall India.
2. I.N. Herstein: Topics in Algebra, Wiley-India.
3. R.B.J.T. Allenby: Rings, fields and Groups, An Introduction to Abstract Algebra, Elsevier (Indian edition).
4. J. B. Fraleigh, A first Course in Abstract Algebra, Narosa.
5. David Dummit, Richard Foot: Abstract Algebra, Wiley-India.

PSMT202 / PAMT202 TOPOLOGY

Course Outcomes

1. To understand a formation of new spaces from old one using product, box and quotient topology.
2. This course create a building block for analysis as well as algebraic geometry.
3. Students will understand extension theorems (e.g. Tietze extension theorem) which is useful in Functional Analysis.
4. This course covers the Tychonoff theorem which is a mile-stone of this subject.

Unit I.Topology and Topological spaces (15 Lectures)
Topological spaces, basis, topology generated by basis, sub-basis, order topology, product topology, subspace topology, closed sets, limit points, closure, interior, continuous functions, homeomorphism, box topology, comparison of the box and product topologies, T_0, T_1 spaces, For a T_1 space X, $x \in A \subset X$ is limit point of A if and only if every neighbourhood of x contains infinitely many points if A. Hausdorff space.

Unit II.Connected topological spaces (15 Lectures)
Quotient spaces. Connected topological spaces, separation of a topological space, continuity and connected-ness, path-connected topological spaces, topologist's sine curve, the order square I_0^2 is connected but not path connected. For $\mathbb{R}$ equipped with usual topology, the infinte cartesian product $\mathbb{R}^\omega$ in the product topology is connected but in box topology it is not. Connected components of a topological space, Path components of a topological space. Countability Axioms, first and second countable spaces, Separable spaces, Lindeloff spaces.

Unit III. Compact topological spaces (15 Lectures)
Compact spaces, continuity and compactness, tube lemma, finite product of compact topological spaces is compact. Finite intersection property, the Lebesgue number lemma, uniform continuity theorem, compact Hausdorff space with no isolated points is uncountable. Limit point compact spaces, local compactness, one point compactification.

Unit IV. Metrizable spaces and Tychonoff theorem (15 Lectures)
Metrizable spaces, separation axioms (regular and normal spaces). Every metrizable space is normal. A compact T_2 space is a normal space. Urysohn lemma, Urysohn metrization theorem, Tietze extension theorem. Tychonoff theorem,

Recommended Text Books

1. J. F. Munkres: Topology, Pearson; 2 edition (January 7, 2000).
2. G. F. Simmons: INtroduction to Topology and Modern Analysis, Tata McGraw Hill, 2004.

PSMT 203/PAMT 203: ANALYSIS II

Course Outcome:

1. In this course students are expected to understand the basic concepts of measure on an arbitrary measure space X as well as on $\mathbb{R}^n$.

2. They are also expected to study Lebesgue outer measure of sets and measurable sets, measurable functions.

3. Students will be able to understand the concepts of integrals of measurable functions in an arbitrary measure space $(X, \mathcal{A}, \mu)$. Lebesgue integration of complex valued functions and basic concepts of signed measures.

Unit-I: Measures and Measurable Sets (15 Lectures)
Additive set functions, σ-algebra countable additivity, Outer measure, constructing measures, μ^* measurable sets (Definitions due to Carathéodory), μ^* measurable subsets of X forms a σ algebra, measure space $(X, \sum, \mu)$. Lebesgue outer measure in $\mathbb{R}^d$, properties of exterior measure, monotonicity property and countable sub-additivity property of Lebesgue measure, translation invariance of exterior measure, example of set of measure zero. Measurable sets and Lebesgue measure, properties of measurable sets. Existence of a subset of $\mathbb{R}$ which is not Lebesgue measurable.
[Reference for unit I: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India]

Unit-II: Measurable functions and their Integration (15 Lectures)
Measurable functions on $(X, \sum, \mu)$, simple functions, properties of measurable functions. If $f \geq 0$ is a measurable function, then there exists a monotone increasing sequence (s_n) of non-negative simple measurable functions converging to point wise to the function f. Egorov's theorem, Lusin's theorem. Integral of nonnegative simple measurable functions defined on the measure space $(X, \sum, \mu)$ and their properties. Integral of a non-negative measurable function.
[Reference for unit II: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India]

Unit-III: Convergence Theorems on Measure space (15 Lectures)
Monotone convergence theorem. If $f \geq 0$ and $g \geq 0$ are measurable functions, then $\int (f + g) d\mu = \int f d\mu + \int g d\mu$, Fatou's lemma, summable functions, vector space of summable functions, Lebesgue's dominated convergence theorem. Lebesgue integral of bounded functions over a set of finite measure, Bounded convergence theorem. Lebesgue and

Riemann integrals: A bounded real valued function on [a, b] is Riemann integrable if and only if it is continuous at almost every point of [a, b]; in this case, its Riemann integral and Lebesgue integral coincide.
[Reference for unit III: 1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.
2. Royden H. L. Real Analysis, PHI]

Unit-IV: Space of Integrable functions (15 Lectures)
Borel set, Borel algebra of $\mathbb{R}^d$. Any closed subset and any open subset of $\mathbb{R}^d$ is Lebesgue measurable. Every Borel set in $\mathbb{R}^d$ is Lebesgue measurable. For any bounded Lebesgue measurable subset E of $\mathbb{R}^d$, given any $\epsilon > 0$ there exist a compact set K and open set U in $\mathbb{R}^d$ such that $K \subseteq E \subseteq U$ and $m(U \setminus K) < \epsilon$. For any Lebesgue measurable subset E of $\mathbb{R}^d$, there exist Borel sets F, G in $\mathbb{R}^d$ such that $F \subseteq E \subseteq G$ and $m(E \setminus F) = 0 = m(G \setminus E)$. Signed Measures, positive set, negative set and null set. Hahn decomposition theorem. Complex valued Lebesgue measurable functions on $\mathbb{R}^d$. Lebesgue integral of complex valued measurable functions, Approximation of Lebesgue integrable functions by continuous functions. The space $L^1(\mu)$ of integrable functions, properties of L^1 integrable functions, Riesz-Fischer theorem.
[Reference for unit IV: 1. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India
2. Royden H. L. Real Analysis, PHI
3. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.]

Recommended Text Books

1. Andrew Browder, Mathematical Analysis, An Introduction, Springer Undergraduate Texts in Mathematics.

2. Elias M. Stein and Rami Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert Spaces, New Age International Limited, India

3. Royden H. L. Real Analysis, PHI.

4. Terence Tao, Analysis II, Hindustan Book Agency (Second Edition).

PSMT 204 / PAMT 204 : PARTIAL DIFFERENTIAL EQUATIONS

Course Outcomes

1. Students are expected to understand the basic concepts and method of finding the solution of first and second order Partial Differential Equations (PDEs).
2. Students will be able to know the classification of second order PDEs, singularity and fundamental solution.
3. Students will be able to know the role of Green's function in the solution of Partial Differential Equations.
4. Through this course students will understand existence and uniqueness of solutions to Diffusion and Parabolic equations.

Unit-I: First Order Partial Differential Equations (15 Lectures)
First order partial differential equations in two independent variables, Semilinear and Quasilinear equations in two independent variables, method of characteristics, the Characteristics Cauchy Problem, General solutions.
Non-linear equations in two independent variables: Monge Strip and Charpit Equations, Solution of Cauchy problem, Determination of Complete integral, solution of Cauchy problem,
[Reference Unit-I (1.1,1.2, 1.3, 1.4)of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Unit-II: Second Order Partial Differential Equations (15 Lectures)
Classifications of second order partial differential equations in two and more than two independent variables, method of reduction to normal form, the Cauchy problem. Potential theory and elliptic differential equations, boundary value problems and Cauchy problem, Poisson's theorem, the mean value and the Maximum-Minimum properties
[Reference Unit-II (2.1, 2.2) of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Unit-III: Green's Functions and Integral Representations (15 Lectures)
Singularity functions and the fundamental solution, Green functions, Greens identities, Green's function for m-dimensions sphere of radius R, Green's functions Dirichlet problem in the plane, Neumann's function in the plane.
[Reference Unit-II (2.2.2):1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India and 2. Unit VIII of Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press]

Unit-IV: The Diffusion Equation & Parabolic Differential Equations (15 Lectures)

Existence and Uniqueness theorem for initial value problem in an infinite domain, semi-infinite domain, one dimensional Heat equation, maximum and minimum principle for the heat equation and for some parabolic equations, one dimensional wave equation, boundary value problem for the one dimensional heat and wave equations, method of separation of variables.
[Reference Unit-II (2.3.1, 2.3.2, 2.3.3, 2.3.4, 2.4.1 2.4.8)of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Recommended Text Books

1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India.

2. Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press.

3. T.Amaranath, An Elemetary Course in Partial Differential Equations, Narosa.

4. F. John, Partial Differential Equations, Springer publications.

5. G.B. Folland, Introduction to partial differential equations, Prentice Hall.

PSMT205/PAMT205 PROBABILITY THEORY

Course Outcomes

1. Students will understand the concept of Modelling Random Experiments, Classical probability spaces, -fields generated by a family of sets, -field of Borel sets, Limit superior and limit inferior for a sequence of events.

2. Students will be able to know about probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on -field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction.

3. Students will be able to earn knowledge of discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula.

4. Students will learn distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions,

5. Students will be able to understand Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers, Central limit theorem and Application of Probability Theory.

Unit I. Probability basics (15 Lectures)
Modelling Random Experiments: Introduction to probability, probability space, events. Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II. Probability measure (15 Lectures)
Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ-field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction. Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III. Random variables (15 Lectures)
Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV. Limit Theorems (15 Lectures)
Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Central limit theorem (statement only).

Recommended Text Books

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.
2. J. F. Rosenthal: A First Look at Rigorous Probability Theory, World Scientic.
3. Kai Lai Chung, Farid AitSahlia: Elementary Probability Theory, Springer Verlag.
4. Ross, Sheldon M. A first course in probability(8th Ed), Pearson.

Scheme of Examination
The scheme of examination for the syllabus of Semesters I & II of M.A./M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows.
Scheme of Evaluation R8435 for M. Sc /M. A.

1. A) 80: 20 for distance education (external evaluation of 80 marks and internal evaluation of 20 marks) under the choice based credit system (CBCS).
2. B) 60:40 for university affiliated PG centers (external evaluation of 60 marks and internal evaluation of 40 marks).
3. C) 100 percent internal evaluation scheme for University department of mathematics (One mid semester test of 30 marks, 05 marks for attendance, 05 marks for active participation and one end semester test of 60 marks, both tests will be conducted by the department and answer book will be shown to the students).

Duration:- Examination shall be of 2 and 1/2 Hours duration.
Theory Question Paper Pattern for Scheme B and Scheme C :-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, and the allocation of marks depend on the weight-age of the topic.
5. Each question will be of maximum 18 marks when marks of all the sub-questions are added (including the options) in that question.
6. For scheme A: 60 marks will be converted into 80 marks.

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UNIVERSITY OF MUMBAI

Syllabus

For M.Sc. / M.A. Semester III and IV CBCS

Program: M.Sc. / M.A.

Course: Mathematics

With effect from the academic year 2021-22

Semester III

Sr. No.	Subject code	Units	Subject	Credits	L/W
			Algebra III		
01	PSMT/ PAMT 301	Unit I	Groups	06	04
		Unit II	Rings and Ideals		
		Unit III	Modules		
		Unit IV	Modules over PID		
			Functional Analysis		
02	PSMT/ PAMT 302	Unit I	Baire Spaces and Hilbert Spaces	06	04
		Unit II	Normed Linear Spaces		
		Unit III	Bounded Linear Transformations		
		Unit IV	Basic Theorems		
			Differential Geometry		
03	PSMT/ PAMT 303	Unit I	Isometries of R^n	06	04
		Unit II	Curves		
		Unit III	Regular Surface		
		Unit IV	Curvature		
			Elective Courses		
04	PSMT/ PAMT 304	Elective I (Any one from 1 to 10)		03	04
05	PSMT/ PAMT 305	Elective II (Any one from 1 to 10)		03	04

Note:

- PSMT301/PAMT301, PSMT302/PAMT302, PSMT303/PAMT303 are compulsory courses for Semester III.
- 2. PSMT 304/PAMT 304 and PSMT 305/PAMT305 are Elective Courses for Semester III.
- 3. Elective course Courses I and II will be any TWO of the following list of ten courses:

1. Algebraic Topology

2. Advanced Complex Analysis
3. Commutative Algebra
4. Algebraic Number Theory
5. Advanced Partial Differential Equations
6. Integral Transforms
7. Numerical Analysis
8. Graph Theory
9. Coding Theory
10. Design Theory

Teaching Pattern for Semester III

1. Four lectures per week for each of the courses: PSMT301/PAMT301, PSMT302/ PAMT302, PSMT303/PAMT303, PSMT304/PAMT304 and PSMT305/PAMT305.
2. Each lecture is of 60 minutes duration.
3. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

Semester IV

Sr. No.	Subject code	Units	Subject	Credits	L/W
			Algebra IV		
01	PSMT/ PAMT 401	Unit I	Algebraic Extensions	06	04
		Unit II	Normal and Separable Extensions		
		Unit III	Galois Theory		
		Unit IV	Applications		
			Fourier Analysis		
02	PSMT/ PAMT 402	Unit I	Fourier Series	05	04
		Unit II	Dirichlet's Theorem		
		Unit III	Fejer's Theorem and Applications		
		Unit IV	Dirichlet Problem in the Unit Disc		
			Calculus on Manifolds		
03	PSMT/ PAMT 403	Unit I	Multivariable Algebra	05	04
		Unit II	Differential Forms		
		Unit III	Basics of Submanifolds of R^n		
		Unit IV	Stoke's theorem		
04	PSMT/ PAMT 404	**Skill Based Course** (Any one from I to V)		04	04
05	PSMT/ PAMT 405	**Project**		04	04

Note:

- PSMT401/PAMT401, PSMT402/PAMT402, PSMT403/PAMT403 are compulsory courses for Semester IV.
- PSMT 404/PAMT 404 is Skill Based Course for Semester IV.
- Skill Based Course will be any ONE of the following list of **FIVE** courses:

1 Business Statistics

2 Statistical Methods

3. Computer Science
4. Linear and Non Linear Programming
5. Computational Algebra

- PSMT 405/PAMT 405 is a project based Course for Semester IV.

The projects for this course are to be guided by the Faculty members of the Department of Mathematics of the concerned college. Each project shall have maximum of 08 (eight) students. The workload for each project is 2 L/W.

List of Equivalent courses

Sr. No.	Old Course (60:40 &100) CBSGS/Choice Based/ CBCGS	New Courses (2020-21&2021-22) CBCS
1	Algebra I	Algebra I
2	Analysis I	Analysis I
3	Complex Analysis I	
4	Discrete Mathematics	Discrete Mathematics
5	Soft Skill/ Set Theory and Logic	
6	Algebra II	Algebra II
7	Complex Analysis II/ Complex Analysis	Complex Analysis
8	Topology	Topology
9	Differential Equations	Ordinary Differential Equations
10	Elementary Probability Theory/ Probability Theory	Probability Theory
11	Algebra III	Algebra III
12	Analysis II	Analysis II
13	Differential Geometry	Differential Geometry
14	Numerical Analysis I	
15	Graph Theory I	Graph Theory
16	Commutative Algebra	Commutative Algebra
17	Algebraic Number Theory	Algebraic Number Theory
18	Algebraic Topology	Algebraic Topology
19	Coding Theory	Coding Theory
20	Field Theory	Algebra IV
21	Fourier Analysis	Fourier Analysis
22	Functional Analysis	Functional Analysis
23	Calculus of Manifolds	Calculus of Manifolds
24	Operations Research/ OC 1& OC II	Linear and Non Linear Programming
25	Numerical Analysis II	Numerical Analysis
26	Integral Transform	Integral Transform
27	Graph Theory II	

Teaching Pattern for Semester III and IV
1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

Semester-III

All Results have to be done with proof unless and otherwise stated.

PSMT 301 /PAMT 301 Algebra III

Course Outcomes:

1. Students will learn about classical groups like Simple groups, Solvable groups and Nilpotent groups and applications of these classical groups.

2. Motive of an introduction of the Zariski topology is to take a glance at algebraic geometry which is like ocean in its own right. This topic provides geometric point of view of algebra. It gives students a wider perspective to rethink the nature of a prime ideal, maximal ideal and very precise the geometric position of prime and maximal ideals. It also helps to visualize radical of an ideal in geometric setting and local nature of a prime ideal.

3. Students will learn Finitely generated modules, Free modules, Free module of rank n.

4. Students will understand the Structure theorem for finitely generated modules over a PID and Applications to the Structure theorem for finitely generated Abelian groups and linear operators.

Unit I. Groups (15 Lectures)
Simple groups, A_5 is simple. Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus Lemma, Jordan-Holder theorem,
Direct and Semi-direct products, Examples such as
(i) The group of affine transformations $x \mapsto ax + b$ as semi-direct product of the group of linear transformations acting on the group of translations.
(ii) Dihedral group D_{2n} as semi-direct product of $\mathbb{Z}_2$ and $\mathbb{Z}_n$.
Classification of groups of order 12.

Unit II. Rings and Ideals (15 Lectures)
Nilradical and relation to prime ideals, Jacobson radical and maximal ideals, Radical of an ideal, Annihilator ideal, Local rings and equivalent conditions for a local ring, Prime spectrum of a ring and Zariski topology, idempotents and connectedness, ring homomorphisms and induced map on Spec. Hilbert Nullstellensatz (only statement) and its corollaries.

Unit III. Modules (15 Lectures)
Modules over rings, Submodules. Module homomorphisms, kernels. Quotient modules. Isomorphism theorems. Generation of modules, finitely generated modules, (internal) direct sums and equivalent conditions. Free modules, free module of rank n. For a commutative ring R, R^n is isomorphic to R^m if and only if $n = m$. Matrix representations of homomorphisms between free modules of finite rank. Dimension of a free module over a P.I.D.

Unit IV. Modules over PID (15 Lectures)
Noetherian modules and equivalent conditions. Rank of an R-module. Torsion submodule of an R-module M, torsion free modules, annihilator ideal of a submodule. Finitely generated modules over a PID: If N is a submodule of free module M (over a P.I.D.) of finite rank n, then N is free of rank $m \leq n$. Any submodule of a finitely generated module over a P.I.D. is finitely generated. Structure theorem for finitely generated modules over a PID: Fundamental theorem, Existence (Invariant Factor Form and Elementary Divisor Form), Fundamental theorem, Uniqueness. Applications to the Structure theorem for finitely generated Abelian groups and linear operators.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, Indian Edition 2007.
3. D. S. Dummit and R. M. Foote, Abstract Algebra.
4. N. Jacobson, Basic Algebra, Volume 1, Dover, 1985.
5. S. Lang, Algebra, Springer Verlag, 2004

PSMT 302 / PAMT 302 Functional Analysis

Course Outcomes:

1. Students will learn Hilbert spaces and Banach spaces.
2. Students will be able to understand the concept of dimension of a Hilbert space, bounded linear transformations, norms, inner products, dual spaces and their difference from the finite dimensional cases.
3. Students should know about ℓ^p, L^p spaces, dual spaces and their properties;
4. Students should understand the fundamental theorems as mentioned in the syllabus.

Unit I: Baire spaces and Hilbert spaces (15 Lectures)
Baire spaces. Open subspace of a Baire space is a Baire space. Complete metric spaces are Baire spaces and application to a sequence of continuous real valued functions converging point-wise to a limit function on a complete metric space. Hilbert spaces, examples of Hilbert spaces such as ℓ^2, $L^2[-\pi, \pi]$, $L^2(\mathbb{R}^n)$ (with no proofs). Inner product induced by norm, Bessel's inequality. Equivalence of complete orthonormal set and maximal orthonormal set, orthogonal decomposition, Parseval's identity. Existence of a maximal orthonormal set, separability of Hilbert space.

Unit II: Normed Linear Spaces (15 Lectures)
Normed Linear spaces, Banach spaces. Examples of Normed linear spaces, Arzela-Ascoli theorem, ℓ^p $(1 \leq p \leq \infty)$ spaces are Banach spaces, $L^p(\mu)(1 \leq p \leq \infty)$ spaces: Holder's inequality, Minkowski's inequality, $L^p(\mu)(1 \leq p \leq \infty)$ are Banach spaces. Quotient space of a normed linear space. Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to infinite dimension of a normed linear spaces.

Unit III: Bounded Linear Transformations (15 Lectures)
Bounded linear transformations, Equivalent characterizations. The space $\mathcal{B}(X, Y)$. Completeness of $\mathcal{B}(X, Y)$ when Y is complete. Dual space of a normed linear space, Dual space of ℓ^p $(1 \leq p < \infty)$, Riesz Representation theorem for Hilbert spaces. Dual of $L^p(\mu)(1 \leq p < \infty)$ spaces: Riesz-Representation theorem for $L^p(\mu)(1 \leq p < \infty)$ spaces. Separable spaces, examples of separable spaces such as ℓ^p $(1 \leq p < \infty)$. If the dual space X' of X is separable, then X is separable.

Unit IV: Basic Theorems (15 Lectures)
Hahn-Banach theorem (Extension and Separation), applications of Hahn-Banach theorem. Open mapping theorem, Closed graph theorem, Uniform boundedness principle and application.

Recommended Text Books:

1. Andrew Browder, Mathematical Analysis, An Introduction, Springer International Edition, 1996.
2. E. Keryszig, Introductory Functional Analysis with Applications, Wiely India, 1978.
3. B. V. Limaye, Functional Analysis, New Age International, 1996.
4. J. R. Munkres, Topology, Prentice Hall, 2000.
5. M. T. Nair, Functional Analysis, Prentice Hall, India
6. H. L. Royden, Real Analysis, Pearson, 4th edition, 2017.
7. G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.

PSMT303/PAMT303: Differential Geometry

Course Outcomes:

1. Students will be able to grasp parametrization of curves and surfaces.
2. Students will be able to understand the various geometrical aspects like tangent, arc length, curvature, torsion etc of plane and space curves.
3. Students will be able to understand the role of first fundamental theorem and second fundamental theorem in the computation of Gaussian curvature, mean curvature and principal curvature.
4. Students will aware about properties of various special types of curves and surfaces.

Unit I: Isometries of $\mathbb{R}^n$ (15 Lectures)
(Pre-requisite: Lines and planes) Orthogonal transformations of $\mathbb{R}^n$ and Orthogonal matrices. Reflection, Rotations and Translations of $\mathbb{R}^2$ and $\mathbb{R}^3$, Euler's theorem, Hyperplanes, Refection map about a hyperplane W of $\mathbb{R}^n$ through the origin, Isometry of $\mathbb{R}^n$, Isometries of the plane, Orientation preserving and reversing isometries of $\mathbb{R}^n$, Glide reflection.

Unit II: Curves (15 Lectures)
Parametrized curves, Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Curvature and torsion of curves in $\mathbb{R}^3$, Plane curves, Signed curvature for plane curves, Fundamental theorem for plane curves, Space curves, Serret-Frenet equations. Fundamental theorem for space curves.

Unit III: Regular Surfaces (15 Lectures)
Regular surfaces in $\mathbb{R}^3$, Examples. Surfaces as graphs, Surfaces as level sets, Surfaces of revolution. Tangent space to a surface at a point, Equivalent definitions. Smooth functions on a surface, Differential of a smooth function defined on a surface. Orientable surfaces. Mobius band is not orientable.

Unit IV: Curvature (15 Lectures)
The first fundamental form. Isometries of surfaces, Surface area, The Gauss map, The shape operator of a surface at a point, Self-adjointness of the shape operator, The second fundamental form,Normal curvature, Principle curvatures and directions, Euler's formula, Meusnier's Theorem, Gaussian curvature and mean curvature, Computation of Gaussian curvature. Geodesics.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. C. Bar, Elementary Differential geometry, Cambridge University Press, 2010.
3. M. DoCarmo, Differential geometry of curves and surfaces, Prentice Hall Inc., 1976.
4. S. Kumaresan, Linear Algebra, A Geometric Approach, 2000.
5. A. Pressley, Elementary Differential Geometry, Springer UTM.

PSMT304/PAMT304 & PSMT305/PAMT305 ELECTIVE COURSES I & II
The Elective Courses I and II will be any TWO of the following list of ten courses:

1. Algebraic Topology

Course Outcomes:

1. Students will learn about homotopy of maps, homotopy of paths and the fundamental group and its applications.
2. Students will learn about universal covering spaces.
3. Students will understand the Singular homology and Excision theorem.

Unit I. Fundamental Group (15 Lectures)
Homotopy. Path homotopy. The fundamental group. Simply connected spaces. Coveringspaces. Path lifting and homotopy lifting lemma. Fundamental group of the circle.

Unit II. Fundamental group, Applications (15 Lectures)
Deformation retracts and homotopy types. Fundamental group of S^n. Fundamental group of the projective space. Brower fixed point theorem. Fundamental theorem of algebra. Borsuk-Ulam theorem. Seifert-Van Kampen Theorem (without proof). Fundamental group of wedge of circles. Fundamental group of the torus.

Unit III. Universal Covering Spaces (15 Lectures)
Equivalence of covering maps and Equivalences of covering spaces. The general lifting lemma. The necessary and sufficient condition of equivalence of two covering maps. Universal covering space. Conditional existence of Universal covering space. Example of a space with no universal covering space.

Unit IV. Singular Homology (15 lectures)
Singular p-simplex, singular chain complex, singular homology group, reduced singular homology group, acyclic space, zero dimensional singular homology group, induced homomorphism. Axioms for singular theory: Identity axiom, composition axiom, homotopy axiom, exactness axiom and commutativity axiom. Excision Theorem.

Recommended Text Books:

1. Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. John Lee, Introduction to Topological Manifolds, Springer GTM, 2000.
3. James Munkres, Topology, Prentice Hall of India, 1992.
4. James Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.2.

2. Advanced Complex Analysis

Course Outcomes:

1. Students will learn about monodromy and the monodromy theorem.

2. Students will gain knowledge of Elliptic Functions and Zeta functions.

3. Students will understand Uniform convergence, Ascoli's theorem, Riemann mapping theorem.

Unit I. Monodromy (15 Lectures)
Holomorphic functions of one variable. Germs of holomorphic functions. Analytic continuation along a path. Examples including $z^{1/n}$ and $\log(z)$, Homotopy between paths. The monodromy theorem.

Unit II. Riemann Mapping Theorem (15 Lectures)
Uniform convergence, Ascoli's theorem, Riemann mapping theorem.

Unit III. Elliptic Functions (15 Lectures)
Lattices in $\mathbb{C}$. Elliptic functions (doubly periodic meromorphic functions) with respect to a lattice. Sum of residues in a fundamental parallelogram is zero and the sum of zeros and poles (counting multiplicities) in a fundamental parallelogram is zero, Weierstrass $\mathcal{P}$-function, Relation between $\mathcal{P}$ and $\mathcal{P}'$, Theorem that $\mathcal{P}$ and $\mathcal{P}'$ generate the field of elliptic functions.

Unit IV. Zeta Function (15 Lectures)
Gamma and Riemann Zeta functions, Analytic continuation, Functional equation for the Zeta function.

Recommended Text Books:

1. L. V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.

2. John Conway, Functions of one complex variable, Narosa India, 1978.

3. S. Lang, Complex Analysis, Springer, 1999 .

4. Narasimhan, Complex Analysis in one Variable, Birkhauser, 1985.

5. E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press, 2003.

3. Commutative Algebra

Course Outcomes:

1. Students will learn about basics of rings and modules, primary decomposition and associated primes.
2. Students will gain knowledge of integral extensions, valuation rings, discrete valuation rings, Dedekind domains.
3. Students will understand the Going up theorem and Going down theorem.

Unit I. Basics of rings and modules (15 Lectures)
Basic operations with commutative rings and modules, Polynomial and power series rings, Prime and maximal ideals, Extension and contractions, Nil and Jacobson radicals, Chain conditions, Hilbert basis theorem, Local rings, Localization, Nakayama's lemma, Tensor products.

Unit II. Primary decomposition (15 Lectures)
Primary ideal, primary decomposition of an ideal, examples, example of an ideal for which primary decomposition does not exist. Minimal decomposition, First uniqueness theorem for minimal primary decomposition of a decomposible ideals. Associated prime ideals, minimal prime ideals, containment of minimal primes in associated prime ideals, Behavior of primary ideals under localization. Second uniqueness theorem for minimal primary decomposition of a decomposible ideals.

Unit III. Integral Extensions (15 Lectures)
Integral extensions, Going up and going down theorems, The ring of integers in a quadratic extension of rationals, Noether normalization, Hilbert's nullstellensatz.

Unit IV. Dedekind Domains(15 Lectures)
Artinian rings, Discrete valuation rings, Alternative characterizations of discrete valuation rings, Dedekind domains, Fractional ideals, Factorization of ideals in a Dedekind domain, Examples.

Recommended Text Books:

1. M. F. Atiyah and I. G. McDonald, Introduction to Commutative Algebra, Addison Wesley, 2007.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons, 20011.
3. 1. S. Lang, Algebra, Springer, 2004.
4. O. Zariski and P. Samuel, Commutative Algebra, Volume 1, Princeton , NJ, 1965.

4. Algebraic Number Theory

Course Outcomes:

1. Students will learn about algebraic numbers, algebraic integers and further properties of rings of integers.
2. Students will understand the class group .
3. Students will gain knowledge of Ramification theory and Diophantine equations

Unit I. Algebraic Numbers and algebraic integers (15 Lectures)
Number fields, Algebraic numbers, Integral extensions, Ring of integers in a number field, Quadratic fields, Real and imaginary quadratic fields, Ring of integers in a quadratic field, Examples like the ring of Gaussian integers.

Unit II. Rings of integers (15 Lectures)
Norms and traces, basis for the ring of algebraic integers, Norm of an ideal, prime factorization of ideals, Norm of a principal ideal, Definition of Dedekind domain, Fractional ideals, Existence and uniqueness of factorization

Unit III. Class group (15 Lectures)
Principal fractional ideals, Norm map is multiplicative on integral ideals, Minkowski lemma, Finiteness of the class group, Explicit example of factorization in quadratic number fields, Legendre symbol, Jacobi symbol and quadratic reciprocity.

Unit IV. Ramification theory and Diophantine equations (15 Lectures)
Ramification, residue degree, transitivity of ramification and residue degrees, Proof of ramification theorem, Examples, Group of units, Applications to Diophantine equations.

Recommended Text Books:

1. M. Artin, Algebra, Prentice-Hall, India, 2000.
2. S. R. Ghorpade, Lectures on Field Theory and Ramification Theory, IITB.
3. Marcus, Number Fields, Springer.
4. Niven and Zuckermann, An Introduction to the Theory of Numbers, 1980.
5. Algebraic Number Theory, T.I.F.R. Lecture Notes, 1966

5. Advanced Partial Differential Equations

Course Outcomes:

1. Students will be able to grasp nature of the differential operator viz parabolic, hyperbolic and elliptic.
2. Students will be able to understand the solution and various properties of the Laplacian operator, heat operator and wave operator.
3. Students will aware about applications of the Laplacian operator, heat operator and wave operator.

Unit-I: Local existence theory (15 Lectures)
The differential operator, Real first order equations, the general Cauchy problem, Cauchy-Kowalevsky theorem, Local solvability: the Lewy example, the fundamental solution.

Unit-II: Laplace operator (15 Lectures)
Symmetry properties of the Laplacian, basic properties of the Harmonic functions, Green's identities, The mean value theorem, Liouville's theorem, the Fundamental solution, the Dirichlet and Neumann boundary value problems, the Green's function. Applications to the Dirichlet problem in a ball in $\mathbb{R}^n$ and in a half space of $\mathbb{R}^n$.

Unit-III: Heat operator (15 Lectures)
The properties of the Gaussian kernel, solution of initial value problem for heat equation: homogeneous and non-homogeneous, The fundamental solution for heat operator, Heat equation in a bounded domains, Maximum principle for the heat equation and applications.

Unit-IV: Wave operator (15 Lectures)
Wave operator in dimensions 1, 2 & 3; Cauchy problem for the wave equation. D'Alemberts solution,of the one dimensional wave equation, Poisson formula of spherical means, Hadamards method of descent, Inhomogeneous Wave equation, Wave equation in a bounded domain.

Recommended Text Books:

1. G. B. Folland, Introduction to partial differential equations, Overseas Press, 1995.
2. F. John, Partial Differential Equations, Springer publications, 1964.

6. Integral Transforms

Course Outcomes:

1. Students will be able to grasp the concept of integral transforms and development of corresponding kernels.

2. Students will be able to understand various properties of the Laplace transform, Fourier transform, Mellin transform and $Z-$transform.

3. Students will aware about applications of the integral transform in the solution of initial and boundary value problems.

Unit I: The Laplace Transform (15 Lectures)
Definition of Laplace Transform, Laplace transforms of some elementary functions, Existence theorem, Properties of Laplace transform, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Convolution Theorem, Inversion theorem, Laplace transform of special functions: Heaviside unit step function, Dirac-delta function and Periodic function, Application of Laplace transform to evaluation of integrals and solutions of ODEs & PDEs: One dimensional heat equation & wave equation.

Unit II: The Fourier Transform (15 Lectures)
Fourier integral representation, Fourier integral theorem, Fourier Sine & Cosine integral representation, Riemann-Lebesgue lemma, Fourier transform pairs, Fourier Sine & Cosine transform pairs, Fourier transform of elementary functions, Properties of Fourier Transform, Convolution Theorem, Convolution integrals of Fourier, Parseval Identity, Cosine & Sine convolution integrals, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems, Heat conduction in solids (one dimensional problems in infinite & semi infinite domain).

Unit III: The Mellin Transform (15 Lectures)
Derivation for Mellin transform & its inversion by Fourier integral theorem, Properties and evaluation of Mellin transforms, Complex variable method, Convolution theorem for Mellin transform, Applications of Mellin transform: Summation of series, Products of random variables, Application to boundary value problems.

Unit IV: The Z-Transform (15 Lectures)
Definition of Z-transform, Z-transform of some elementary sequences, Properties of Z-transform Convolution Theorem, Inversion of the Z-transform, Applications of Z-transform to solutions of difference equations and summation of series.

Recommended Text Books:

1. L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India, 1999.
2. R. Bracemell, Fourier Transform and its Applications, MacGraw hill, 1963.
3. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their Applications, CRC Press Taylor & Francis, 2014.
4. Brian Davies, Integral transforms and their Applications, Springer, 1985.
5. I. N. Sneddon, Use of Integral Transforms, Tata-McGraw Hill, 1972.

7. Numerical Analysis

Course Outcomes:

1. Students will be able to grasp the concept of numerical solution of various mathematical problems and corresponding erorrs.
2. Students will be able to understand the approximation of functions by least square method.
3. Students will aware about applications of various numerical techniques in the solution of difference equations, ordinary and partial differential equations.

Note: Numerical methods be taught with error estimate, convergence and stability

Unit I: Approximation of functions (15 Lectures)
Least squares approximation, Weighted least squares method, Gram-Schmidt orthogonalizing process, Least squares approximation by Chebyshev polynomials. Discrete Fourier Transform and Fast Fourier Transform.

Unit II: Differential and Difference Equations (15 Lectures)
Differential equations: Solutions of linear differential equations with constant coefficients by Predictor corrector methods and Milne's method. Galerkin's method for two point linear boundary value problems. Difference Equations: Linear difference equation with constant coefficients and methods of solving them.

Unit III: Numerical Integration (15 Lectures)
Derivation of a formula for numerical integration in terms of finite difference and its special cases viz Trapezoidal, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule. Boole's and Weddle's rule, Guass Legendre numerical integration, Gauss-Chebyshev numerical integration, Gauss-Hermite numerical integration, Gauss-Laguree numerical integration with the derivation of all methods using the method of undetermined coefficients. Romberg's method. Gaussian quadratures, Multiple integrals.

Unit IV: Numerical solutions of Partial Differential Equations (15 Lectures)
Classifications of Partial Differential Equations, Finite Difference approximations to derivatives. Numerical methods of solving elliptic, parabolic and hyperbolic equations.

Recommended Text Books

1. H. M. Antia, Numerical Analysis, Hindustan Publictions.
2. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and sons, 2008.

3. Jain, Iyengar, Numerical Methods for Scientific and Engineering Problems, New Age International, 2009.

4. Erwin Kreyszig, Advanced Engineering Mathematics, Wiley John Wiley & Sons, 1999.

5. S.S. Sastry, Introductory Methods of Numerical Analysis, Prentice-Hall, India, 2012.

8. Graph Theory

Course Outcomes: Students should know the following:

1. Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem. Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa' \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

2. Cut vertices, Cut edges, Bond.

3. Trees, Characterizations of Trees, Spanning trees, Fundamental cycles.

4. Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

5. Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness.

6. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-Hamiltonian graph, Traveling salesman problem.

7. Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Hall's theorem (Necessary and sufficient condition for complete Matching), Konig's theorem (Maximum matching is same as minimum vettex cover), Tutte's theorem, Personal assignment problem,

8. Independent sets and covering- $\alpha + \beta = p$, Gallai's theorem.

9. Ramsey theorem-Existence of $r(k, l)$, Upper bounds of $r(k, l)$, Lower bound for $r(k, l) \geq 2m/2$ where $m = \min\{k, l\}$, Generalize Ramsey numbers-$r(k_1, k_2, \ldots, k_n)$, Graph Ramsey Theorem, Evaluation of $r(G, H)$ when for simple graphs $G = P_3, H = C_4$.

Unit I. Connectivity(15 Lectures)
Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa' \leq \delta$; Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

Unit II. Trees (15 Lectures)
Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula, Connector problem-Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)
Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese post- man problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisa- tion, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

Unit IV. Matching and Ramsey Theory (15 Lectures)
Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Halls theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering-$\alpha + \beta = p$; Gallai's theorem, Ramsey theorem-Existence of $r(k; l)$; Upper and Lower bounds of $r(k; l)$. $r(k; l) \geq 2^{m/2}$ where $m = \min\{k, l\}$; Generalize Ramsey numbers- $r(k_1, k_2, \ldots, k_n)$; Graph Ramsey theorem, Evaluation of $r(G; H)$ when for simple graphs $G = P_3; H = C_4$:

Recommended Text Books:

1. M. Behzad and A. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.

2. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier.

3. J. A. Bondy and U.S. R. Murty, Graph Theory, GTM 244 Springer, 2008.

4. Reinhard Diestel, Graph Theory GTM 173, 5th edition, Springer.

5. K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.

6. D.B.West, Introduction to Graph Theory, PHI, 2009.

9. Coding Theory

Course Outcomes: Students should know the following:

1. Error detection, Correction and Decoding. Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

2. Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbor decoding for linear codes, Syndrome decoding.

3. Definition of cyclic codes, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

4. Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

Unit I. Error detection, Correction and Decoding (15 Lectures)
Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding. Relation between minimum distance of a code and the error-detecting and error-correcting capabilities of the code.

Unit II. Linear codes (15 Lectures)
Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbour decoding for linear codes, Syndrome decoding.

Unit III. Bonds in coding theory (15 Lectures)
Sphere-covering bound, Gilbert–Varshamov bound, Hamming bound and perfect codes, Binary Hamming codes, q-ary Hamming codes, Golay codes, Singleton bound and MDS codes, Plotkin bound, Nonlinear codes, Hadamard matrix codes, Nordstrom–Robinson code, Preparata codes, Kerdock codes, Griesmer bound, Linear programming bound.

Unit IV. Constructions of codes (15 Lectures)
Propagation rules, Reed–Muller codes, Subfield codes, Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes. Alternant codes Goppa codes, Reed-Solomon codes. Quadratic-residue codes, Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Hamming code and simplex code.

Recommended Text Books:

1. S. R. Ghorpade, Aspects of coding theory, Lecture Notes IITB.
2. San Ling and Chaoing xing, Coding Theory- A First Course, Cambridge University Press, 2004.
3. Lid and Pilz, Applied Abstract Algebra, 2nd Edition, Springer, 1984.

10. Design Theory

Course Outcomes: Students should know the following:

1. Balanced Incomplete Block Designs, Basic Definitions and Properties, I incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.
2. Symmetric BIBDs An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.
3. Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.
4. Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Matrices and Hadamard Matrices, A Product Construction, Williamsons Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

Unit I. Introduction to Balanced Incomplete Block Designs (15 Lectures)
What Is Design Theory? Basic Definitions and Properties, Incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.

Unit II. Symmetric BIBDs (15 Lectures)
An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

Unit III. Difference Sets and Automorphisms (15 Lectures)
Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

Unit IV. Hadamard Matrices and Designs (15 Lectures)
Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Ma- trices and Hadamard Matrices, A Product Construction, Williamson's Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Ma- trices, Bent Functions.

Recommended Text Books:

1. T. Beth, D. Jungnickel and H. Lenz, Design Theory, Volume 1 (Second Edition), Cambridge University Press, Cambridge, 1999.
2. D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cambridge, 1985.
3. D. R. Stinson, Combinatorial Designs: Constructions and Analysis, Springer,2004.
4. W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall.

Semester-IV

PSMT401/PAMT401: Algebra IV

Course Outcomes:

1. Students will learn about algebraic extensions and their properties.
2. Splitting fields and their degrees can be computed. The notion of normal extension is introduced and its equivalent properties are discussed.
3. Finite fields as splitting fields are visualized and notion of algebraic closure is discussed in detail.
4. Galois extensions are studied and the fundamental theorem of Galois theory is established.
5. Cyclotomic extensions are studied in detail and order of its Galois group is computed.
6. Examples of fixed fields, field automorphisms and fundamental theorem are studied in special cases.

Unit I. Algebraic Extensions (15 lectures)
Prime subfield of a field, definition of field extension K/F, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element. Algebraic extensions, Finite extensions, degree of an algebraic element, degree of a field extension. If α is algebraic over the filed F and $m_\alpha(x)$ is the minimum polynomial of α over F, then $F(\alpha)$ is isomorphic to $F[X]/(m_\alpha(x))$. If $F \subseteq K \subseteq L$ are fields, then $[L : F] = [L : K][K : F]$. If K/F is a field extension, then the collection of all elements of K which are algebraic over F is a subfield of K. If L/K, K/F are algebraic extensions, then so is L/F. Composite filed K_1K_2 of two subfields of a field and examples. Classical Straight-edge and Compass constructions: definition of Constructible points, lines, circles by Straight-edge and Compass starting with $(0, 0)$ and $(1, 0)$, definition of constructible real numbers. If $a \in \mathbb{R}$ is constructible, then a is an algebraic number and its degree over $\mathbb{Q}$ is a power of 2. $\cos 20^0$ is not a constructible number. The regular 7-gon is not constructible. The regular 17-gon is constructible. The Constructible numbers form a subfield of $\mathbb{R}$. If $a > 0$ is constructible, then so is $\sqrt{a}$. Impossibility of the classical Greek problems: 1) Doubling a Cube, 2) Trisecting an Angle, 3) Squaring the Circle is possible.

Unit II. Normal and Separable Extensions (15 lectures)
Splitting field for a set of polynomials, normal extension, examples such of splitting fields of $x^p - 1$ (p prime), uniqueness of splitting fields, existence and uniqueness of finite fields. Algebraic closure of a field, existence of algebraic closure. Separable elements, Separable extensions. Perfect Fields. Frobenius automorphism of a finite field. Every irreducible polynomial over a finite field is separable. Primitive element theorem.

Unit III. Galois Theory(15 Lectures)
Galois group $G(K/F)$ of a field extension K/F, Galois extensions, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory.

Unit IV. Applications (15 Lectures)
Cyclotomic field $Q(\zeta_n)$ (splitting field of $x^n - 1$ over $\mathbb{Q}$), Cyclotomic polynomial, degree of Cyclotomic field $Q(\zeta_n)$. Galois group for an irreducible cubic polynomial, Galois group for an irreducible quartic polynomial. Solvability by radicals in terms of Galois group and Abel's theorem on the insolvability of a general quintic polynomial.

Rcommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.

2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons, 2011.

3. C. R. Handlock, Field Theory and its classical problems, Cambridge University Press, 2000.

4. Ireland K. & Rosen M., A Classical Introduction to Modern Number Theory, Springer, 1990.

5. N. Jacobson, Basic Algebra, Dover, 1985.

6. S. Lang, Algebra, Springer Verlag, 2004

PSMT402/PAMT402: Fourier Analysis

Course Outcomes:

1. Students will be able to understand the Fourier series expansion of a periodic function and their convergence.

2. Students will be able to grasp properties of the Dirichlet kernel, Fejer kernel, Poisson kernel and the concept of a good kernel.

3. Students will aware about application of a Fourier series in the solution of the Dirichlet problem and heat equation.

Unit I: Fourier series (15 Lectures)
The Fourier series of a periodic function, Bessel's inequality for a 2π periodic Riemann integrable function, Dirichlet kernel, Convergence theorem for the Fourier series of a 2π periodic and piecewise smooth function, Uniqueness theorem. Derivatives, Integrals and Uniform Convergence properties, Fourier series on intervals, Even and odd extensions, Fourier series of a periodic function of an arbitrary period.

Unit II: Dirichlet's theorem (15 Lectures)
Review: Lebesgue measure of $\mathbb{R}$; Lebesgue integrable functions, Dominated Convergence theorem, Bounded linear maps (no questions be asked). Fourier coefficients of integrable and square integrable periodic functions, The Riemann-Lebesgue lemma and its converse, Bessel's inequality for a L^2 periodic functions, Dirichlet's theorem, Concept of Good kernels, Dirichlet's kernel is not good kernel.

Unit III: Fejer's Theorem and applications (15 Lectures)
Cesaro summability, Cesaro mean and Cesaro sum of the Fourier series, Fejer's Kernel, Fejer's kernel is a good kernel, Fejer's Theorem, Parseval's identity. Convergence of Fourier series of an L^2 periodic function w.r.t the L^2-norm, Riesz-Fischer theorem on Unitary isomorphism from $L^2(-\pi, \pi)$ onto the sequence space l^2 of square summable complex sequences.

Unit IV: Dirichlet Problem in the unit disc(15 Lectures)
Abel summability, Abel sum of the Fourier series, The Poisson kernel, The Poisson kernel is a good kernel, Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The solution of Dirichlet problem for the unit disc.The Poisson integral, Applications of Fourier series to heat equation on the circle.

Recommended Text Books:

1. R. Beals, Analysis An Introduction,Cambridge University Press, 2004

2. R. Bhatia, Fourier Series, MAA Press AMS, 2005.

3. G.B. Folland, Fourier Analysis and its Applications, American Mathematical Society, Indian Edition 2010.

4. E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003.

5. E. M. Stein and R. Shakarchi, Real Analysis an Introduction, New age International.

PSMT 403/PAMT 403: Calculus on Manifolds

Course Outcomes:

1. Students will be able to grasp the concept of tensor, alternating tensor, wedge product and differential forms.
2. Students will be able to understand fields and forms on manifolds.
3. Students will be able to understand the application of Classical theorems: Stoke's theorem, Green's theorem, Gauss divergence theorem.

Unit I: Multilinear Algebra (15 Lectures)
Multilinear map on a finite dimensional vector space V over $\mathbb{R}$ and $k-$ tensors on V, the collection $\tau^k(V)$ (or $\otimes^k(V^*)$) of all $k-$ tensors on V, tensor product $S \otimes T$ of $S \in \tau^k(V)$ and $T \in \tau^k(V)$. Alternating tensor and the collection $\wedge^k V^*$ of $k-$tensors on V. The exterior product (or wedge product), basis of $\wedge^k V^*$, orientation of a finite dimensional vector space V over $\mathbb{R}$.

Unit II: Differential Forms (15 Lectures)
Differential forms: $k-$forms on $\mathbb{R}^n$, wedge product $\omega \wedge \eta$ of a $k-$ form ω and $l-$ forms η, the exterior derivative and its properties, Pull back forms and its properties, closed and exact forms, Poincare's lemma.

Unit III: Basics of Submanifolds of $\mathbb{R}^n$ (15 Lectures)
Submanifolds of $\mathbb{R}^n$, submanifolds of $\mathbb{R}^n$ with boundary, Smooth functions defined on Submanifolds of $\mathbb{R}^n$, Tangent vector and Tangent space of Submanifolds of $\mathbb{R}^n$. $p-$ forms and differential $p-$forms on a submanifolds of $\mathbb{R}^n$, exterior derivative $d\omega$ of any differential $p-$forms on a submanifolds of $\mathbb{R}^n$, Orientable submanifolds of $\mathbb{R}^n$ and Oriented submanifolds of $\mathbb{R}^n$, Orientation preserving map, Vector fields on submanifolds of $\mathbb{R}^n$, outward unit normal on the boundary of a submanifolds of $\mathbb{R}^n$ with non-empty boundary, induced orientation of the boundary of an oriented submanifolds of $\mathbb{R}^n$ with non-empty boundary.

Unit IV: Stoke's Theorem (15 Lectures)
Integral $\int_{[0,1]^k} \omega$ of a $k-$form on cube $[0,1]^k$, Integral $\int_c \omega$ of a $k-$ form on an open subset A of $\mathbb{R}^k$ where c is a singular $k-$ cube in A, Theorem (Stoke's Theorem for $k-$ cube): If ω is $k-1$ form on an open subset A of $\mathbb{R}^k$ and c is a singular $k-$ cube in A then $\int_c d\omega = \int \partial c\omega$.

Integration of a differentiable $k-$ form on oriented k dimensional submanifolds M of $\mathbb{R}^n$: Change of variables theorem: If $c_1, c_2 : [0,1]^k \longrightarrow M$ are two Orientation preserving maps in M and ω is any $k-$ form on M such that $\omega = 0$ outside of $c_1([0,1]^k) \cap c_2([0,1]^k)$ then $\int_{c_1} \omega = \int_{c_2} \omega$, Stokes' theorem for submanifolds of $\mathbb{R}^k$, Volume element, Integration of functions on a submanifold of $\mathbb{R}^k$, Classical theorems: Green's theorem, Divergence theorem of Gauss, Green's identities.

Recommended Text Books:

1. A. Browder, Mathematical Analysis, Springer International Edition, 1996.

2. V. Guillemin and A. Pollack, Differential Topology, AMS Chelsea Publishing, 2010.

3. J. Munkers, Analysis on Manifolds, Addision Wesley, 1997.

4. M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc., 1965.

PSMT404/PAMT404 ELECTIVE COURSE
Elective Course will be any ONE of the following Skill Courses:

Skill Course
Skill Course I: Business Statistics

Course Outcomes: Students should know the following:

1. Classification of Data: Requisites of Ideal Classification, Basis of Classification.

2. Organizing Data Using Data Array: Frequency Distribution, Methods of Data Classification, Bivariate Frequency Distribution, Types of Frequency Distributions.

3. Tabulation of Data: Objectives of Tabulation, Parts of a Table, Types of Tables, General and Summary Tables, Original and Derived Tables.

4. Graphical Presentation of Data: Functions of a Graph, Advantages and Limitations of Diagrams(Graph), General Rules for Drawing Diagrams.

5. Types of Diagrams: One-Dimensional Diagrams, Two-Dimensional Diagrams, Three-Dimensional Diagrams, Pictograms or Ideographs, Cartograms or Statistical Maps.

6. Exploratory Data Analysis: Stem-and-Leaf Displays.

7. Objectives of Averaging, Requisites of a Measure of Central Tendency, Measures of Central Tendency,

8. Mathematical Averages: Arithmetic Mean of Ungrouped Data, Arithmetic Mean of Grouped (Or Classified) Data, Some Special Types of Problems and Their Solutions, Advantages and Disadvantages of Arithmetic Mean, Weighted Arithmetic Mean.

9. Geometric Mean: Combined Geometric Mean, Weighted Geometric Mean, Advantages, Disadvantages and Applications of G.m.

10. Harmonic Mean: Advantages, Disadvantages and Applications of H.M. Relationship Between A.M., G.M. and H.M.

11. Averages of Position: Median, Advantages, Disadvantages and Applications of Median.

12. Partition Values quartiles, Deciles and Percentiles: Graphical Method for Calculating Partition Values.

13. Mode: Graphical Method for Calculating Mode Value. Advantages and Disadvantages of Mode Value.

14. Relationship Between Mean, Median and Mode, Comparison Between Measures of Central Tendency.

15. Significance of Measuring Dispersion (Variation):Essential Requisites for a Measure of Variation.

16. Classification of Measures of Dispersion.

17. Distance Measures: Range, Interquartile Range or Deviation.

18. Average Deviation Measures: Mean Absolute Deviation, Variance and Standard Deviation,

19. Mathematical Properties of Standard Deviation, Chebyshev's Theorem, Coefficient of Variation.

20. Measures of Skewness: Relative Measures of Skewness.

21. Moments: Moments About Mean, Moments About Arbitrary Point, Moments About Zero or Origin, Relationship Between Central Moments and Moments About Any Arbitrary Point, Moments in Standard Units, Sheppard's Corrections for Moments.

22. Kurtosis: Measures of Kurtosis.

Unit I. Data Classification, Tabulation and Presentation (15 Lectures)
Classification of Data: Requisites of Ideal Classification, Basis of Classification. Organizing Data Using Data Array: Frequency Distribution, Methods of Data Classification, Bi-variate Frequency Distribution, Types of Frequency Distributions. Tabulation of Data: Objectives of Tabulation, Parts of a Table, Types of Tables, General and Summary Tables, Original and Derived Tables. Graphical Presentation of Data: Functions of a Graph, Advantages and Limitations of Diagrams (Graph), General Rules for Drawing Diagrams. Types of Diagrams: One-Dimensional Diagrams, Two-Dimensional Diagrams, Three-Dimensional Diagrams, Pictograms or Ideographs, Cartograms or Statistical Maps. Exploratory Data Analysis: Stem-and-Leaf Displays.

Unit II. Measures of Central Tendency (15 Lectures)
Objectives of Averaging, Requisites of a Measure of Central Tendency, Measures of Central Tendency. Mathematical Averages: Arithmetic Mean of Ungrouped Data, Arithmetic Mean of Grouped (or classified) Data, Some Special Types of Problems and Their Solutions, Advantages and Disadvantages of Arithmetic Mean, Weighted Arithmetic Mean. Geometric Mean: Combined Geometric Mean, Weighted Geometric Mean, Advantages, Disadvantages and Applications of G.m. Harmonic Mean: Advantages, Disadvantages and Applications of H.M. Relationship Between A.M., G.M. and H.M. Averages of Position: Median, Advantages, Disadvantages and Applications of Median. Partition Values quartiles, Deciles and Percentiles: Graphical Method for Calculating Partition Values. Mode: Graphical Method for Calculating Mode Value. Advantages and Disadvantages of Mode Value. Relationship Between Mean, Median and Mode, Comparison Between Measures of Central Tendency.

Unit III. Measures of Dispersion (15 Lectures)
Significance of Measuring Dispersion (Variation):Essential Requisites for a Measure of Variation. Classification of Measures of Dispersion. Distance Measures: Range, Interquartile Range or Deviation. Average Deviation Measures: Mean Absolute Deviation, Variance and Standard Deviation, Mathematical Properties of Standard Deviation, Chebyshev's Theorem, Coefficient of Variation.

Unit IV. Skewness, Moments and Kurtosis (15 Lectures)
Measures of Skewness: Relative Measures of Skewness. Moments: Moments About Mean, Moments About Arbitrary Point, Moments About Zero or Origin, Relationship Between Central Moments and Moments About Any Arbitrary Point, Moments in Standard Units, Sheppard's Corrections for Moments. Kurtosis: Measures of Kurtosis.

Practicals

Four practicals, one on each unit on realistic data and its analysis should be conducted with the help of R-statistical package.

Reference Book: J. K. Sharma , Business Statistics, Pearson Education India, 2012.

Skill Course II: Statistical Methods

Course Outcomes: Students should know the following:

1. Measures of central tendencies: Mean, Median, Mode.
2. Measures of Dispersion: Range, Mean deviation, Standard deviation. Measures of skewness. Measures of relationship: Covariance, Karl Pearson's coefficient of Correlation, Rank Correlation. Basics of Probability.
3. Sampling Distribution, Student's t-Distribution, Chi-square (χ^2) Distribution, Snedecor's F-Distribution. Standard Error. Central Limit theorem. Type I and Type II Errors, Critical Regions. F-test, t-test, χ^2 test, goodness of Fit test.
4. The Anova Technique. The basic Principle of Anova. One Way ANOVA, Two Way ANOVA. Latin square design. Analysis of Co-variance.
5. R as Statistical software and language, methods of Data input, Data accessing, usefull built-in functions, Graphics with R, Saving, storing and retrieving work.

Unit I. Basic notions of Statistics (15 Lectures)
Measures of central tendencies: Mean, Median, Mode. Measures of Dispersion: Range, Mean deviation, Standard deviation. Measures of skewness. Measures of relationship: Covariance, Karl Pearson's coefficient of Correlation, Rank Correlation. Basics of Probability.

Unit II. Sampling and Testing of Hypothesis (15 Lectures)
Sampling Distribution, Student's t-Distribution, Chi-square (χ^2) Distribution, Snedecor's F- Distribution. Standard Error. Central Limit theorem. Type I and Type II Errors, Critical Regions. F-test, t-test, χ^2 test, goodness of Fit test. Testing of Hypothesis.

Unit III. Analysis of Variance (15 Lectures)
The Anova Technique. The basic Principle of Anova. One Way ANOVA, Two Way ANOVA. Latin square design. Analysis of Co-variance.

Unit IV. Use of package R (15 Lectures)
R as Statistical software and language, methods of Data input, Data accessing, useful built-in functions, Graphics with R, Saving, storing and retrieving work.

Practicals

Four practicals, one on each unit on realistic data and its analysis should be conducted with the help of R-statistical package.

Recommended Text Books:

1. S. C. Gupta And V. K. Kapoor, Fundamentals Of Mathematical Statistics, Sultan Chand & Sons, 1994.

2. C. R. Kothari and G. Garg, Research Methodology Methods and Techniques, New Age International, 2019.

3. S. G. Purohit, S.D. Gore and S.R. Deshmukh, Statistics using R, Alpha Science Int., 2008.

Skill Course III: Computer Science

Aim: Mathematics students are well versed in logic. This Skill course aims at giving input of necessary skills of algorithms and data structures and relational database background so that the students are found suitable to be absorbed as trainee software professional in industry. Prerequisite for this course: Good knowledge of C, C++ or java or python.

Course Outcomes: Students should know the following:

1. Basics of object oriented programming principles, templates, reference operators NEW and delete in C++, the java innovation which avoids use of delete, classes polymorphism friend functions, inheritance, multiple inheritance operator overloading basics

2. Basic algorithms, selection sort, quick sort, heap sort, priory queses, radix sort, merge sort, dynamic programming, app pairs, shortest paths, image compression, topological sorting, single source shortest paths reference, hashing intuitive evaluation of running time.

3. Stacks queues, linked lists implementation and simple applications, trees implementation and tree traversal (stress on binary trees),

4. Concept of relational databases , normal forms BCNF and third normal forms. Armstrongs axioms. Relational algebra and operations in it.

Unit I. OOPS Concepts (15 lectures)
Basics of object oriented programming principles, templates, reference operators NEW and delete in C++, the java innovation which avoids use of delete, classes polymorphism friend functions, inheritance, multiple inheritance operator overloading basics only.

Unit II. Basic Algorithms (15 lectures)
Basic algorithms, selection sort, quick sort, heap sort, priory queues, radix sort, merge sort, dynamic programming, app pairs, shortest paths, image compression, topological sorting, single source shortest paths reference, hashing intuitive evaluation of running time.

Unit III. Data Structures (15 lectures)
Stacks queues, linked lists implementation and simple applications, trees implementation and tree traversal (stress on binary trees),

Unit IV. Relational Databases (15 lectures)
concept of relational databases, normal forms BCNF and third normal forms. Armstrong's axioms. Relational algebra and operations in it.

Recommended Text Books:

1. T. Aron and others, Data structure using C, Pratibha Publications, 2020.
2. Balaguruswamy, Programming in C++, Tata McGraw Hill, 2008
3. S. Sahani, Data Structures and applications, TMH.
4. Programming in Java, Schaum Series.
5. J.D. Ullam, Principles of Database systems, Computer Science Press, 1982.

Skill Course IV: Linear and Non-linear Programming

Course Outcomes:

1. Understand the concept of an objective function, a feasible region, and a solution set of an optimization problem.
2. Understand the broad classification of optimization problems, and where they arise in simple applications.
3. Use the simplex method to find an optimal vector for the standard linear programming problem and the corresponding dual problem.
4. Use Lagrange multipliers to solve nonlinear optimization problems.
5. Write down and apply Kuhn-Tucker conditions for constrained nonlinear optimization problems.
6. Apply approximate methods for constraint problems.
7. Understand the importance of convexity in nonlinear optimization prob- lems.
8. Apply basic line search methods to one-dimensional optimization problems, gradient methods to optimization problems, ,conjugate gradient methods to optimization problems

Unit I. Linear Programming (15 Lectures)
Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

Unit II.Transportation Problems(15 Lectures)
Transportation and Assignment problems of linear programming, Sequencing theory and Travelling salesperson problem.

Unit III. Unconstrained Optimization(15 Lectures)
First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newtons Method, Secant Method, Gradient Methods, Steep-est Descent Methods.

Unit IV. Constrained Optimization Problems(15 Lectures)
Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

Recommended Text Books:

1. E. K. P. Chong and S. H. Zak, Introduction to Optimization, Wiley-Int., 1996.
2. F. S. Hillier and G.J. Lieberman, Introduction to Operations Research (Sixth Edition), McGraw Hill, 1990.
3. G. Hadley, Linear Programming, Narosa Publishing House, 1995.
4. S. S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd, New Delhi, 1984.
5. Rangarajan and K. Sundaram, A First Course in Optimization Theory, Cambridge University Press, 1996.
6. K. Swarup, P. K. Gupta and Man Mohan, Operations Research, S. Chand andsons, New Delhi, 2010.
7. H. A. Taha, Operations Research-An introduction, Macmillan Publishing Co. Inc., 1997.

Skill Course V: Computational Algebra

Course Outcomes:

1. Students will learn to balance between theory and practicals via the use of computers.

2. Previously learnt concepts can be strengthened by allowing them to explore topics using the mathematical softwares.

Unit I. Representation Theory (15 lectures)
Linear representations of a finite group on a finite dimensional vector space over $\mathbb{C}$. If ρ is a representation of a finite group G on a complex vector space V, then there exits a G-invariant positive definite Hermitian inner product on V. Complete reducibility (Maschke's theorem). The space of class functions, Characters and Orthogonality relations. For a finite group G, there are finitely many isomorphism classes of irreducible representations, the same number as the number of conjugacy classes in G. Two representations having same character are isomorphic. Regular representation. Schur's lemma and proof of the Orthogonality relations. Every irreducible representation over $\mathbb{C}$ of a finite Abelian group is one dimensional. Character tables with emphasis on examples of groups of small order.

Unit II. Group theory software (15 lectures)
Introduction to Sage Math and GAP softwares. Permutation groups, examples, Groups with generators, center of a group, derived series examples, Character tables, Matrices over finite fields.

Unit III. Ideals, Varieties and Algorithms (15 lectures)
Polynomials in one variable, Affine spaces, Parameterizations of Affine varieties, Polynomial rings in more variables, Dickson's lemma, Hilbert basis theorem, Basics of invariant theory, Groebner basis, Buchberger Algorithm and Applications.

Unit IV. Commutative Algebra software (15 lectures)
Introduction to Singular and Macaulay. Polynomials in more than two variables over fields, quotient rings, localizations, Groebner bases.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.

2. David A. Cox, John Little and Donal O'Shea, Ideals, Varieties, and Algorithms, Springe, 2015.

3. S. Sternberg, Group theory and Physics, Cambridge University Press, 1994.

4. Bernd Sturmfeles, Algorithms in Invariant Theory, Springer, 2008.

PSMT405/PAMT405: PROJECT

Evaluation of Project work
The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the respective college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee.

Each project output shall be displayed on the website of the University.
The Marks for the project are detailed below:

1. Contents of the project: 50 Marks
2. Attendance: 10 Marks
3. Presentation of the project: 20 Marks
4. Viva of the project: 20 Marks
5. Total Marks= 100 Marks per project per student

Scheme of Examination
The scheme of examination for the syllabus of Semesters III & IV of M.A./M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows.

Scheme of Evaluation R8435 for M. Sc /M. A.–

1. A) 80: 20 for distance education (external evaluation of 80 marks and internal evaluation of 20 marks) under the choice based credit system (CBCS).
2. B) 60:40 for university affiliated PG centers (external evaluation of 60 marks and internal evaluation of 40 marks).
3. C) 100 percent internal evaluation scheme for University department of mathematics (One mid semester test of 30 marks, 05 marks for attendance, 05 marks for active participation and one end semester test of 60 marks, both tests will be conducted by the department and answer book will be shown to the students).

Duration:- Examination shall be of 60 Marks and $2\frac{1}{2}$ Hours duration.
Theory Question Paper Pattern for above Schemes:-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.

3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, ·· and the allocation of marks depend on the weight-age of the topic.
5. Each question will be of maximum 18 marks when marks of all the sub-questions are added (including the options) in that question.
6. For scheme A: 60 marks will be converted into 80 marks.

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