UM-DAE Centre for Excellence in Basic Sciences
Course Structure and Syllabus
5-year Integrated MSc - Mathematics Stream

P: Physics, M: Mathematics, C: Chemistry, B: Biology, G: General, H: Humanity,
ME: Math Elective, MPR : Math Project

April 25, 2014

Semester I

Subject Code	**Subject**	**Contact Hours per Week** Theory + Tutorials	**Credits**
B101	Biology I	[2+1]	3
C101	Chemistry I	[2+1]	3
M101/100	Mathematics I	[2+1]	3
P101	Physics I (Mechanics & Thermodynamics)	[2+1]	3
G101	Computer Basics	[2+1]	3
H101	Communication Skills	[2+1]	3
		Lab Hours per Week	
BL101	Biology Lab	[4]	2
CL101	Chemistry Lab	[4]	2
PL101	Physics Lab	[4]	2
GL101	Computer Lab	[4]	2
		Semester Credits	**26**
		Subtotal	**26**

Semester II

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
B201	Biology II	[2+1]	3
C201	Chemistry II	[2+1]	3
M201/200	Mathematics II	[2+1]	3
P201	Physics II (Optics, Electricirty & Magnetism)	[2+1]	3
G201	Electronics & Instrumentation	[2+1]	3
G202	Glimpses of Contemporary Science	[2+1]	3
		Lab Hours per Week	
BL201	Biology Lab	[4]	2
CL201	Chemistry Lab	[4]	2
PL201	Physics Lab	[4]	2
GL201	Electronics Lab	[4]	2
		Semester Credits	**26**
		Subtotal	**52**

M201 : Mathematics II (Calculus and Linear Algebra)

1. Recollection and rigorous treatment of continuity and differentiability of a function of one variable.
2. Riemann integration, proof of the Fundamental Theorem of Calculus.
3. Functions of two and three variables, double and triple integrals.
4. Line integrals.
5. Parametrized surfaces, oriented surfaces.
6. Stokes Theorem, Gauss Divergence Theorem (both without proof).
7. Recollection of the algebra of matrices (mainly over the field of real numbers, but mention other fields also), linear equations, row-echelon form, Gauss-Jordan elimination.
8. Determinants, rank of a matrix, rank and invertibility.
9. Vector spaces (mainly over the field of real numbers, but mention other fields also), span, linear independence, basis, dimension and its uniqueness (without proof).
10. Linear transformations, kernel and image, the rank-nullity formula.
11. Eigenvalues and eigenvectors of a square matrix or a linear operator.

References

[1] D.J.S. Robinson, A Course in Linear Algebra with Applications, World Scientific.

[2] G. B. Thomas and R.L. Finney, Calculus and Analytic Geometry, 9th ed., Addison-Wesley/Narosa, 1998.

[3] J. Marsden, A. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer

[4] Inder K. Rana, Calculus@iitb, Concepts and Examples, Version 1.2, math4all 2009.

Semester III

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M301	Foundations	[3+1]	4
M302	Analysis I	[3+1]	4
M303	Algebra I	[3+1]	4
M304	Discrete Mathematics	[3+1]	4
M305	Computational Mathematics I	[3+1]	4
H301	World Literature	[2+0]	2
H302	History and Philosophy of Science	[2+0]	2
		Lab Hours per Week	
GL301	Applied Electronics Lab	[4]	2
		Semester Credits	**26**
		Subtotal	**78**

M301 : Foundations

1. Logic : Quantifiers, negations, examples of various mathematical and non-mathematical statements. Exercises and examples.
2. Set Theory : Definitions, subsets, unions, intersections, complements, symmetric difference, De Morgan's laws for arbitrary collection of sets. Power set of a set.
3. Relations and maps :
   (i) Cartesian product of two sets. Relations between two sets. Examples of relations. Definition of a map, injective, surjective and bijective maps. A map is invertible if and only if it is bijective.
   (ii) Inverse image of a set with respect to a map. Relation between inverse images and set theoretic operations. Equivalence relations (with lots of examples).
4. Schroeder-Bernstein theorem.
5. Finite and Infinite sets :
   (i) Finite sets, maps between finite sets, proof that number of elements in a finite set is well-defined.
   (ii) Definition of a countable set (inclusive of a finite set). Countably infinite and uncountable sets. Examples.
   (iii) Proof that every infinite set has a proper, countably infinite subset.
   (iv) Uncountability of $\mathcal{P}(\mathbb{N})$.
6. Partially Ordered Sets :
   (i) Concept of partial order, total order, examples.
   (ii) Chains, Zorn's Lemma.
7. (i) Peano's Axioms.
   (ii) Well-Ordering Principle.
   (iii) Weak and Strong Principles of Mathematical Induction.
   (iv) Transfinite Induction.
   (v) Axiom of Choice, product of an arbitrary family of sets.
   (vi) Equivalence of Axiom of Choice, Zorn's Lemma and Well-ordering principle.
8. Additional Topics (Optional)
   (i) Dedekind's Construction of Real Numbers.
   (ii) Decimal, dyadic, triadic expansions of real numbers.
   (iii) Cantor Sets.

References

[1] Naive Set Theory, P. Halmos.

[2] Set Theory and Logic, R. Stoll.

A lot of the material can be found in the beginning sections of the following books:

[3] Methods of Real Analysis, R. Goldberg.

[4] Topology, J. Munkres.

[5] Elementary Number Theory, D. Burton.

[6] Real Analysis, Bartle and Sherbert.

M302 : Analysis I

1. Real Number System : Concept of a field, ordered field, examples of ordered fields, supremum, infimum. Order completeness of $\mathbb{R}$, $\mathbb{Q}$ is not order complete. Absolute values, Archimedean property of $\mathbb{R}$. $\mathbb{C}$ as a field, and the fact that $\mathbb{C}$ cannot be made into an ordered field. Denseness of $\mathbb{Q}$ in $\mathbb{R}$. Every positive real number has a unique positive n-th root.

2. Sequences : Sequences, limit of a sequence, basic properties like $lim_n(x_ny_n) = (lim_nx_n)(lim_ny_n)$.Bounded sequences, monotone sequences, a monotone increasing sequence bounded above converges to it's supremum. Sandwich theorem and its applications. Using the Arithmetic mean-Geometric mean inequality to prove results like $lim_n\left(1+\frac{1}{n}\right)^n$ and $lim_n\left(1-\frac{1}{n}\right)^n$ and are equal, $lim_n\sqrt[n]{n} = 1$ and $lim_na^{\frac{1}{n}} = 1$. Cauchy's first limit theorem, Cauchy's second theorem.

3. Subsequences and Cauchy sequences : Every sequence of real numbers has a monotone subsequence. Definition of a Cauchy sequence. Cauchy completeness of $\mathbb{R}$, $\mathbb{Q}$ is not Cauchy complete.

4. Infinite Series : Basic notions on the convergence of infinite series. Absolute and conditional convergence. Comparison test, ratio test, root test, alternating series test, Theorem of Dirichlet, Statement of Riemann's rearrangement theorem, Cauchy product of two series. Power series, radius of convergence via examples.

5. Continuous functions : Continuity, sequential and neighbourhood definitions, basic properties such as sums and products of continuous functions are continuous. Intermediate Value Theorem, Continuous functions on closed and bounded intervals, Monotone continuous functions, inverse functions, Uniform Continuity, examples and counter-examples.

6. Differentiable functions : Definition : as a function infinitesimally approximable by a linear map, equivalence with Newton ratio definition, basic properties. One-sided derivatives, The O, o and $\sim$ notations with illustrative examples. Chain rule with complete proof (using above definition). Local monotonicity, relation between the sign of f' and local monotonicity. Proofs of Rolle's theorem and the Cauchy-Lagrange Mean value theorem. L'Hospital's rule and applications. Higher derivatives and Taylor's theorem, estimation of the remainder in Taylor's theorem, example : $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Convex functions.

7. Riemann Integration : Definition via upper and lower Riemann sums, basic properties. Riemann integrability, Thm : $f : [a,b] \longrightarrow \mathbb{R}$ continuous implies f is Riemann integrable, examples of Riemann integrable functions which are not continuous on $[a,b]$. Thm : if $f : [a,b] \longrightarrow \mathbb{R}$ is Riemann integrable then so is $|f|$ and $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$. Cauchy-Schwartz inequality : $\left|\int fg\right| \leq \sqrt{\int f^2}\sqrt{\int g^2}$, $\left|\int fg\right| \leq (\int f^p)^{\frac{1}{p}}(\int g^q)^{\frac{1}{q}}$ where $\frac{1}{p}+\frac{1}{q} = 1$. Mean value theorem for integrals.

8. Improper integrals, power series and elementary functions : Cauchy's condition for existence of improper integrals, test for convergence. Examples : $\int \frac{\sin x dx}{x}$, $\int \cos x^2 dx$, $\int \sin x^2 dx$. Power series and basic properties, continuity of the sum, validity of term by term differentiation. Binomial theorem for arbitrary real coefficients. Elementary transcendental functions e^x, $\sin x$, $\cos x$ and their inverse functions, $\log x$, $\tan^{-1} x$, Gudermannian and other examples.

References

[1] Introduction to Real Analysis : R. Bartle & D. Sherbert, Wiley.

[2] A First Course in Analysis : G. Pedrick

M303 : Algebra I (Groups, rings, fields)

1. Recollection of equivalence relations and equivalence classes, congruence classes of integers modulo n.
2. Definition of a group, examples including matrices, permutation groups, groups of symmetry, roots of unity.
3. First properties of a group, laws of exponents, finite and infinite groups.
4. Subgroups and cosets, order of an element, Lagrange theorem, normal subgroups, quotient groups.
5. Detailed look at the group S_n of permutations, cycles and transpositions, even and odd permutations, the alternating group, simplicity of A_n for $n \geq 5$.
6. Homomorphisms, kernel, image, isomorphism, the fundamental theorem of group homomorphisms.
7. Cyclic groups, subgroups and quotients of cyclic groups, finite and infinite cyclic groups.
8. Cayleys theorem on representing a group as a permutation group.
9. Conjugacy classes, centre, class equation, centre of a p-group.
10. Sylow theorems, solvable and nilpotent groups.
11. Definition of a ring, examples including congruence classes modulo n, ideals and homomorphisms, quotient rings, polynomial ring in one variable over a ring, units, fields, non-zero divisors, integral domains.
12. Rings of fractions, field of fractions of an integral domain.
13. PID, unique factorization in the ring of integers and in the polynomial ring over a field, Gauss Lemma.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D.S. Dummit and R.M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, 1983.

[4] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

M 304 : Discrete Mathematics

1. Combinatorics: Permutations and combinations. Linear equations and their relation to distribution into boxes.
2. Distributions with repetitions and non-repetitions. Combinatorial derivation of these formulae. Pigeonhole Principle and applications.
3. Binomial and multinomial theorems. Inclusion-Exclusion Principle and Applications. Recurrence Relations and Generating Functions.

4. Partitions of a number. Number of partitions. Brief introduction to the combinatorics of Young tableaux.

5. Graph theory: Vertices and edges. Graphs and special types like complete graph, bipartite graph. Degree of a vertex, weighted graphs. Traveling Salesman's Problem. Koenigsberg Seven-bridge puzzle. Walks, Paths, Circuits.

6. Euler Graphs, Hamiltonian Paths and Circuits. Trees and algorithms to find trees in a given graph. Planar Graphs.

7. Spaning trees and cut sets. Minimal spanning trees and algorithms for their computer implementation: the Kruskal's algorithm.

8. Coloring in graph theory. The four colour problem.

References

[1] Richard Stanley, Enumerative Combinatorics.

[2] Alan Tucker, Applied Combinatorics.

[3] F. Harrary, Graph Theory.

[4] Narsingh Deo, Graph Theory.

M305 : Computational Mathematics I

1. Basics of Spreadsheet Programmes (such as Libreoffice/gnumeric).

2. Inroduction to Mathematica including writing simple programmes.

3. Detailed exploration of notion of calculus of one variable, and simple multivariable calculus using Mathematica.

4. Basic Linear Algebra Using Mathematica.

5. Numerical Solutions of Linear and Non-linear equations using Mathematica. Developing programmes for each of these methods.

References

[1] Selwyn Hollis, CalcLabs with Mathematica for Single Variable Calculus, Fifth Edition.

[2] Selwyn Hollis, CalcLabs with Mathematica for Multivariable Calculus, Fifth Edition.

[3] Kenneth Shiskowski, Karl Frinkle, Principles of Linear Algebra with Mathematica.

H301 : World Literature

1. What is literature? - A discussion; introduction to literary terms, genres, and forms of various periods, countries, languages, etc.

2. The novel: Class study of 'Brave New World' by Aldous Huxley; group discussions and student presentations on other genres such as the graphic novel, detective fiction, children's literature, etc.

3. Plays: Introduction to the history of theatre, class study of (mainly) two plays: 'Pygmalion' by G. B. Shaw and 'Fire and Rain' by Girish Karnad, the setting up of a play-reading group through which the students can be introduced to several other plays.

4. Poetry: Brief introduction, study of poetic genres, forms, topics, figures of speech, poetic language, etc. by analyzing various poems from around the world.
5. Short stories, essays, and other types of writing by various authors.
6. Screening of films based on literary works, such as Pygmalion (My Fair Lady), Fire and Rain (Agnivarsha), Persepolis (a graphic novel), and many others.

Semester IV

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M401	Analysis II	[3+1]	4
M402	Algebra II	[3+1]	4
M403	Elementary Number Theory	[3+1]	4
M404	Topology I	[3+1]	4
G401	Statistical Techniques and Applications	[3+1]	4
		Lab Hours per Week	
GL401	Computational Laboratory & Numerical Methods	[4]	2
		Semester Credits	**22**
		Subtotal	**100**

M401 : Analysis II (Multivariable Calculus)

1. Linear maps from $\mathbb{R}^n$ to $\mathbb{R}^m$, Directional derivative, partial derivative, total derivative, Jacobian, Mean value theorem and Taylors theorem for several variables, Chain Rule.
2. Parametrized surfaces, coordinate transformations, Inverse function theorem , Implicit function theorem, Rank theorem.
3. Critical points, maxima and minima, saddle points, Lagrange multiplier method.
4. Multiple integrals, Riemann and Darboux integrals, Iterated integrals, Improper integrals, Change of variables.
5. Integration on curves and surfaces, Greens theorem, Differential forms, Divergence, Stokes theorem.

References

[1] M. Spivak, Calculus on Manifolds.

[2] W. Fleming, Functions of Several Variables, 2nd Ed., Springer-Verlag, 1977.

[3] J.E.Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus.

[4] W. Rudin, Principles of Mathematical Analysis, 3rd ed., McGraw-Hill, 1984.

[5] A Modern Approach to Classical Theorems of Advanced Calculus, W. A. Benjamin, Inc., 1965.

M402 : Algebra II (Linear Algebra)

1. Modules over a commutative ring, submodules and quotient modules, generators, homomorphisms, exact sequences, finitely generated free modules.
2. Vector spaces as modules over a field, subspaces, quotient spaces.
3. Span and linear independence, basis, dimension.
4. Linear maps and their corrrespondence with matrices with respect to given bases, change of bases.
5. Eigenvalues, eigenvectors, eigenspaces, characteristic polynomial, Cayley-Hamilton.
6. Bilinear forms, inner product spaces, Gram-Schmidt process, diagonalization, spectral theorem.

Note: Jordan and rational canonical forms to be done in M602 in Semester VI as an application of the structure of finitely generated modules over a PID.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] K. Hoffman and R. Kunze, Linear Algebra, Prentice Hall, 1992.

[4] N. Jacobson, Basic Algebra II, Hindustan Publishing Corporation, 1983.

[5] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

M403 : Elementary Number theory

1. Fundamental theorem of arithmetic, divisibility in integers.
2. Prime numbers and infinitude of primes. Infinitude of primes of special types. Special primes like Fermat primes, Mersenne primes, Lucas primes etc.
3. Euclidean algorithm, greatest common divisor, least common multiple.
4. Equivalence relations and the notion of congruences. Wilson's theorem and Fermat's little theorem. Chinese remainder theorem.
5. Continued fractions and their applications.
6. Primitive roots, Euler's Phi function.
7. Sum of divisors and number of divisors, Möbius inversion.
8. Quadratic residues and non-residues with examples.
9. Euler's Criterion, Gauss' Lemma.
10. Quadratic reciprocity and applications.
11. Applications of quadratic reciprocity to calculation of symbols.
12. Legendre symbol: Definition and basic properties.
13. Fermat's two square theorem, Lagrange's four square theorem.
14. Pythagorean triples.
15. Diophantine equations and Bachet's equation. The duplication formula.

References

[1] D. Burton, Elementary Number Theory.

[2] Kenneth H. Rosen, Elementary number theory and its applications.

[3] Niven, Ivan M.; Zuckerman, Herbert S.; Montgomery, Hugh L, An Introduction to the Theory of Numbers.

M404 : Topology I

1. Recollection of some set theory, particularly the following topics:

   (i) Equipotence of sets, Schroeder-Bernstein theorem, countable and uncountable sets, countability of $\mathbb{Q}$ and uncountability of $\mathbb{R}$.

   (ii) Equivalence relations, Zorn's lemma, axiom of choice.

2. Metric spaces: Definition and basic examples including the following:

   (i) The discrete metric on any set.

   (ii) $\mathbb{R}$ and $\mathbb{R}^n$ with Euclidean metrics, Cauchy-Schwarz inequality, definition of a norm on a finite dimensional $\mathbb{R}$-vector space and the metric defined by a norm.

   (iii) The set $\mathcal{C}[0,1]$ with the metric given by $\sup|f(t) - g(t)|$ (resp. $\int_0^1 |f(t) - g(t)|dt$).

   (iv) Metric subspaces, examples.

3. Topology generated by a metric: Open and closed balls, open and closed sets, complement of an open (closed) set, arbitrary unions (intersections) of open (closed) sets, finite intersections (unions) of open (closed) sets, open (closed) ball is an open (closed) set, a set is open if and only if it is a union of open balls, Hausdorff property of a metric space.

4. Equivalence of metrics, examples, the metrics on $\mathbb{R}^2$ given by $|x_1 - y_1| + |x_2 - y_2|$ (resp. $\max\{|x_1 - y_1|, |x_2 - y_2|\}$) is equivalent to the Euclidean metric, the shapes of open balls under these metrics.

5. Limit points, isolated points, interior points, closure, interior and boundary of a set, dense and nowhere dense sets.

6. Continuous maps: ε-δ definition and characterization in terms of inverse images of open (resp. closed) sets, composite of continuous maps, pointwise sums and products of continuous maps into $\mathbb{R}$, homeomorphism, isometry, an isometry is a homeomorphism but not conversely, uniformly continuous maps, examples.

7. Complete metric spaces: Cauchy sequences and convergent sequences, a subspace of a complete metric space is complete if and only if it is closed, Cantor intersection theorem, Baire category theorem and its applications, completion of a metric space.

8. General topological spaces, stronger and weaker topologies, continuous maps, homeomorphisms, bases and subbases, finite products of topological spaces.

9. Compactness for general topological spaces: Finite subcoverings of open coverings and finite intersection property, continuous image of a compact set is compact, compactness and Hausdorff property.

10. Compactness for metric spaces: Bolzano-Weierstrass property, the Lebesgue number for an open covering, sequentially compact and totally bounded metric spaces, Heine-Borel theorem, compact subsets of $\mathbb{R}$, a continuous map from a compact metric space is uniformly continuous.

11. Connectedness: definition, continuous image of a connected set is connected, characterization in terms of continuous maps into the discrete space $\mathbb{N}$, connected subsets of $\mathbb{R}$, intermediate value theorem as a corollary, countable (arbitrary) union of connected sets, connected components,

References

[1] E. T. Copson, *Metric spaces.*

[2] M. Eisenberg, *Topology.*

[3] R.H. Kasriel, *Undergraduate topology.*

[4] W. Rudin, *Principles of mathematical analysis.*

[5] G. F. Simmons, *Topology and modern analysis.*

[6] W. A. Sutherland, *Introduction to metric and topological spaces.*

Semester V

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M501	Analysis III	[3+1]	4
M502	Algebra III	[3+1]	4
M503	Topology II	[3+1]	4
M504	Probability Theory	[3+1]	4
G501	Earth Science & Energy & Environmental Sciences	[3+1]	4
PM501	Numerical Analysis	[3+1]	4
		Lab Hours per Week	
PML501	Numerical Methods Laboratory	[4]	2
		Semester Credits	**26**
		Subtotal	**126**

M501 : Analysis III (Measure Theory and Integration)

1. Sigma algebra of sets, measure spaces. Lebesgues outer measure on the Real line. Measurable set in the sense of Caratheodory. Translation invariance of Lebesgue measure. Existence of a non-Lebesgue measurable set. Cantor set- uncountable set with measure zero.
2. Measurable functions, types of convergence of measurable functions. The Lebesgue integral for simple functions, nonnegative measurable functions and Lebesgue integrable function, in general.
3. Convergence theorems- monotone and dominated convergence theorems.
4. Comparison of Riemann and Lebesgue integrals. Riemanns theorem on functions which are continuous almost everywhere.
5. The product measure and Fubinis theorem.
6. The L^p spaces and the norm topology. Inequalities of Hölder and Minkowski. Completeness of L^p and L^∞ spaces.

References

[1] H.L. Royden, Real Analysis, Pearson Education.

[2] G. DeBarra, Introduction to Measure Theory, Van Nostrand Reinhold.

[3] I. K. Rana, An Introduction to Measure and Integration, Narosa.

[4] H.S. Bear, A Primer on Lebesgue Integration, Academic press.

M502 : Algebra III (Galois Theory)

1. Prime and maximal ideals in a commutative ring and their elementary properties.
2. Field extensions, prime fields, characteristic of a field, algebraic field extensions, finite field extensions, splitting fields, algebraic closure, separable extensions, normal extensions,
3. Finite Galois extensions, Fundamental Theorem of Galois Theory.
4. Solvability by radicals.
5. Extensions of finite fields.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D. S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[4] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

[5] R. Lidl and H. Niederreiter, Introduction to Finite Fields and Their Applications, Cambridge University Press, 1986.

M503 : Topology II

1. Review of some notions from Topology I. Basic Separation axions and first and second countability axioms. Examples.
2. Products and quotients. Tychonoff's theorem. Product of connected spaces is connected. Weak topology on X induced by a family of maps $f_\alpha : X \longrightarrow X_\alpha$ where each X_α is a topological space. The coherent topology on Y induced by a family of maps $g_\alpha : Y_\alpha \longrightarrow Y$ where Y_α are given topological spaces. Examples of quotients to illustrate the universal property such as embeddings of $\mathbb{R}P^2$ and the Klein's bottle in $\mathbb{R}^4$.
3. Completely regular spaces and its embeddings in a product of intervals. Compactification, Alexandroff and Stone-Cech compactifications.
4. Normal spaces and the theorems of Urysohn and Tietze. The metrization theorem of Urysohn.
5. Local compactness, local connectedness and local path-connectedness and their basic properties. If $q : X \longrightarrow Y$ is a quotient map and Z is locally compact Hausdorff space then $q \times \mathrm{id} : X \times Z \longrightarrow Y \times Z$ is also a quotient map.
6. Locally finite families of sets and Partitions of unity. Baire Category theorem for locally compact Hausdorff spaces.

References

[1] G. F. Simmons, *Topology and modern analysis*

[2] W. A. Sutherland, *Introduction to metric and topological spaces.*

[3] S. Willard, *General Topology*, Dover, New York.

M 504 : Probability Theory

1. Probability as a measure, Probability space, conditional probability, independence of events, Bayes formula. Random variables, distribution functions, expected value and variance. Standard Probability distributions: Binomial, Poisson and Normal distribution.
2. Borel-Cantelli lemmas, zero-one laws. Sequences of random variables, convergence theorems, various modes of convergence. Weak law and the strong law of large numbers.
3. Central limit theorem: DeMoivre-Laplace theorem, weak convergence, characteristic functions, inversion formula, moment generating function.
4. Random walks, Markov Chains, Recurrence and Transience.
5. Conditional Expectation, Martingales.

References

[1] Marek Capinski and Tomasz Zastawniak, Probability through Problems, Springer, Indian Reprint 2008.

[2] P. Billingsley, Probability and Measure, 3rd ed., John Wiley & Sons, New York, 1995.

[3] J. Rosenthal, A First Look at Rigorous Probability, World Scientific, Singapore, 2000.

[4] A.N. Shiryayev, Probability, 2nd ed., Springer, New York, 1995.

[5] K.L. Chung, A Course in Probability Theory, Academic Press, New York, 1974.

Semester VI

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M601	Analysis IV	[3+1]	4
M602	Algebra IV	[3+1]	4
M603	Differential Geometry & Applications	[3+1]	4
M604	Differential Equations & Dynamical Systems	[3+1]	4
M605	Computational Mathematics II	[3+1]	4
H601	Ethics of Science and IPR	[2+0]	2
		Semester Credits	**22**
		Subtotal	**148**

M601 : Analysis IV (Complex Analysis)

1. Complex numbers and Riemann sphere. Möbius transformations.
2. Analytic functions. Cauchy-Riemann conditions, harmonic functions, Elementary functions, Power series, Conformal mappings.
3. Contour integrals, Cauchy theorem for simply and multiply connected domains. Cauchy integral formula, Winding number.
4. Moreras theorem. Liouvilles theorem, Fundamental theorem of Algebra.
5. Zeros of an analytic function and Taylors theorem. Isolated singularities and residues, Laurent series, Evaluation of real integrals.
6. Zeros and Poles, Argument principle, Rouchs theorem.

References

[1] L.Ahlfors, Complex Analysis.

[2] R.V. Churchill and J. W. Brown, Complex Variables and Applications, International Student Edition,Mc-Graw Hill, 4th ed., 1984.

[3] B.R. Palka, An Introduction to Complex Function Theory, UTM Springer-Verlag, 1991.

M602 : Algebra IV (Rings and Modules: Some Structure Theory)

1. Recollection of modules, submodules, quotient modules, homomorphisms.
2. External and internal direct sums of modules.
3. Tensor product of modules over a commutative ring. Functorial properties of $\otimes$ and Hom.
4. Definitions and elementary properties of projective and injective modules over a commutative ring.
5. Structure of finitely generated modules over a PID. Applications to matrices and linear maps over a field: rational and Jordan canonical forms.
6. Simpl modules over a not necessarily commutative ring, modules of finite length, Jordan-Hölder Theorem, Schur's lemma.
7. (Optional, if time permits) Semisimple modules over a not necessarily commutative ring, Wedderburn Structure Theorem for semisimple rings.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[3] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[4] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

M603 : Differential Geometry & Applications

1. Curvature of curves in $\mathbb{E}^n$: Parametrized Curves, Existence of Arc length parametrization, Curvature of plane curves, Frennet-Serret theory of (arc-length parametrized) curves in $\mathbb{E}^3$, Curvature of (arc-length parametrized) curves in $\mathbb{E}^n$, Curvature theory for parametrized curves in $\mathbb{E}^n$. Significance of the sign of curvature, Rigidity of curves in $\mathbb{E}^n$.
2. Euler's Theory of curves on Surfaces : Surface patches and local coordinates, Examples of surfaces in $\mathbb{E}^3$, curves on a surface, tangents to the surface at a point, Vector fields along curves, Parallel vector fields, vector fields on surfaces, normal vector fields, the First Fundamental form, Normal curvature of curves on a surface, Geodesics, geodesic Curvature, Christoffel symbols, Gauss' formula, Principal Curvatures, Euler's theorem.
3. Gauss' theory of Curvature of Surfaces : The Second Fundamental Form, Weingarten map and the Shape operator, Gaussian Curvature, Gauss' *Theorema Egregium*, Guass-Codazzi equations, Computation of First/Second fundamental form, curvature etc. for surfaces of revolution and other examples.
4. More Surface theory : Isoperimetric Inequality, Mean Curvature and Minimal Surfaces (introduction), surfaces of constant curvature, Geodesic coordinates, Notion of orientation, examples of non-orientable surfaces, Euler characteristic, statement of Gauss-Bonnet Theorem.
5. Modern Perspective on Surfaces : Tangent planes, Parallel Transport, Affine Connections, Riemannian metrics on surfaces.

References

[1] Elementary Differential Geometry : Andrew Pressley, Springer Undergraduate Mathematics Series.

[2] Elementary Differential Geometry : J. Thorpe, Elsevier.

[3] Differential Geometry of Curves and Surfaces : M. do Carmo.

[4] Elements of Differential Geometry : R. Millman & G. Parker.

M604 : Differential Equations & Dynamical Systems

1. Basic existence and uniqueness of systems of ordinary differential equations satisfying the Lipschitz' condition. Examples illustrating non-uniqueness when Lipschitz or other relevant conditions are dropped. Gronwall's lemma and its applications to continuity of the solutions with respect to initial conditions. Smooth dependence on initial conditions and the variational equation. Maximal interval of existence and global solutions. Proof that if (a, b) is the maximal interval of existence and $a < \infty$ then the graph of the solution must exit every compact subset of the domain on the differential equation.
2. Linear systems and fundamental systems of solutions. Wronskians and its basic properties. The Abel Liouville formula. The dimensionality of the space of solutions. Fundamental matrix. The method of variation of parameters.

3. Linear systems with constant coefficients and the structure of the solutions. Matrix exponentials and methods for computing them. Solving the in-homogeneous system. The Laplace transform and its applications.
4. Second order scalar linear differential equations. The Sturm comparison and separation theorems and regular Sturm-Liouville problems.
5. Series solutions of ordinary differential equations and a detailed analytic study of the differential equations of Bessel and Legendre.
6. Dynamical systems and basic notions of dynamical systems such as flows, rectification theorem, rest-points and its stability. Liouville's theorem on the preservation of phase volume. First integrals and their applications.

References

[1] R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume - I

[2] W. Hurewicz, *Lectures on ordinary differential equations*, Dover, New York.

[3] . F. Simmons, *Differential equations with applications and historical notes,* McGraw Hill.

M605 : Computational Mathematics II

1. Introduction to SAGE. Using SAGE to explore basics notions of Linear algebra, Number theory, Group Theory
2. Solving linear and non-linear optimization problems using Mathematica. Developing programmes for various numerical optimization techniques.
3. Exploration of Galois theory and Finite Fields Using Sage/Singular/Kash etc.
4. Basics of discrete mathematics using Sage/Mathematica.
5. Exploring advanced notions of Complex Analysis and Differential Equations using Mathematica.
6. Applied Linear Algebra using Mathematica, various matrix factorizations and their applications.

Semester VII

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M701	Functional Analysis	[3+1]	4
M702	Commutative Algebra	[3+1]	4
M703	Differential Topology	[3+1]	4
M704	Partial Differential Equations	[3+1]	4
M705	Representation Theory of Finite Groups	[3+1]	4
MPr701	Project		4
		Semester Credits	**24**
		Subtotal	**172**

M701 : Functional Analysis

1. Normed linear spaces. Riesz lemma. Heine-Borel theorem. Continuity of linear maps.
2. Hahn-Banach extension and separation theorems.
3. Banach spaces. Subspaces, product spaces and quotient spaces. Standard examples of Banach spaces like ℓ^p, L^p, $C([0,1])$ etc.
4. Uniform boundedness principle. Closed graph theorem. Open mapping theorem. Bounded inverse theorem.
5. Spectrum of a bounded operator. Eigenspectrum. Gelfand-Mazur theorem and spectral radius formula.
6. Dual spaces. Transpose of a bounded linear map. Standard examples.
7. Hilbert spaces. Bessel inequality, Riesz-Schauder theorem, Fourier expansion, Parseval's formula.
8. In the framework of a Hilbert space: Projection theorem. Riesz representation theorem. Uniqueness of Hahn-Banach extension.

References

[1] J.B. Conway, A course in Functional Analysis, Springer-Verlag, Berlin, 1985.

[2] G. Goffman and G. Pedrick, First course in functional analysis, Prentice-Hall, 1974.

[3] E. Kreyszig, Introductory Functional Analysis with applications, John Wiley & Sons, NY, 1978.

[4] B.V. Limaye, Functional Analysis, 2nd ed., New Age International, New Delhi, 1996.

[5] A. Taylor and D. Lay, Introduction to functional analysis, Wiley, New York, 1980.

M702 : Commutative Algebra

1. Prime and maximal ideals in a commutative ring, nil and Jacobson radicals, Nakayamas lemma, local rings.
2. Rings and modules of fractions, correspondence between prime ideals, localization.
3. Modules of finite length, Noetherian and Artinian modules.
4. Primary decomposition in a Noetherian module, associated primes, support of a module.
5. Graded rings and modules, Artin-Rees, Krull-intersection,
6. Hilbert-Samuel function of a local ring, dimension theory, principal ideal theorem.
7. Integral extensions, Noethers normalization lemma, Hilberts Nullstellensatz (algebraic and geometric versions).

References

[1] M.F Atiyah and I.G MacDonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.

[2] D. Eisenbud, Commutative Algebra with a view toward algebraic geometry, Springer-Verlag, Berlin, 2003.

[3] H. Matsumura, Commutative ring theory, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, Cambridge, 1980.

[4] S. Raghavan, B. Singh and R. Sridharan, Homological methods in commutative algebra, TIFR Math. Pamphlet No.5, Oxford, 1975.

[5] B. Singh, Basic Commutative Algebra, World Scientific, 2011.

M703 : Differential Topology

1. Differentiable functions on $\mathbb{R}^n$: Review of differentiable functions $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, Implicit and Inverse function theorems, Immersions and Submersions, critical points, critical and regular values.

2. Manifolds : Level sets, sub-manifolds of $\mathbb{R}^n$, immersed and embedded sub-manifolds, tangent spaces, differentiable functions between sub-manifolds of $\mathbb{R}^n$, abstract differential manifolds and tangent spaces.

3. Differentiable functions on Manifolds : Differentiable functions $f : M \longrightarrow N$, critical points, Sard's theorem, non-degenerate critical points, Morse Lemma, Manifolds with boundary, Brouwer fixed point theorem, *mod 2* degree of a mapping.

4. Transversality : Orientation of Manifolds, oriented intersection number, Brouwer degree, transverse intersections.

5. Integration on Manifolds : Vector field and Differential forms, integration of forms, Stokes' theorem, exact and closed forms, Poincar Lemma, Introduction to de Rham theory.

References

[1] Topology from a Differentiable Viewpoint : J. Milnor.

[2] Differential Topology : V. Guellemin & A. Pollack.

[3] Differential Topology : M. Hirsch.

M704 : Partial Differential Equations

1. Generalities on the origins of partial differential equations. Generalities on the Cauchy problem for a scalar linear equation of arbitrary order. The concept of characteristics. The Cauchy-Kowalevskya theorem and the Holmgren's uniqueness theorem. The fundamental equations of mathematical physics as paradigms for the study of Elliptic, Hyperbolic and Parabolic equations.

2. Quasilinear first order scalar partial differential equations and the method of characteristics. Detailed discussion of the inviscid Burger's equation illustrating the formation of discontinuities in finite time. The fully nonlinear scalar equation and Eikonal equation. The Hamilton-Jacobi equation.

3. Detailed analysis of the Laplace and Poisson's equations. Green's function for the Laplacian and its basic properties. Integral representation of solutions and its consequences such as the analyticity of solutions. The mean value property for harmonic functions and maximum principles. Harnack inequality.

4. The wave equation and the Cauchy problem for the wave equation. The Euler-Poisson-Darboux equation and integral representation for the wave equation in dimensions two and three. Properties of solutions such as finite speed of propagation. Domain of dependence and domain of influence.
5. The Cauchy problem for the heat equation and the integral representation for the solutions of the Cauchy problem for Cauchy data satisfying suitable growth restrictions. Infinite speed of propagation of signals. Example of non-uniqueness.
6. Fourier methods for solving initial boundary value problems.

References

[1] R. Courant and D. Hilbert, Methods of Mathematical Physics, Volume - II

[2] R. C. McOwen, *Partial differential equations*, Pearson Education, 2004.

M705 : Representation Theory of Finite Groups

1. Recollection of left and right modules, direct sums, tensor products.
2. Semi-simplicity of rings and modules, Schur′s lemma, Maschke′s Theorem, Wedderburn′s Structure Theorem.
3. The group algebra.
4. Representations of a finite group over a field, induced representations, characters, orthogonality relations.
5. Representations of some special groups.
6. Burnside's $p^a q^b$ theorem.

References

[1] M. Artin, Algebra, Prentice Hall of India, 1994.

[2] M. Burrow, Representation Theory of Finite Groups, Academic Press, 1965.

[3] D.S. Dummit and R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 2002.

[4] N. Jacobson, Basic Algebra I & II, Hindustan Publishing Corporation, 1983.

[5] S. Lang, Algebra, 3rd ed. Springer (India) 2004.

[6] J.P. Serre, Linear Representation of Groups, Springer-Verlag, 1977.

Semester VIII

Subject Code	Subject	Contact Hours per Week Theory + Tutorials	Credits
M801	Fourier Analysis	[3+1]	4
M802	Algebraic Number Theory	[3+1]	4
M803	Algebraic Topology	[3+1]	4
M804	Stochastic Analysis	[3+1]	4
M805	Computational Mathematics III	[3+1]	4
MPr801	Project		4
		Semester Credits	**24**
		Subtotal	**196**

M801 : Fourier Analysis

1. Fourier series. Discussion of convergence of Fourier series.

2. Uniqueness of Fourier Series, Convolutions, Cesaro and Abel Summability, Fejer‘s theorem, Dirichlets theorem, Poisson Kernel and summability kernels. Example of a continuous function with divergent Fourier series.

3. Summability of Fourier series for functions in L^1, L^2 and L^p spaces. Fourier-transforms of integrable functions. Basic properties of Fourier transforms, Poisson summation formula, Hausdorff-Young inequality, Riesz-Thorin Interpolation theorem.

4. Schwartz class of rapidly decreasing functions, Fourier transforms of rapidly decreasing functions, Riemann Lebesgue lemma, Fourier Inversion Theorem, Fourier transforms of Gaussians, Plancheral theorem, Paley-Weiner theorem.

5. Distributions and Fourier Transforms: Calculus of Distributions, Tempered Distributions: Fourier transforms of tempered distributions, Convolutions, Applications to PDEs.

References

[1] Y. Katznelson, Introduction to Harmonic Analysis, Dover.

[2] R. E. Edwards, Fourier Series, Academic Press.

[3] E. M. Stein and R. Shakarchi, Fourier Analysis: An Introduction, Princeton University Press, Princeton 2003.

[4] W. Rudin, Fourier Analysis on groups, Interscience.

M802 : Algebraic Number Theory

1. Field extensions and examples of field extensions of rational numbers, real numbers and complex numbers. Monic polynomials, Integral extensions, Minimal polynomial, Characteristic polynomial.

2. Integral closure and examples of rings which are integrally closed. Examples of rings which are not integrally closed. The ring of integers. The ring of Gaussian integers. Quadratic extensions and description of the ring of integers in quadratic number fields.

3. Units in quadratic number fields and relations to continued fractions.

4. Noetherian rings, Rings of dimension one. Dedekind domains. Norms and traces. Derive formulae relating norms and traces for towers of field extensions.

5. Discriminant and calculations of the discriminant in the special context of quadratic number fields. Different and it's applications.
6. Cyclotomic extensions and calculation of the discriminant in this case. Factorization of ideals into prime ideals and its relation to the discriminant.
7. Ramification theory, residual degree and its relation to the degree of the extension. Ramified primes in quadratic number fields.
8. Ideal class group. Geometric ideas involving volumes. Minkowski's theorem and its application to proving finiteness of the ideal class group.
9. Real and complex embeddings. Structure of finitely generated abelian groups. Dirichlet's Unit Theorem and the rank of the group of units. Discrete valuation rings, Local fields.

References

[1] Janusz, Algebraic Number Fields.

[2] Neukirch, Algebraic Number Theory.

[3] Marcus, Number Fields.

M803 : Algebraic Topology

1. Review of quotient spaces and its universal properties. Examples on $\mathbb{R}P^n$, Klein's bottle, Möbius band, $\mathbb{C}P^n$, $SO(n,\mathbb{R})$. Connectedness and path connectedness of spaces such as $SO(n,\mathbb{R})$ and other similar examples. Topological groups and their basic properties. Proof that if H is a connected subgroup such that G/H is also connected (as a topological space) then G is connected. Quaternions, S^3 and $SO(3,\mathbb{R})$. Connected, locally path connected space is path connected.
2. Paths and homotopies of paths. The fundamental group and its basic properties. The fundamental group of a topological group is abelian. Homotopy of maps, retraction and deformation retraction. The fundamental group of a product. The fundamental group of the circle. Brouwer's fixed point theorem. Degree of a map. Applications such as the fundamental theorem of algebra, Borsuk-Ulam theorem and the Perron Frobenius theorem.
3. Covering spaces and its basic properties. Examples such as the real line as a covering space of a circle, the double cover $\eta : S^n \longrightarrow \mathbb{R}P^n$, the double cover $\eta : S^3 \longrightarrow SO(3,\mathbb{R})$. Relationship to the fundamental group. Lifting criterion and Deck transformations. Equivalence of covering spaces. Universal covering spaces. Regular coverings and its various equivalent formulations such as the transitivity of the action of the Deck group. The Galois theory of covering spaces.
4. Orbit spaces. Fundamental group of the Klein's bottle and torus. Relation between covering spaces and Orientation of smooth manifolds. Non orientabilty of $\mathbb{R}P^2$ illustrated via covering spaces.
5. Free groups and its basic properties, free products with amalgamations. Concept of push outs in the context of topological spaces and groups. Seifert Van Kampen theorem and its applications. Basic notions of knot theory such as the group of a knot. Wirtinger's algorithm for calculating the group of a knot illustrated with simple examples.

References

[1] E. L. Lima, *Fundamental groups and covering spaces*, A. K. Peters, 2003.

[2] W. Massey, *Introduction to algebraic topology.* Springer Verlag.

M804 : Stochastic Analysis

1. Preliminaries :
   - (i) Martingales and properties.
   - (ii) Brownian Motion- definition and construction, Markov property, stopping times, strong Markov property zeros of one dimensional Brownian motion.
   - (iii) Reflection principle, hitting times, higher dimensional Brownian Motion, recurrence and transience, occupation times, exit times, change of time, Levys theorem.
2. Stochastic Calculus :
   - (i) Predictable processes, continuous local martingales, variance and covariance processes.
   - (ii) Integration with respect to bounded martingales and local martingales, Kunita Watanabe inequality, Ito s formula, stochastic integral, change of variables.
   - (iii) Stochastic differential equations, weak solutions, Change of measure , Change of time, Girsanovs theorem.

References

[1] Richard Durrett, Stochastc Calculus A Practical Introduction, CRC Press 1996.

[2] Karatzas I. and Steven Shreve, Brownian Motion and Stochastic Calculus, Springer.

[3] Oksendal Bernt, Stochastic Differential Equations, Springer.

[4] J.Michael Steele, Stochastic Calculus and Financial Applications, Springer, 2000

M805 : Computational Mathematics III

1. Differential Geometry of curves and surfaces using Mathematica.
2. Exploring Differential Equation and Dynamical System using XPPAUT or some other specialized sofware.
3. Design of Experiments and Statistics Quality control using R.
4. Project/Math Modeling problem using any Mathematical Software and developing Mathematica packages for various specific methods.
5. Exploring solutions of Partial Differential equations using Mathematica. Developing programmes to solve problems numerically.
6. Exploring basic Notions of Commutative algebra using Sage/Singular /Kash etc.
7. Advanced notion of optimization techniques using Mathematica.
8. Project/Math Modeling problem using any Mathematical Software and developing Mathematica packages for various specific methods.

References

[1] Alfred Gray, Elsa Abbena, Simon Salamon, Modern Differential Geometry of Curves and Surfaces with Mathematica, Third Edition.

Semester IX

Subject Code	**Subject**	**Contact Hours per Week** Theory + Tutorials	**Credits**
MPr901	Project		24
		Semester Credits	**24**
		Subtotal	**220**

Semester X

Subject Code	**Subject**	**Contact Hours per Week** Theory + Tutorials	**Credits**
ME1001	Elective 1	[3+1]	4
ME1002	Elective 2	[3+1]	4
ME1003	Elective 3	[3+1]	4
ME1004	Elective 4	[3+1]	4
ME1005	Elective 5	[3+1]	4
ME1006	Elective 6	[3+1]	4

Total Credits (with 5 Electives): 240
Total Credits (with 6 Electives): 244

(Minimum required: 240)

Electives

Electives taught in the semester Jan-Apr 2014

1. Advanced Commutative Algebra & Applications.
2. Advanced Differential Topology.
3. Advanced Numerical Techniques.
4. Combinatorics & Enumeration.
5. Lie Groups & Geometry
6. Topics in Algebraic Geometry.

Other possible electives for future

1. Advanced Algebraic Topology & Applications.
2. Advanced Differential Geometry & Applications.
3. Algebraic curves.
4. Analytic number theory.
5. Class field theory.
6. Coding Theory & Cryptography.
7. Combinatorial Design Theory.
8. Econometrics.
9. Elliptic curves.
10. Financial Mathematics.
11. Finite Fields & Applications.
12. Fluid Mechanics.
13. Fractals & Applications.
14. Geometric algebra.
15. Homological Algebra & Applications.
16. Industrial Mathematics.
17. Introduction to algebraic groups.
18. Mathematical Applications to Engineering.
19. Mathematics & Nano Technology.
20. Modular forms.
21. Operator Theory.
22. Perturbation Theory.
23. Quantum Computing.
24. Wavelet Analysis & Applications.