

CIRCULAR :

In partial modification of this office Circular No.UG/476 of 2006 dated 1st December, 2006, the Principals of the affiliated colleges in Science are hereby informed that the Appendix sent alongwith the above mentioned circular is treated as cancelled and that the correct Appendix of the guidelines for practical in the subject of Statistics at the TY.B.Sc. is enclosed herewith for your information.

Mumbai 400 032
19th January, 2007

for REGISTRAR.

To,

The Principals of the affiliated colleges in Science.

A.C.4.7/18.08.06

No.UG/ 22-A of 2007

19th January, 2007

Copy forwarded with compliments to :-

- 1) the Dean, Faculty of Science,
- 2) the Chairman, Board of Studies in Statistics.

for REGISTRAR.

Copy to :

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18.8.06

UNIVERSITY OF MUMBAI



GUIDELINES FOR PRACTICAL IN THE SUBJECT OF STATISTICS AT THE T.Y.B.SC.

UNIT-VI (Sections I & II)
UNIT-VII (Sections I & II)
UNIT-VIII (Sections I & II)
UNIT-IX (Sections I & II)

(With effect from the academic year ~~2006-2007~~)

2007-2008

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PROBABILITY

- 1) In 'Maharaja' hotel there are 10 rooms, each having capacity of maximum 40 persons. On one fine Sunday 40 passengers arrived at the hotel and manager places them at random in 10 rooms. What is the probability that
 - a) no room is empty?
 - b) exactly 7 rooms remain empty?
- 2) An elevator starts with 5 passengers and stops on 7 floors. What is the probability that
 - a) all passengers leave on same floor?
 - b) each passenger leaves on different floor?
 - c) on at least 3 floors, no passenger leaves?
- 3) Five different letters are distributed at random in 5 self addressed envelopes. Find the probability that
 - a) exactly 2 letters
 - b) at least 3 letters and
 - c) none
 are placed in correct envelope.
- 4) A student is asked to guess an unknown order of 10 distinct cards. What is the probability of exactly 4 correct guesses if he calls each card only once at random? What is the probability that all his guesses are wrong?
- 5) A pack of cards consists of 5 identical series, each containing 10 cards numbered 1 to 10. A random sample of size 15 is drawn from the pack
 - a) without replacement
 - b) with replacement.
 Calculate the probability that each number is represented in the sample.
- 6) Twenty pairs of shoes are in a closet. Six shoes are selected at random. Find the probability that there will be
 - a) at least one pair
 - b) exactly 2 pairs
 among the 6 shoes selected.
- 7) In a game of bridge, what is the chance that
 - a) exactly 2 players
 - b) at least 3 players
 get complete suit?
- 8) In a lottery, 2 tickets are drawn at a time out of the total number of 5 tickets numbered 1 to 5 and returned before the next drawing is made. Find the chance that in 3 drawings, each of the numbers 1, 2, ..., 5 will appear at least once.

	0	1	2
0	$1/16$	$1/8$	$1/16$
1	$1/8$	$1/4$	$1/8$
2	$1/16$	$1/8$	$1/16$

- 1) The probability distribution function of a random variable X is
- $$p(x) = \begin{cases} (1/6)(5/6)^{x-1} & ; x = 1, 2, 3, \dots \\ 0 & ; \text{otherwise} \end{cases}$$

Find

- probability generating function of X .
 - $E(X)$ and $V(X)$.
- 2) The probability of error in the transmission of a bit over a communication channel is $p = 10^{-4}$. Let X denotes the number of errors in transmitting a block of 1000 bits. Find
- probability generating function of X .
 - variance of the distribution of X .
- 3) A random variable Y is defined as number of tosses required to obtain 5th head when an unbiased coin is tossed repeatedly under identical conditions. Find
- probability generating function of Y .
 - $E(Y)$ and $V(Y)$.
- 4) Let $X_1, X_2, X_3, \dots, X_{12}$ be mutually independent random variables having common probability mass function
- $$p(x) = \begin{cases} (1/4)^x (3/4)^{1-x} & ; x = 0, 1 \\ 0 & ; \text{otherwise} \end{cases}$$

Find

- probability generating function of X_i .
 - probability generating function of Y .
 - $E(Y)$ and $V(Y)$
where $Y = X_1 + X_2 + X_3 + \dots + X_{12}$
- 5) Three fair six sided dice are tossed simultaneously. Let X denote the sum of the numbers observed on the uppermost faces of the 3 dice. Find
- probability generating function of X .
 - $P(X = 17)$.
 - $P(X = 6)$.
- 6) Four tickets are drawn at random with replacement from a bag containing ten tickets numbered 1, 2, ..., 10. Let X denote the sum of numbers on the tickets drawn. Find:
- probability generating function of X .
 - $P(X = 38)$.
- 7) In a lottery, tickets are numbered from 000000 to 999999 and they are equally probable. What is the probability that the sum of the digits of a chosen number will be 21?
- 8) The joint p.d.f. of X and Y is given below

	0	1	2
0	1/16	1/8	1/16
1	1/8	1/4	1/8
2	1/16	1/8	1/16

Find

- probability generating function of X .
- probability generating function of Y .
- probability generating function and probability distribution of $Z = X + Y$.

- 1) The time between arrivals at L and J Restaurant is exponential with mean 5 minutes. The restaurant opens for business at 11.00 a.m. Determine the following
 - a) The probability of having 10 arrivals in restaurant at 11.12 a.m. given that there were 8 at 11.05 a.m.
 - b) The probability that a new customer will arrive between 11.28 and 11.33 a.m. given that the last customer arrived at 11.25 a.m.
 - c) The probability that time between two successive arrivals is at least 2 minutes.
- 2) A Yule type of birth process with initially 2 members, experiences an average birth rate of 3 members per hour. Obtain
 - a) probability of having 4 or more members in the population after 1 hour 15 minutes.
 - b) expected size and standard deviation of population at 2 hours and 30 minutes.
- 3) Demand for an item occurs according to a Poisson distribution with mean 2 per day. The maximum stock level is 10 items, which occurs on each Monday immediately after a new order is received. The order size depends on number of units left at the end of the week on Saturday (business is closed on Sunday). Determine the following
 - a) The average weekly size of order.
 - b) The probability of incurring shortage when the business opens on Friday morning.
- 4) A population consists of elements which can split or die. The rate of splitting in a two is 3 per unit time and rate of dying is 2 per unit time. If there is no interaction among the elements, obtain the mean and standard deviation of population after 2 time units if initial population size is 100.
- 5) An office opens at 11 a.m. daily. There are 2 particular employees who come before 11 a.m. and other employees arrive at the rate of 2 employees per minute after 11 a.m. Obtain the probability that there are
 - a) less than 3 employees at 11.01 a.m.
 - b) 10 employees at 11.05 a.m. Also obtain expected number of employees at 11.02 a.m.
- 6) A machine shop has just stocked 10 spare parts for the repair of a machine. Stock replenishment of size 10 pieces occurs every 7 days. The time between breakdowns is exponential with mean 1 day. Determine the probability that the machine will remain broken for 2 days because of the unavailability of parts.

QUEUEING THEORY-I

- 1) Customers arrive at the first class ticket counter of a railway station at a rate of 12 per hour. There is one clerk serving the customers at a rate of 30 per hour. Obtain a) the probability that the system is empty. b) the probability that there are more than 2 customers in the system. c) the probability that there is no customer waiting to be served. d) the probability that a customer is being served and nobody is waiting. Assume that the arrival of customers is in Poisson fashion and service time is exponentially distributed.
- 2) A tailor specializes in ladies' dresses. The number of customers approaching the tailor appears to be Poisson distributed with a mean of 6 customers per hour. The tailor attends the customers on a first-come-first-served basis and the customers wait if the need be. The average time taken by the tailor to attend one customer is 6 minutes. Obtain a) the average time that the tailor is free on a 10-hour working day. b) the expected number of customers in the tailor's shop. c) the expected number of customers waiting for tailor's services. d) the probability that an arriving customer will be waiting in the queue for more than 10 minutes. e) the probability that an arriving customer will be in the shop for more than 15 minutes.
- 3) At Dr.Prachi's clinic, on an average, 8 patients arrive per hour. Some patients require only the repeat prescription; others come for minor check-up, while some others require thorough inspection for the diagnosis. This takes Dr.Prachi six minutes per patient on an average. It can be assumed that arrivals follow a Poisson distribution and the doctor's inspection time follows an exponential distribution. Obtain a) the percent of times a patient can walk inside the doctor's cabin without having to wait. b) the average number of patients waiting for their turn. c) the average time that a patient spends in the clinic.
- 4) A post office has 3 windows providing the same service. It receives on an average 30 customers per hour. Arrivals are in Poisson type and service time is exponentially distributed. Each window serves on an average 12 customers per hour. Obtain a) the probability that an arriving customer will be served immediately. b) the probability that a customer will have to wait. c) the average number of customers in the system. d) the average total time that a customer spend in the post office.
- 5) A bakery is staffed by 2 clerks, each of them capable of handling an average of 30 customers an hour. Customers arrive at the bakery according to a Poisson process, at a mean rate of 50 per hour and the service time is exponentially distributed. Obtain a) the probability that there are more than 3 customers in the bakery at the same time. b) the probability that a given clerk is idle.
- 6) A to Z Groceries operates with three check-out counters and the operation is set up such that the customer will go to the first empty counter. Customers arrive at the store according to a Poisson distribution, with a mean of 10 per hour. The service time per customer is exponential with mean 12 minutes. Obtain a) the expected number of customers waiting to be served. b) the expected number of busy servers. c) probability that all the three counters are busy.
- 7) On an average, 96 patients per 24-hour day require the service of an emergency clinic. Also, on an average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs.100 per patient treated. Each minute of decrease in the average service time would cost Rs.10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $4/3$ patients to $1/2$ patients?
- 8) A company currently has two tool cribs, each having a single clerk, in its manufacturing area. One tool crib handles only the tools for heavy machinery, while the second one handles all other tools. It is observed that for each tool crib the arrivals follow a Poisson distribution with a mean of 20 per hour and the service time distribution is exponential with a mean of 2 minutes. The tool manager feels that if the tool cribs are combined in such a way that either clerk can handle any kind of tools as demand arises, it would reduce the waiting time in the system. Analyze the data and comment on the management's opinion.

- 1) A small barber shop, operated by a single barber, has a room for at most 2 customers. Potential customers arrive in a Poisson fashion at an average rate of 3 per hour. Service time follows exponential distribution with mean $\frac{1}{4}$ hour. Obtain
 - a) the average time spent in the shop by an entering customer.
 - b) the probability that an arriving customer can enter the system.
 - c) the probability that the barber is busy.
- 2) Customers arrive at a beauty parlour in a Poisson fashion at an average rate of 6 per hour. Any arriving customer is denied service if there are already 7 customers in all at the parlour. A single beautician attends to arriving customers on a FCFS basis at an average rate of 40 minutes per customer. The actual service time is exponential. Obtain
 - a) the percentage of time the beautician is busy.
 - b) the expected number of customers at the parlour.
 - c) the average time a customer will spend at the parlour.
 - d) the average number of customers who are denied entry per hour.
- 3) A service center is manned by 5 servers each of whom work independently at an average rate of 15 minutes per customer. The actual distribution of service time for each server is exponential. Customers arrive at the center in a Poisson fashion at an average rate of 10 per hour. Any arriving customer is forced to leave the system if all the servers are busy. Obtain
 - a) the steady state probability distribution of the number of customers in the system.
 - b) the probability that 2 or more servers are free.
 - c) the expected waiting time in the system for any arrival.
- 4) A self service car wash has four stalls in which customers can clean and wax their automobiles and room to accommodate maximum of three additional cars when all stalls are full. Customers arrive at the car wash according to Poisson process at a mean rate of 15 per hour. If there is no room for them on the grounds of the car wash, arriving customers must go elsewhere. The time required to service a car appears to be exponentially distributed with a mean of 12 minutes. Obtain
 - a) the average number of cars at the car wash at any given time.
 - b) the expected rate at which cars are denied entrance to the car wash.
- 5) One repairman is attending 5 machines. The rate of breakdown per machine is one per hour while the service rate is ten per hour. Obtain
 - a) the probability that the repairman is idle.
 - b) the expected number of machines waiting for the service.
- 6) A company has five machines, each of which suffers breakdown at an average rate of 0.5 per hour. There are two repairmen and average repair time taken by each of them is 30 minutes. Obtain
 - a) the probability that all machines are in good condition.
 - b) the average time an out of order machine has to spend waiting for repairs to start.
- 7) A doctor's clinic can accommodate at most 10 patients (including the one being examined). Patients arrive at the clinic in a Poisson fashion at a rate of 5 per hour. The time taken by the doctor to examine one patient is exponentially distributed with a mean of 12 minutes per customer. Obtain
 - a) the expected waiting time in the system of a customer.
 - b) the percentage of time the doctor is idle.
 - c) the expected number of customers per hour who are denied the service.
- 8) At a self-service facility arrivals occur according to Poisson fashion with mean 30 per hour. The distribution of service time is exponential with mean 5 minutes. Obtain
 - a) the expected number of customers in service at service facility.
 - b) the percentage of time the facility is not used by customers.

CHEBYCHEV'S INEQUALITY

- 1) The probability distribution of a random variable X is given by
- $$p(x) = 1/8 \quad ; \quad x = -1, 1$$
- $$= 6/8 \quad ; \quad x = 0.$$

Obtain a bound to $P(|X| \geq 1)$. Compare it with the actual probability and comment on the same.

- 2) 10% of a large lot of apples are bad. If X denotes the number of bad apples in a random sample of size 20, obtain a bound to $P(0 < X < 4)$ and compare your answer with the actual probability.
- 3) It is suspected that 40% persons of a large community have high blood pressure. What is the Chebychev's lower bound to the probability that in a random sample of 100 persons from the community, 30% to 50% of them would test positive for high blood pressure? Obtain the above probability using Central limit theorem and Compare the two.
- 4) The number of persons X_A and X_B , lined up respectively at railway booking windows A and B is independent and Geometrically distributed with probability distribution given by
- $$p(x) = q^x p, \quad x = 0, 1, 2, 3, \dots$$
- where $p = 0.2$ for window A and $p = 0.5$ for window B. An arrival joins the queue at any window at random. Obtain a bound to
- $P(|X_A - E(X_A)| \geq 5)$.
 - $P(|X_B - E(X_B)| \geq 3)$.
 - $P(|X - E(X)| \geq 8)$ where $X = X_A + X_B$.
- 5) The marks of any student in T.Y.B.Sc. has mean 60 and standard deviation of 4. Find bound to the
- marks of a given student lies between 55 and 65.
 - the average marks of five given students lie between 55 and 65.
 - the absolute difference in marks of any two given students is greater than or equal to 6.
- 6) (X_1, X_2, X_3) follows multinomial distribution with parameters $(10, 0.2, 0.3, 0.4)$.
- obtain Chebychev's bound to $P(0 < X_1 < 4)$, $P(3 < X_1 + X_2 < 7)$.
 - compare each of the above with actual probability.
- 7) The chance of failure X of an electronic component has p.d.f. given by
- $$f(x) = 504 x^5 (1-x)^3 \quad ; \quad 0 \leq x \leq 1$$
- $$= 0 \quad ; \quad \text{otherwise}$$
- Obtain a bound to $P(0.4 < X < 0.8)$.
- 8) The amount of toll X (in lakhs of rupees) collected per day at a booth is Uniformly distributed over the interval $(20, 40)$. Obtain Chebychev's bound to the probability that the average collection per day during the last one week lies between Rs.26 lakhs and Rs.34 lakhs.

ORDER STATISTICS

- 1) Given a random sample of size seven from a population of continuous type, obtain the probability that
- a) smallest order statistic exceeds the population median.
 - b) second order statistic is less than the population quartile Q_1 .
 - c) sample median lies between the population quartiles Q_1 and Q_3 .
- 2) Five observations are randomly made from a population with p.d.f.

$$f(x) = 2(1-x) ; 0 < x < 1$$

$$= 0 ; \text{otherwise.}$$

Find the probability that

- a) the sample median is more than $\frac{1}{2}$.
 - b) fourth order statistic is more than $\frac{1}{4}$.
- 3) A random sample of size six is drawn from a continuous Uniform distribution over the interval $(0, 2)$. Obtain p.d.f. of the sample range and hence probability that the sample range is less than K for values of $K = 0.75, 1, 1.25$.
- 4) The time taken per telephone call X in minutes follows an Exponential distribution with mean 5. Obtain the probability that the sample range exceeds 4 minutes, based on a random samples of size $n = 10, 15, 20$.
- 5) The monthly income X in thousands of rupees of workers in an area has p.d.f.
- $$f(x) = e^{-(x-\theta)} ; x > \theta, \theta > 0$$
- $$= 0 ; \text{otherwise}$$

A random sample from the above population is as follows 12, 10, 15, 12, 11, 10, 11, 17, 9, 22. Obtain an unbiased estimate of θ based on the smallest order statistic and standard error of your estimate.

- 6) There is a bus service every θ minutes where θ is unknown. The time interval between any two consecutive services is Uniform continuous over the interval $(0, \theta)$. The time intervals between any two consecutive services are recently observed to be as follows: 5, 7, 6.5, 4, 4, 3, 6, 8, 5.5, 6. Obtain
- a) an unbiased estimate of θ based on the order statistics (i) Y_5 (ii) Y_{10} .
 - b) standard errors of your estimates.
 - c) correlation coefficient between Y_5 and Y_{10} .
- 7) A random sample of size two is drawn from a population with p.d.f.
- $$f(x) = 2x ; 0 < x < 1$$
- $$= 0 ; \text{otherwise}$$

Find the probability that one of them is at least twice as large as the other.

- 8) A random sample of size three is drawn from an Exponential distribution with mean one. Obtain
- a) p.d.f. and c.d.f. of Y_1 .
 - b) mean and standard deviation of Y_1 .

BIVARIATE DISCRETE DISTRIBUTIONS

Let W be the weight of laundry soap in 1-kilogram box. $P(W < 1) = 0.02$ and $P(W > 1.072) = 0.08$. We call a box of soap light, good or heavy depending on whether $W < 1$, $1 < W < 1.072$ or $W > 1.072$, respectively. In a random sample of $n = 50$ boxes, let X denote the number of light boxes and Y the number of good boxes.

- what is the joint p.d.f. of X and Y ?
- what are the parameters of the distribution?
- given that $X = 3$, how is Y distributed conditionally?
- determine $E(Y / X = 3)$.
- find ρ , the correlation coefficient between X and Y .

It is observed that the percentage of students securing first class, second class, pass class and failure at a certain examination are 40, 20, 30 and 10 respectively out of 100 students who have appeared for this examination. Let X_1 , X_2 and X_3 be the number of students who have secured 1st, 2nd and pass class respectively. State

- the joint distribution of X_1 , X_2 and X_3 .
- marginal distribution of X_1 , X_2 and X_3 .
- joint marginal distribution of X_1 and X_2 . Hence find correlation between X_1 and X_2 .
- conditional distribution of X_1 and X_2 for given X_3 .
- conditional distribution of X_1 for given X_2 and X_3 hence find $E(X_1 / X_2 = 10 \text{ and } X_3 = 15)$.

The M.G.F. of (X, Y) is given by

$$(0.2 e^{t_1} + 0.5 e^{t_2} + 0.3)^{50}$$

Write the probability distribution of

- (X, Y) .
- Y given $X = 2$.
- X given $Y = 2$.

Also find $E(Y / X = 2)$ and $E(X / Y = 2)$.

One of the numbers 1, 2, ..., 6 is to be chosen by casting an unbiased die. Let this random experiment be repeated five independent times. Let the random variable X_1 denote the number of terminations in the set $\{x; x = 1, 2, 3\}$ and let the random variable X_2 denote the number of terminations in the set $\{x; x = 4, 5\}$. Compute $P(X_1 = 2, X_2 = 1)$.

Four independent observations are made on a random variable X having p.d.f.

$$f(x) = 1 - |x|; -1 < x < 1$$

$$= 0; \text{ otherwise}$$

Suppose that the interval -1 to 1 is divided into 4 distinct classes of the same length. Find the probability that one observation falls in each of the first two classes, two in the third and none in the last class.

T.Y B.Sc. Practical No.6.2.4

BIVARIATE CONTINUOUS DISTRIBUTIONS

- 1) Let X and Y have a Bivariate Normal Distribution with parameters $\mu_1 = 3, \mu_2 = 0, \sigma_1^2 = 16, \sigma_2^2 = 25, \rho = 0.6$
Determine the following probabilities.
 a) $P(2 < Y < 7)$
 b) $P(2 < Y < 7 / X = 7)$
 c) $P(-3 < X < 3)$
 d) $P(-3 < X < 3 / Y = -3)$
- 2) Let X and Y have a Bivariate Normal Distribution with parameters $\mu_1 = 1, \mu_2 = 2, \sigma_1^2 = 16, \sigma_2^2 = 25, \rho = 12/13$
Determine the following probabilities.
 a) $P(X > 2)$
 b) $P(X > 2 / Y = 2)$
- 3) A Bivariate Normal Distribution has the p.d.f.
 $f(x,y) = C \exp [-x^2 + xy - y^2]; -\infty < x,y < \infty$
 Find the constant C and the parameters of the distribution. Also find the conditional p.d.f. of Y given $X = 2$.
- 4) Find the constant k and the parameters of the following Bivariate Normal Distribution.

$$f(x,y) = k \exp \left\{ -\frac{8}{27} \left[(x-7)^2 - 2(x-7)(y-5) + 4(y+5)^2 \right] \right\}$$
- 5) A Bivariate Normal Distribution has the p.d.f.

$$f(x,y) = k \exp \left\{ -\frac{1}{102} \left[(x+2)^2 - 2.8(x+2)(y-1) + 4(y-1)^2 \right] \right\}$$

 Find
 a) $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$.
 b) conditional mean of Y given $X = x$.
 c) $V(Y / x)$
- 6) The marks X and Y scored by candidates in an examination in two subjects are known to follow a Bivariate Normal Distribution. The mean of X is 52 and its s.d. is 15 while Y has mean 48 and s.d. 15. Also $\rho = 0.6$. Write down the joint p.d.f. of X and Y. If 100 marks in the aggregate are needed to pass in the examination, find the proportion of candidates who pass in the examination. Also find $P(|X - Y| < 10)$.

SIGNIFICANCE OF CORRELATION COEFFICIENT

- A random sample of 30 pairs of observation from a normal population gave a correlation coefficient of 0.7. Is it likely that the variables in the population are uncorrelated?
- What is the least value of 'r' in a random sample of 27 pairs that is significant
- at 0.05 level.
 - at 0.01 level.

The height (in cms) and the number of leaves of 15 sunflower plants was obtained and recorded as follows. Calculate correlation coefficient and test its significance.

Ht (in cms)	20	27	29	24	30	37	23	24	28	15	25	31	25	37	21
No. of leaves	16	19	18	20	25	30	16	29	26	12	18	23	24	28	19

A correlation coefficient of 0.72 was obtained from a sample of 29 pairs of observations.

- Can the sample be regarded as drawn from a bivariate normal population with correlation coefficient of 0.8?
- Obtain 95% confidence interval for the population correlation coefficient ρ in the light of the information provided by the sample.

In an air pollution study a researcher obtained the following results with regard to the number of lichen species (X) and the respective distance from each of the 25 chemical factories (Y in kms).

$$\sum X = 1165, \sum Y = 185.20, \sum X^2 = 56947, \sum Y^2 = 1437.24, \sum XY = 9024.40.$$

Test the hypothesis that the corresponding correlation value for the population is 0.8.

The correlation coefficient between body length and body weight of *Anabas scandens* (fresh water fish) for two samples of sizes 40 and 50 was 0.5 and 0.55 respectively. Test whether the correlation coefficients are significantly different.

T.Y.B.Sc. Practical No.6.2.6

MULTIPLE AND PARTIAL CORRELATION AND REGRESSION

- 1) The following table gives the weights X_1 (in pounds), heights X_2 (in inches) and the ages of 12 boys.

Weight X_1	64	71	53	67	55	58	77	57	56	51	76	78
Height X_2	57	59	49	62	51	50	55	48	52	42	61	57
Age X_3	8	10	6	11	8	7	10	9	10	6	12	9

- a) Find the least square regression of X_1 on X_2 and X_3 .
 b) Estimate the weight of a boy who is 9 years old and 54 inches tall.
 2) The manager of a Pizza shack collected the following data for the past 12 months on the total monthly sales X_1 (in 000's of rupees), number of advertisements appearing every month (X_2) and monthly cost of advertising (in 00's of rupees).

$$\begin{aligned}\sum X_1 &= 432.4 & \sum X_2 &= 110 & \sum X_3 &= 133.1 \\ \sum X_1^2 &= 16034 & \sum X_2^2 &= 1072 & \sum X_3^2 &= 1513.27 \\ \sum \sum X_1 X_2 &= 40963 & \sum \sum X_1 X_3 &= 4902.23 & \sum \sum X_2 X_3 &= 1263.5\end{aligned}$$

- a) Find the plane of regression of X_1 on X_2 and X_3 .
 b) Estimate the total monthly sales of pizza if the cost of advertisement is Rs.15.200 and the number of advertisement monthly is 16.
 3) An instructor wished to determine the relationship of marks on a final examination to those on the two unit tests conducted during the year. Calling X_1 , X_2 and X_3 as the marks of a student on the final exam, first unit test and second unit test respectively, he obtained the following computations for a total of 120 students.

$$\begin{aligned}\bar{X}_1 &= 73, \bar{X}_2 = 6.8, \bar{X}_3 = 7.0, \sigma_1 = 9.0, \sigma_2 = 1.0, \sigma_3 = 0.8 \\ r_{12} &= 0.65, r_{13} = 0.60, r_{23} = 0.70\end{aligned}$$

- a) Find the least square equation of X_1 on X_2 and X_3 .
 b) Estimate the final marks of two students who secured respectively (9, 7) and (4, 8) in the two unit tests.
 c) Obtain multiple correlation coefficient $R_{1.23}$ and test its significance.
 d) Obtain partial correlation coefficient $r_{12.3}$ and test its significance.
 4) In a tri-variate distribution, it is given that
 $\sigma_1 = 3$; $\sigma_2 = 4$; $\sigma_3 = 5$; $r_{23} = 0.9$; $r_{13} = 0.6$; $r_{12} = 0.65$
 a) Obtain regression line of X_1 on X_2 and X_3 .
 b) Find Partial correlation coefficients $r_{12.3}$, $r_{13.2}$ and $r_{23.1}$
 c) Find Multiple correlation coefficients $R_{1.23}$, $R_{2.13}$ and $R_{3.12}$
 d) Calculate $\sigma_{1.23}^2$, $\sigma_{2.13}^2$, $\sigma_{3.12}^2$
 (Note the variates are measured from their respective means.)

METHOD OF MAXIMUM LIKELIHOOD, METHOD OF MOMENTS AND METHOD OF MODIFIED MINIMUM CHI-SQUARE

Let probability density function (p.d.f.) of a random variable (r.v.) X be $f(x, \theta)$, and a random sample of observations from a population with this p.d.f. is given. Estimate the unknown parameter θ by

- method of moments
- method of maximum likelihood

and estimate standard error of the m.l.e, for each of the following examples (a),(b),(c):-

a) $f(x, \theta) = \theta e^{-\theta x}$; $x > 0, \theta > 0$
 $= 0$; otherwise.

Observed Sample values : 1.0, 0.7, 2.5, 8.9, 12.0, 7.5, 11.75, 15.5, 14.6 and 15.5.
 b) $f(x, \theta) = \theta x^{\theta-1}$; $0 < x < 1, \theta > 0$.
 $= 0$; otherwise.

Observed sample values : 0.41, 0.57, 0.75, 0.78, 0.65, 0.82, 0.48, 0.39, 0.65, 0.51, 0.46, and 0.83.
 c) $f(x, \theta) = \frac{x}{\theta^2} e^{-x/\theta}$; $x > 0, \theta > 0$.

Observed Sample values : 5.4, 7.1, 5.6, 6.2, 6.4, 4.9, 5.8, 6.3, 5.5 and 4.8
 Given the p.d.f. $f(x, \theta)$ of a r.v. X , and a random sample of observations from a population with this p.d.f., estimate the unknown parameter θ by

- method of moments
- method of maximum likelihood

for each of the following examples (a),(b),(c):-

a) $f(x, \theta) = \frac{1}{\theta}$; $0 < x < \theta$
 $= 0$; otherwise.

Observed sample values are: 1.3, 0.6, 1.7, 2.2, 0.3 and 1.1.

b) $f(x, \theta) = e^{-(x-\theta)}$; $\theta < x < \infty, -\infty < \theta < \infty$
 $= 0$; otherwise.

Observed sample values are: 1.3, 0.7, 3.6, 5.4, 7.4, 2.5, 3.2, 4.1, 2.5 and 2.6.

c) $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$; $-\infty < x < \infty, -\infty < \theta < \infty$
 $= 0$; otherwise.

Observed sample values are: 1.9, -1.8, 2.9, -2.2, 1.5, 2.2, -1.4, -1.5, 1.6 and 1.7.

The following table gives probabilities and observed frequencies in four classes AB, Ab, aB and ab in a genetic experiment.

CLASS	AB	Ab	aB	ab	Total
Probability	$\frac{2+\theta}{4}$	$\frac{1-\theta}{4}$	$\frac{1-\theta}{4}$	$\frac{\theta}{4}$	1
Observed Frequency	108	30	25	7	170

where $0 < \theta < 1$. Estimate the unknown parameter θ by

- method of modified minimum chi-square
- method of maximum likelihood and obtain its standard error.

- 4) The p.m.f. of a discrete random variable X is given by

$$p(x) = \begin{cases} 1 & x=1 \\ \frac{2pq}{1-q^2} & x=2 \\ \frac{p^2}{1-q^2} & x=3 \end{cases}$$

where $q = 1 - p$, $0 < p < 1$. A sample of 30 observations gave the frequencies for the two cases as 11 and 19 respectively. Estimate p using

- a) method of modified minimum chi-square
b) method of maximum likelihood
c) method of moments.
- 5) 1000 observations and their respective probabilities (as per a genetic model) are classified according to sex and color-blindness as follows:

Obs. frequencies	MALE	FEMALE
NORMAL	440	515
COLOUR-BLIND	40	5

Probabilities	MALE	FEMALE
NORMAL	$(p/2)$	$(p^2/2) + pq$
COLOR-BLIND	$(q/2)$	$(q^2/2)$

Given $p + q = 1$ and $0 < p < 1$, estimate parameter p by method of maximum likelihood. Also find expected frequencies.

- 9) A random sample of 10 observations from a population with p.d.f.

$$f(x, \mu) = \begin{cases} 2\mu x e^{-\mu x^2} & ; x > 0, \mu > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

is given as follows: 1.2, 3.8, 6.8, 2.87, 0.75, 3.19, 0.72, 1.05, 5.4 and 7.82. Estimate unknown parameter μ by

- a) method of maximum likelihood and find its standard error
b) method of moments.

METHOD OF MAXIMUM LIKELIHOOD AND METHOD OF MOMENTS

- 1) Let a random sample of 5 observations 8.1, 0.2, 1.6, 5.2, 2.1 be taken from a population with p.d.f.
- $$f(x, a, b) = \frac{1}{\beta - \alpha} : \alpha < x < \beta, -\infty < \alpha, \beta < \infty$$

Estimate parameters α and β by

a) method of moments and b) method of maximum likelihood.

- 2) Let sample observations 2.3, 1.2, 0.9, 3.2, 1.6, 2.0, 2.9 and 3.4 be taken from a population with p.d.f
- $$f(x, a, b) = \frac{1}{b} e^{-(x-a)/b} : a < x < \infty, -\infty < a < \infty, b > 0$$

Estimate parameters a and b by

a) method of moments and b) method of maximum likelihood.

- 3) A sample of size 10 was taken from a normally distributed population having mean μ and variance σ^2 , i.e. $X \sim N(\mu, \sigma^2)$. The observations are as follows: 6.18, 5.02, 5.55, 6.32, 3.57, 4.82, 4.95, 5.80, 5.22 and 4.99. Find

a) maximum likelihood estimate (m.l.e) of (i) μ when $\sigma^2 = 0.4$ (ii) σ^2 when $\mu = 5.25$.
b) m.l.e's of μ and σ^2 and $P(X > 4.25)$.

- 4) The following data is drawn from a Negative binomial distribution with parameters k and p ,

$x : 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \text{Total}$

Frequency: 214 123 45 13 4 1 400.

Given $k \in \mathbb{N}$, $0 < p < 1$.

Estimate parameters k and p by method of moments.

- 5) A random variable X takes values 0, 1, 2, 3, 4 and 5 with probabilities:

$$p(x) = (1 + pm) e^{-m} \quad \text{when } x = 0$$

$$= m(1 - p) e^{-m} \quad \text{when } x = 1$$

$$= \frac{e^{-m} m^x}{x!} \quad \text{when } x = 2, 3, 4, 5. \quad \text{Given } m > 0 \text{ and } 0 < p < 1,$$

The observed frequencies are 37, 19, 22, 15, 6 and 1 respectively. Find maximum likelihood estimates of unknown parameters p and m .

BAYES' ESTIMATION

- 1) The proportion θ of defective items in a large shipment is unknown and the prior distribution of θ is Beta with parameters $\alpha = 5$ and $\beta = 10$. Suppose that 20 items are selected at random from the shipment and that exactly one item is found to be defective. If the squared error loss function is used, what is the Bayes' estimate of θ ?
- 2) The daily number of incoming calls at a small city telephone exchange has a Poisson distribution with mean θ . The mean θ , however, varies from day to day. This variation is represented by a gamma density with mean $\alpha\beta = 100$ and variance 200. Five days are selected at random and the number of incoming calls found was 90, 95, 108, 100 and 106. Find Bayes' estimate of the average number of daily incoming calls θ , using squared error loss function.
- 3) White pine is one of the best known species of pines in the Northern United States and Canada. White pine is susceptible to a blister rust, which develops canker in the bark. These cankers swell, resulting in the death of twigs and small trees. A forester wishes to estimate the average number of diseased pine trees per acre in a forest. The number of diseased trees can be modeled by a Poisson distribution with parameter θ . Since θ changes from area to area, the forester believes that θ has an exponential distribution with mean $1/\alpha = 5$. Find Bayes' estimate of the mean number of diseased pine trees per acre, if the forester takes a random sample of 50 and $\sum_{i=1}^{50} x_i = 200$. (Use squared error loss function).
- 4) The distribution of lifetimes of bulbs made by a certain manufacturer is exponential with mean $1/\theta$. Suppose θ has a priori exponential distribution with mean 1. Using squared error loss function, find Bayes' estimate of θ if the total of 25 observations is 29.
- 5) The time (in minutes) required to serve a customer at a certain facility has an exponential distribution, for which the value of the parameter θ is unknown. The prior distribution of θ is a gamma distribution for which the mean is 0.2 and the variance is 1. The average time required to serve a random sample of 20 customers is observed to be 3.8 minutes. If squared error loss function is used, what is the Bayes' estimate of θ ?
- 6) The brain weight (in grams) of adults is assumed to have a normal distribution, that is $N(\theta, \sigma^2 = 6)$. In order to estimate θ , a random sample of 150 gave $\sum_{i=1}^{150} x_i = 241380$ grams. The experimenter believes that θ itself has a normal distribution $N(1800, 9)$. Find Bayes' estimate of θ using squared error loss function. What is the Bayes' estimate if absolute error loss function is used?
- 7) Given that 1, 0, 1, 1, 0, 0, 1, 0, 0, 1 is a random sample from a Bernoulli distribution with parameter θ where θ has prior uniform distribution on $[0, 1]$, obtain Bayes' estimate of θ using squared error loss function. Also find the Bayes' risk.
- 8) $X = 6$ is a single observation from a negative binomial, that is, $NB(10, \theta)$ distribution. If the prior distribution of θ is Beta $B(3, 5)$ obtain Bayes' estimate of θ using quadratic error loss function.

INTERVAL ESTIMATION

- 1) The results of 64 determination of strength (x) of a magnetic field field = 0.1812 units with $\sum (x_i - \bar{x})^2 = 0.0063$. Obtain 90% confidence interval for mean μ assuming Normal distribution of X having an unknown variance.
- 2) Yield of strawberry plant follows Normal distribution with average yield μ and variance σ^2 . Yields of 10 strawberry plants in an uniformity trial are given below.
- | Plant no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Yield | 239 | 176 | 235 | 217 | 234 | 216 | 318 | 190 | 181 | 225 |
- Compute 95% confidence limits for average yield μ when
- σ^2 is known as 1600.
 - σ^2 is unknown.
- 3) The following are the data on time X (in seconds) required to format 1.2 mb floppy diskettes on PC 386.
- | Floppy no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------|-----|----|----|-----|----|----|----|----|
| Time | 100 | 90 | 97 | 101 | 93 | 95 | 90 | 99 |
- Assuming Normal distribution of X compute 99% confidence interval for variance of time for formatting floppy diskettes when
- mean time is known as 90 seconds.
 - mean time is unknown.
- 4) A random sample of size 9 from $N(\mu, \sigma^2)$ population gives sample mean $\bar{x} = 68$ and $\sum (x_i - \bar{x})^2 = 30$. Find 90% confidence interval for σ^2 if
- μ is unknown.
 - μ is known as 72.
- 5) A nutritionist examined the basal metabolism of 30 volunteers in two groups of $m = 16$ and $n = 14$. He reported $\bar{x} = 36$ cal/sqm/hr, $\bar{y} = 32$ cal/sqm/hr. $\sum (x_i - \bar{x})^2 = 72.86$, $\sum (y_j - \bar{y})^2 = 14.46$. Obtain 95% confidence interval for $\frac{\sigma_x^2}{\sigma_y^2}$ assuming Normal distribution of the basal metabolism in two groups with unknown means.
- 6) The oil contents of two brands of soap measured (in mg per gm) were found to be as follows
- | X | 22 | 24 | 25 | 26 | 20 | 23 | 21 |
|---|----|----|----|----|----|----|----|
| Y | 24 | 21 | 20 | 26 | 22 | 25 | 28 |
- Assuming that oil contents X, Y are following $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ find 99% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$.

- 7) Let X denote the permeability of toroids in upper deck of furnace and Y that in the lower deck. An experiment was performed and observations on permeability of toroids were obtained. Permeability of toroids is as follows

X	1919.5	1916.7	1891.6	1964.2	1848.0	2129.6	2218.4	2236.8
Y	2143.4	2055.0	1922.0	2068.1	1911.0	1974.4	2055.1	2154.8

Compute 95% confidence interval for difference between averages of two populations (assume normality).

- 8) Let $f(x, \theta) = \theta e^{-\theta x} : x \geq 0, \theta > 0$. Using m.l.e of θ and asymptotic distribution of m.l.e, obtain 99% confidence interval for θ if sample mean of 48 observations is 13.73.

LINEAR MODELS

- 1) A soft drink bottler is analyzing the vending machine service routes in his distribution system. The engineer responsible for the study has suggested that the two important variables affecting the delivery time(y) are the number of cases of product stocked(x_1), and the distance walked by the route driver(x_2). The engineer has collected 10 observations on delivery time which are shown below.

obs ⁿ no.	1	2	3	4	5	6	7	8	9	10
delivery time:y	16.68	11.50	12.03	14.88	13.75	18.11	8.00	17.83	79.24	21.50
no. of cases: x_1	7	3	3	4	6	7	2	7	30	5
distance (ft): x_2	560	220	340	80	150	330	110	210	1460	605

- a) Fit a linear statistical model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$, to the delivery time data, such that $y_i \sim N(E(y_i), \sigma^2)$ and $e_i \sim N(0, \sigma^2)$ for $i = 1, 2, \dots, 10$. $\beta_0, \beta_1, \beta_2$ are unknown parameters.
- b) Find best linear unbiased estimate of $\beta_0, \beta_1, \beta_2$.
- c) Also find an estimate of σ^2 and of $\text{cov}(\hat{\beta}_1, \hat{\beta}_2)$.
- d) Obtain 95% confidence interval for β_1 when (i) $\sigma^2 = 8.8$ (ii) σ^2 is unknown.
- 2) An experiment was conducted to determine the influence of acid bath temperature(x), on an appropriate measure of the whiteness of rayon (y). X is the controlled variable and Y is the dependent variable.

Whiteness of rayon(y)			Temperature
y_1	y_2	y_3	x
76.5	80.2	76.0	30
83.5	89.1	79.9	35
84.2	77.2	89.5	40
85.7	85.1	99.5	25
97.5	71.5	89.4	45
103.2	84.5	108.7	50
115.2	77.5	111.5	35
108.1	79.2	102.3	65

If $\bar{y}_i$ is the average whiteness at acid bath temperature(x_i), and if we assume that

$$E(\bar{y}_i) = \beta_0 + \beta_1 x_i \text{ for } i = 1, 2, \dots, 8.$$

where β_0 and β_1 are unknown parameters Find

- a) the least square estimate of β_0 and β_1 .
- b) 99% confidence interval for β_0 .
- c) 98% confidence interval for σ^2 .

The following gives a property evaluation data, where y : sale price of the house (in Rs 1000):

x_1 : living space(square feet)	x_2 : age of houses (in years)											
y	25.9	29.5	27.9	25.9	29.9	29.9	30.9	28.9	35.9	31.5	31.0	30.9
x_1	998	1500	1175	1232	1121	988	1240	1501	1225	1552	975	1121
x_2	41	62	40	54	42	56	51	32	32	30	30	32

- a) Fit the linear statistical model $y = \beta_0 + \beta_1(x_1 - \bar{x}_1) + \beta_2(x_2 - \bar{x}_2) + e$ where $y_i \sim N(E(y_i), \sigma^2)$ and $e_i \sim N(0, \sigma^2)$ for $i = 1, 2, \dots, 12$.
- b) Find best linear unbiased estimate of $\beta_0, \beta_1, \beta_2$.
- c) Obtain the best linear unbiased estimate of $\beta_0 + 2\beta_1 - \beta_2$ and its standard error.
- d) Obtain 95% confidence interval for $\beta_0 + 2\beta_1 - \beta_2$.
- e) Test the hypothesis $H_0: \beta_0 + 2\beta_1 - \beta_2 = 33$.

TESTING OF HYPOTHESES-I

- The length of a tube has exponential p.d.f. with mean 6. In order to test $H_0: \theta = 10$ against $H_1: \theta \neq 10$, consider the critical region of the form $C = \{X: X < c_1 \text{ or } X > c_2\}$, where $c_1 < c_2$. Find c_1 and c_2 such that each tail has same probability and the size of the test is 0.05. What is the power function of the test? Find power at $\theta = 2, 8, 15, 20$.
- An urn contains 20 marbles of which an unknown number θ are white. To test the hypothesis $H_0: \theta \leq 3$ against $H_1: \theta > 3$, following procedure is used. Select two marbles at random from the urn without replacement and reject H_0 if one of them is white. Find the level of significance and power of the test at $\theta = 5, 10, 18$.
- The identification of a cell as of type A or type B is long and expensive, but recent series of experiments have shown that the quantity (in certain units) of an easily quantifiable chemical contained in the cell is well described by a $N(\mu = 10, \sigma^2 = 1)$ random variable for type A and by a $N(\mu = 15, \sigma^2 = 4)$ random variable for type B. In future it is decided to classify a cell as type A if it contains not more than 12 units of chemical and as type B otherwise. Obtain suitable measures of the liabilities to misclassifying. Where should the point of division be drawn if it is required to set the probability of misclassifying a type A cell equal to 0.01? What then is the probability of other kind of misclassification?
- A random sample of size 2 is drawn from exponential distribution with mean θ to test $H_0: \theta = 2$ against $H_1: \theta = 3$. The critical region is $C = \{(X_1, X_2) / 0.5 \leq X_1 + X_2 \leq \infty\}$. Find the level of significance and power of the test.
- A random variable X has the following probability density function
- $$f(x, \theta) = \theta x^{\theta-1}; 0 < x < 1$$
- $$= 0; \text{ otherwise}$$
- To test $H_0: \theta = 1$ against $H_1: \theta > 1$ the critical region is $C = \{(X_1, X_2) / 0.75 \leq X_1 X_2\}$. Find the size of the test and values of power function at $\theta = 4, 6, 10, 15$. Sketch the power curve.
- An item is packed in lots of 100 each. Let X denote the number of defectives in a packet of 100 and is a Poisson variable with mean m . To test $H_0: m = 0.5$ against $H_1: m < 0.5$ it is decided to select 12 packets and inspect the items in them. If they contain less than two defectives, it is proposed to reject H_0 . Find the level of significance of the test. Obtain the $K(m)$ power function of the test. Sketch the power curve, considering $m = 0.3, 0.25, 0.16, 0.08, 0.02$.
- Consider a random variable following a Normal distribution with mean θ and variance 4. The simple hypothesis $H_0: \theta = -1$ is rejected and the alternative $H_1: \theta = 1$ is accepted if $X_1 + 2X_2 + 3X_3 + 4X_4 + \dots + 10X_{10} > 0$. Find the power and size of the test.
- The proportion p of swimmers in a large population is unknown and it is desired to test $H_0: p = 0.5$ against $H_1: p = 0.75$. A random sample of 5 people is drawn from the population. Let $Y =$ number of swimmers in the sample. The following critical regions are proposed.
- i) $C_1 \equiv \{Y \leq 1\}$, ii) $C_2 \equiv \{Y \leq 4\}$, iii) $C_3 \equiv \{Y = 1, 5\}$.
- In each case obtain probability of type I and type II errors and hence suggest the best critical region from the given three.

TESTING OF HYPOTHESES-II

Suppose X_1, X_2, X_3, X_4, X_5 is a random sample from Bernoulli distribution with parameter p . Find BCR of size 0.1035 for testing $H_0 : p = 1/4$ against $H_1 : p = 1/3$. What will be the conclusion if the sample observations are 1, 0, 0, 0, 1?

Let $X_1, X_2, \dots, X_{10}$ be a random sample of size 10 from Poisson distribution with mean θ . Obtain MP test of size 0.0404 for testing $H_0 : \theta = 0.5$ against $H_1 : \theta = 0.1$. Also obtain power of the test.

A random sample of size 10 is drawn from $N(\mu = 60, \sigma^2)$ distribution. Obtain BCR of size 0.05 for testing $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 4$. What will be the conclusion if the sample observations are 60.3, 60.0, 61.2, 59.3, 59.7, 62.3, 56.5, 57.6, 53.4, 62.0?

A sample of size n is taken from $N(\mu, \sigma^2 = 25)$ population. Obtain BCR for testing $H_0 : \mu = 50$ against $H_1 : \mu = 60$. Also, find the value of n such that probabilities of type I and type II errors are 0.025 and 0.01 respectively.

A random variable X has Poisson distribution with parameter θ . To test $H_0 : \theta = 0.75$ against $H_1 : \theta < 0.75$, a random sample of size 8 is taken. Obtain UMP test of size 0.1 assuming normal approximation to Poisson distribution. Also, determine values of power function $K(\theta)$ at $\theta = 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$.

Design Neymann-Pearson's uniformly most powerful test for testing $H_0 : p = 0.2$ against $H_1 : p > 0.2$ based on a single observed value of x from a population with probability distribution

$$p(x) = q^{x-1} p; x = 1, 2, 3, \dots; (q = 1 - p, 0 < p < 1)$$

$$= 0; \text{ otherwise.}$$

Assume that level of significance of the test is approximately 0.36. Evaluate the power function $K(p)$ for $p = 0.25, 0.30, 0.35, 0.40, 0.45, 0.50$ and draw the power curve.

A sample of size 10 is drawn from $N(\mu, \sigma^2 = 10)$ distribution to test $H_0 : \mu = 100$ against $H_1 : \mu < 100$. Derive UMP test of size 0.10. What will be the conclusion if the sample observations are 100.1, 97.6, 133.5, 68.1, 82.3, 93.8, 102.5, 113.7, 79.2, 108.6? Also, find the value of power function when $\mu = 98$.

A random sample of size 9 is drawn from a population having p.d.f.

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}; x \geq 0, \theta > 0$$

$$= 0; \text{ otherwise}$$

Derive UMP test of size $\alpha = 0.05$ for testing $H_0 : \theta = 2$ against $H_1 : \theta > 2$. If a random sample from this population yields values 1.7, 3.5, 2.15, 3.1, 2.21, 1.78, 1.89, 2.17, 1.5; will the null hypothesis be accepted?

LIKELIHOOD RATIO TEST

(Derivations of the tests are not expected.)

- 1) Let a random variable $X \sim N(\mu, \sigma^2)$. To test $H_0: \mu = 0$ against $H_1: \mu \neq 0$ where $\sigma^2 = 4$, what is the critical region of size 0.05 specified by likelihood ratio test criterion? If the sample of size 10 gives mean = 0.6, do you accept or reject H_0 at 5% level of significance?
- 2) The life of a new battery follows Normal distribution with unknown mean μ and unknown standard deviation of 12 hours. The producer claims that on an average a battery will last longer than 70 hours. Is the average life of battery 70 hours or producer's claim is true at 5% level of significance?
- 3) A random sample of size 10 is drawn from a Normal population with mean $\mu = 70$ and unknown variance σ^2 . In order to test $H_0: \sigma^2 = 90$ against $H_1: \sigma^2 \neq 90$, use likelihood ratio test at 5% level of significance and state your conclusion if the sample observations are 86, 77, 71, 76, 64, 96, 72, 66, 77, 91.
- 4) A random sample of 25 people at a local weight-loss center is taken and the mean weight loss is found to be 5 kg with a standard deviation of 1.5 kg. Assuming weight loss follows Normal distribution, can it be concluded that the population standard deviation of weight loss is 2 kg against the alternative that it is less than 2 kg at 1% level of significance?
- 5) Let X and Y be independently distributed normal variables with parameters $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ respectively where the common variance σ^2 is unknown. What is the likelihood ratio test to test $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 < \mu_2$ based on random samples drawn from the two populations? If the observed values are

X	28	31	26	27	23	38	37
Y	37	42	34	37	35		

 What is your conclusion at 1% level of significance?
- 6) A random sample of 7 brand X light bulbs yielded mean life of 891 hours and standard deviation of 10. A random sample of 10 brand Y light bulbs yielded mean life of 592 hours and standard deviation of 8. Use these data to test $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 > \sigma_2^2$ at 1% level of significance. Assume that life of light bulb of both brands follow Normal law.
- 7) Let 1.5, 2.4, 0.9, 1.3, 0.5, 2.6, 1.7, 0.4, 1.4, 0.8 be a random sample from Exponential distribution with mean λ . Can you conclude that $\lambda \neq 2$ at 5% level of significance?

NON-PARAMETRIC TESTS-II

- 1) Six randomly chosen models went on a diet in an attempt to lose weight (in lbs.) with the following results.
- | | | | | | | | | |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Weight Before | 124 | 141 | 138 | 132 | 151 | 138 | 140 | 145 |
| Weight After | 115 | 136 | 133 | 128 | 153 | 131 | 135 | 139 |
- Use sign test at 5% level of significance to check if the diet is an effective means of losing weight.

- 2) Test scores of a group of 15 high-school students before and after a training programme are as given below.
- | | | | | | | | | | | | | | | | |
|--------------|----|----|----|----|----|----|----|----|----|-----|-----|----|-----|----|----|
| Score Before | 63 | 75 | 78 | 84 | 58 | 58 | 70 | 76 | 74 | 88 | 74 | 94 | 99 | 79 | 93 |
| Score After | 84 | 86 | 75 | 94 | 50 | 95 | 97 | 98 | 72 | 100 | 101 | 98 | 105 | 84 | 90 |
- Use sign test at 1% level of significance to check if the training has any effect on the test scores. (Use normal approximation.)

- 3) Twelve children were asked to solve a puzzle under normal conditions and under the conditions of stress. The following data represents time (in seconds) taken to solve the puzzle.
- | | | | | | | | | | | | | |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Normal Condition | 126 | 117 | 115 | 118 | 118 | 128 | 125 | 120 | 125 | 132 | 138 | 124 |
| Stress Condition | 130 | 118 | 125 | 120 | 121 | 125 | 130 | 120 | 119 | 140 | 145 | 130 |

Do the data indicate sufficient evidence to conclude that time taken is more under conditions of stress? Use Wilcoxon test at 5% level of significance. (Use normal approximation.)

- 4) A random sample of size 10 is obtained from one population and a random sample of size 11 is obtained from a second population. The observations are as follows.

Population-I (X) 0.26 1.00 1.29 5.00 8.00 16.00 1.60 8.20 10.20 12.81

Population-II (Y) 0.16 0.08 0.10 0.00 1.28 2.02 0.80 2.56 1.26 5.10 6.24

Test, at 1% level of significance, using run test whether the population distributions represented by these two samples are different. (Use normal approximation.)

- 5) The average monthly expenditures (in Rs.) on cosmetics of working women (X) and housewives (Y) are listed below.

X 770 820 850 860 920 870 880 910 1000 890 930

Y 830 780 660 750 650 970 855 730 900

Use median test at 5% level of significance to check if there is a significant difference between the two sets of data.

- 6) The number of words in emails sent by boys (X) and girls (Y) were found to be as follows.

X 85 89 86 91 76 93 100 82 92 88 84

Y 83 73 65 67 90 77 78 97 87 75

Use Mann-Whitney-Wilcoxon test at 5% level of significance to test whether the average number of words per email is same for boys and girls.

Twenty seven children, who enrolled for a swimming class, were divided into two groups A and B. The children in group A were taught by traditional methods, while group B children were taught by a new method. The instructor claimed that the new method is more effective than the traditional method. At the end of the course, the swimming scores of the children in the two groups were as follows.

Group A (X) 66 86 80 78 77 63 62 87 75 84 61 65

Group B (Y) 95 85 56 57 91 79 94 55 51 74 81 93 95 82 68

Use Mann-Whitney test to check the claim of the instructor at 1% level of significance.

SEQUENTIAL PROBABILITY RATIO TEST

- 1) Let X be a random variable having probability mass function (p.d.f.)

$$f(x, p) = p^x (1-p)^{1-x} ; x = 0, 1$$

$$= 0 ; \text{otherwise}$$
- Derive SPRT of strength $\alpha = 0.05$ and $\beta = 0.10$ for testing $H_0 : p = 0.5$ against $H_1 : p = 0.3$. What will be the conclusion if the first sample observation has value 1?
- 2) A random variable X follows a binomial distribution ($n = 10, p$). Derive SPRT of strength $\alpha = 0.15$ and $\beta = 0.20$ for testing $H_0 : p = 1/4$ against $H_1 : p = 3/4$.
- 3) The number of road accidents per day at a junction is assumed to follow a Poisson distribution with unknown mean λ . In order to test $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$, it was decided to use SPRT of strength $\alpha = 0.10, \beta = 0.10$. What is the decision if the number of accidents observed on 8 days is 1, 1, 0, 2, 0, 1, 2, and 0 respectively? Also represent the test graphically.
- 4) Let X follow $N(\mu, \sigma^2 = 0.5)$ distribution. Derive SPRT to test $H_0 : \mu = 10$ against $H_1 : \mu = 15$ with $\alpha = 0.05, \beta = 0.01$. What is the decision if the sample observations are 12.2, 12.9, 12.5, 13.2, 12.6, 11.9?
- 5) Let X follow $N(\mu = 120, \sigma^2)$ distribution. Derive SPRT to test $H_0 : \sigma = 28$ against $H_1 : \sigma = 20$ with $\alpha = \beta = 0.05$. At what stage and what decision will be taken if the sample observations are 130, 185, 150, 165, 170, 200, 140?
- 6) A random variable X follows an exponential distribution with mean $1/\theta$. Derive SPRT to test $H_0 : \theta = 3$ against $H_1 : \theta = 1$ with $\alpha = 0.20, \beta = 0.10$. What is the decision if the sample observations are 1.2, 0.5, 0.9, 2.3, 1.8?

GENETICS-I

The lack of pigmentation called albinism in man is the result of a recessive allele (a) and normal pigmentation is a result of its dominant allele (A). Two normal parents have an albino child. Determine the probability that

- next child is an albino.
- the next two children are albinos.
- out of two children one is albino and other is normal.

A dominant gene W produces wire-haired texture in dogs; its recessive allele w produces smooth hair. A group of heterozygous wire haired individuals are crossed and their progeny are then test-crossed. Determine expected genotypic and phenotypic ratios among test-crossed progeny.

Shape of radishes may be long (LL), round (ll) and oval (Ll). If long are crossed with oval ones and then F_1 are allowed to cross among themselves; what phenotypic ratios are expected in F_2 ?

Black wool of sheep is due to recessive allele b and white wool due to its dominant allele B. A white male is crossed with a white female, both carrying the allele for black. They produce a white lamb (male offspring), which is then backcrossed to a female parent. What is the probability that backcrossing offspring being black?

An isolated field of corn was found to be segregating for yellow and white endosperm. Yellow colour is governed by a dominant allele and white by its recessive allele. A random sample of 1000 kernels revealed that 910 were yellow. Find the allelic frequencies for this population.

The left handedness is assumed to be an effect of a single recessive trait. In a certain population, the prevalence of left handed people is 10%. If both parents are right handed, what is the probability that their first child is left handed?

Baldness is governed by sex influenced trait which is dominant in men and recessive in women. In a sample of 10,000 men 6233 were found bald. In a sample of equivalent size, how many non bald women are expected?

The presence of horn in a special breed of sheep is dominant in males and recessive in females. If a sample of 900 female sheep is found to contain 225 horned ones

- what percentage of females is expected to be heterozygous?
- what percentage of males are expected to be horned?

A narrow reduced eye called 'bar' is dominant sex linked condition (B) in drosophila. Full wide type eye is produced by its recessive allele b. A homozygous wide type female is mated to a bar eyed male. Determine F_1 and F_2 phenotypic and genotypic expectations. (Assume F_1 are crossed among themselves to produce F_2).

Among New-York Caucasians, the frequency of ABO blood groups were found 49% type O, 36% type A, 12% type B and 3% type AB. Obtain allelic frequencies using

- Bernstein's method
- gene counting method.

GENETICS-II

- In a summer-squash, white fruit colour is governed by a dominant allele W and yellow by its recessive allele w . A dominant allele at another independent assorting locus D produces disc shaped fruit and its recessive allele d yields sphere shaped fruit. Homozygous white disc shaped variety is crossed with homozygous yellow sphere shaped variety to produce F_1 .
- a) If F_1 are allowed to cross at random among themselves, obtain phenotypic ratios in F_2 generation?
 - b) If F_1 are test crossed what phenotypic ratios are expected in F_2 ?
- Short hair in rabbits is governed by a dominant allele L and long by its recessive allele ℓ . Black hair results from the action of dominant genotype B_{-} and brown from recessive genotype bb . Determine expected genotypic and phenotypic ratios in the progeny from the cross
- a) $L\ell Bb \times L\ell bb$ b) $L\ell Bb \times L\ell Bb$ c) dihybrid testcross.
- In a garden pea, yellow cotyledon is dominant to green and inflated pod shape is dominant to constricted form. When both traits are considered jointly in self fertilized dihybrids, the progeny appeared in the following manner: 61 green and constricted, 193 green and inflated, 556 yellow and inflated, 184 yellow and constricted. Test the data for independent assortment.
- The shape of radish may be long (LL), round ($\ell\ell$) or oval ($L\ell$) and its colour may be red (RR), white (rr) or pink (Rr). If long white strain is crossed with round red strain, what phenotypic ratios are expected in F_1 and F_2 ? (F_1 are crossed among themselves to produce F_2)
- A gene locus with co-dominant alleles is known to govern feather colour in chickens such that BB - black, Bb - blue, bb -splashed white. An independently segregating locus L governs leg length such that LL -normal length, $L\ell$ -squatty type leg, $\ell\ell$ - genotype is lethal. Determine phenotypic ratio among the matured progeny if dihybrids are crossed among themselves.
- A dominant allele Y is responsible for yellow seed colour and its recessive allele y produces green. Co-dominant allele at independent assorting locus R specify the shape, such that RR -round, Rr -oval and rr - shrunk. From a cross between yellow-oval and green-oval, obtain the phenotypic ratios expected in the progeny.
- A dominant allele L governs short hair in guinea pigs and its recessive allele ℓ governs long hair. Co-dominant allele at an independently assorting locus specify hair colour. YY -yellow, Yy - cream, yy -white. From crosses between dihybrid short-cream pigs, the following phenotypic distribution is observed. 50-short&cream, 21-short & yellow, 23-short & white, 21-long & cream, 7-long & cream and 6-long & white. Are data consistent with Mendel's laws?
- It was assumed C stands for colour and R (independent of C) is responsible for red colour, with recessive allele r responsible for yellow. cc represents white. (c is recessive). From dihybrid crosses in the offspring generation; out of 160 plants; 90 showed red colour, 25 yellow and rest white. Test the data for recessive epistasis.

Coloured aleurone in corn is due to interaction of two dominant genes in the genotype $A_C_;$ all other genotypes at this locus produce colourless aleurone. A homozygous coloured strain is test-crossed to pure colourless strain, the F_1 are with coloured aleurone. When F_1 are crossed among themselves to produce F_2 , F_2 exhibits 3300 coloured & 2460 colourless. Test the data for duplicate recessive epistasis.

In an experiment on random mating the frequencies of the phenotypes $A_B_$, A_bb , $aaB_$ and $aabb$ were 8125, 275, 975 and 625 respectively. Obtain allelic frequencies.

In a dihybrid testcross it was observed that the phenotypes $A_B_$, A_bb , $aaB_$ and $aabb$ occurred 67, 19, 29 and 77 times respectively. Verify whether there is linkage, if so obtain measure of linkage by m.l.e. method, also obtain its S.E. (Assume genes are in coupling phase)

In dihybrid intercross (genes are in coupling phase) the observed frequencies of the phenotypes $R_Y_$, R_yy , $rrY_$ and $rryy$ in F_1 generation were 187, 35, 37, 31 respectively. Test whether there is a linkage, if so obtain measure of linkage by m.l.e. method, also obtain its S.E.

In dihybrid cross $AB/ab \times AB/ab$ the observed frequencies of the phenotypes $A_B_$, A_bb , $aaB_$ and $aabb$ in F_1 generation were 1221, 219, 246, 243 respectively. Estimate the recombination factor between A and B by

- square root method
- product ratio method

In a cross of the type $Ry/rY \times Ry/rY$ the following results were found.

Phenotype	$R_Y_$	R_yy	$rrY_$	$rryy$
Observed freq	360	120	160	20

Estimate the recombination factor between A and B

- square root method
- product ratio method

- 1) For a simple epidemic with one infected person in a population of size 86 and the infection rate as 0.35, draw the epidemic curve by taking $\tau = 0.03, 0.04, 0.05, 0.06, 0.07, 0.08$. When the curve attains maximum? What are the number of susceptibles and number of infectives at that time?
- 2) In a simple epidemic there were 55 susceptibles and 3 infectives at the start. The infection rate is 0.95, $\tau = t\beta$ obtain values of ω when $\tau = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09$. Draw the epidemic curve. At what value of τ epidemic curve attains its maximum? What are the number of susceptibles and infectives at that time?
- 3) In a general epidemic model with removal, x and y respectively represent the number of susceptibles and the number of infectives at any time t . If the total size of the group is 500 with $x_0 = 450$ and relative removal rate is 300, obtain values of y when x take the values 450, 400, 350, 250, 200, 150. What is the maximum value of y ? At that time how many individuals are expected to be removed? According to threshold theorem what will be lower limit of x ?
- 4) Assuming that an epidemic has started with five carriers in a group of 200 susceptibles, find an estimate of the total size of the epidemic if the relative removal rate for carriers is 4.
- 5) For a carrier model there are 275 susceptibles and 17 carriers at the start, infection rate is $\beta = 0.1$ and removal rate is $\gamma = 0.375$. Find the number of susceptibles that will escape infection.
- 6) For a carrier model with 20 carriers and 450 susceptibles at start, infection rate is $\beta = 0.0075$ and removal rate $\gamma = 0.075$, find the number of susceptibles left uninfected at the end of 5 days, 10 days, 15 days and 20 days. Will the situation change substantially if by intensive drive the removal rate for carriers is doubled?

- 1) The following data is collected from households of size 3 with one primary case of measles. Estimate the value of p , the chance of adequate contact and obtain its S.E.
- | | | | | |
|---------------|-----|-------|---------|-------|
| Type of chain | {1} | {1,1} | {1,1,1} | {1,2} |
| Observed freq | 34 | 25 | 140 | 135 |
- 2) Estimate the value of p , the chance of adequate contact between a susceptible and an infective in conjunctivitis when 500 households each of size 3 with single introduction were considered. It was reported that in 180 households the disease spread to two members and in 197 household all members are affected. Also obtain its Standard error.
- 3) Assuming all households are of size 4, estimate p , the chance of adequate contact. The observed frequencies of the chains {2}, {2,1}, {2,1,1} and {2,2} are 123, 180, 73 and 124 respectively. Use a) Greenwood model
b) Reed Frost model
- 4) Estimate p -the chance of adequate contact between susceptible and infective in an epidemic from a data of 600 households each of size 4, started with two introduction, only two members were infected in 143 households whereas all members were infected in 217 houses. Use Greenwood model.
- 5) Estimate p -the probability of adequate contact between a susceptible and infective. Data is collected from 175 households each of size 4 with single introduction. The observed chain frequencies of the chains {1}, {1,1}, {1,1,1}, {1,2}, {1,1,1,1}, {1,1,2}, {1,2,1} and {1,3} are 10, 9, 6, 20, 11, 8, 20 and 116 respectively. Estimate p using
a) Greenwood model
b) Reed Frost model.
- 6) For a household of size 4, find the probabilities of the chains {1,2,1} and {2,1}, {2,1,1} using
a) Greenwood model
b) Reed Frost model
assuming probability of adequate contact between susceptibles and infective is 0.05.

BIOASSAYS-I

- 1) Lethal doses of two preparations A and B required for a fixed response are given to two different groups of seven experimental animals each. Obtain the relative potency of A with reference to B assuming normality of

a) doses

b) logarithms of doses

A	1.55	1.58	1.71	1.44	1.24	1.89	2.34
B	2.42	1.85	2.00	2.27	1.70	1.47	2.20

Also obtain 95% fiducial limits for the relative potency in both the cases.

- 2) The following data shows the amount of drug A and drug B required to cause cardiac arrest in pigs.

Dose A mg/kg	46.0	46.7	44.9	41.0	45.2	50.4	36.9	40.0
Dose B mg/kg	68.6	63.3	73.6	68.0	69.1	65.5	64.4	65.0

Eight pigs were used for each drug. Using log dose as metameter, estimate the potency of B with respect to A as standard preparation. Also find the 95% fiducial limits for the same.

In a (2,2) point parallel line assay, dosages were
 $T_1 = 28.0$ mg, $T_2 = 42.0$ mg of test preparation

$S_1 = 14.0$ mg, $S_2 = 21.0$ mg of standard preparation

Responses were given as follows.

	S_1	T_1	T_2
S_1	62	46	65
43	62	53	72
48	66	58	74
53	70	61	76
52			

Estimate the potency of test preparation with standard preparation after verifying validity tests. Also obtain 95% fiducial limits for the potency.

The following responses were observed in a (3,3) point assay of vitamin B_{12} .

	S_1	S_2	S_3	T_1	T_2	T_3
Dose	0.8	1.2	1.8	4	6	9
Response	65	75	86	60	78	84
	60	76	83	62	73	84
	61	68	83	67	66	83
	45	55	82	65	75	85
	72	75	82	58	73	79
	62	71	84	70	71	84

Estimate the potency of the test preparation relative to standard preparation after carrying out the necessary validity tests.

In a (3,3) point-parallel line assay, dosages were $S_1 = 0.12$, $S_2 = 0.24$, $S_3 = 0.48$, and $T_1 = 0.25$, $T_2 = 0.5$, $T_3 = 1.00$. (with usual notations)

Responses were given as follows.

Dose	S_1	S_2	S_3	T_1	T_2	T_3
Response	298	250	230	274	176	19
	280	193	228	293	231	17
	262	228	199	325	222	20
	304	259	216	339	189	18
	300	256	201	301	210	20

Test for assay validity.

BIOASSAYS-III

Eight doses of a preparation of insulin were injected to batches of mice and the number of mice showing the symptoms of collapse was recorded. Table below gives dose (z), the number of mice tested (n) and number of convulsed mice(r). Using logarithm of the dose as a dose metameter estimate LD 50 for this preparation using probit method.

z	5.2	7.0	8.5	10.5	13	18	21	28
n	32	38	37	40	37	31	37	30
r	5	11	14	18	21	23	30	27

Estimate the relative potency using probits. Make natural log transformation.

STANDARD PREPARATION		
Z	n	r
3.4	33	2
5.2	32	5
7.0	38	11
8.5	37	14
10.5	40	18
13.0	37	21
18.0	31	23
21.0	37	30
28.0	30	27

TEST PREPARATION		
Z	n	r
6.5	40	2
10.0	30	10
14.0	40	18
21.5	35	21
29.0	47	27

The following table gives the numbers of frogs tested (n) with different doses (z) of a drug and the number of survivals (r). Log transformation ($x = \log_{10} Z + 4$) is used for linearity. Estimate LD50 using probit method.

n	20	20	20	18	10	10
r	20	16	07	04	01	00
$Z \times 10^6$	200	25	250	300	350	400

INVENTORY MODELS-I.

- 1) A shopkeeper has a uniform demand of an item at the rate of 600 items per year. He buys from a supplier at a cost of Rs.8 per item and the cost of ordering is Rs.12 each time. If the stock holding costs are 20% per year of value, how frequency should he replenish his stocks and what is the optimal order quantity? What is the minimum total relevant cost?
- 2) A manufacturing company purchases 18,000 parts of a machine for its annual requirements, ordering one month's usage at a time. Each part costs Rs.20. The ordering cost per order is Rs.30 and the carrying charges are 15% of the average inventory per year. Is the current ordering policy the best for the company? If not, what is the optimal policy? What would be the company's annual saving if it adopted the optimal policy?
- 3) The ABC company sells its product at the rate of 500 units per day throughout a working year of 250 days. To satisfy this demand the company manufactures it at the rate of 800 units per day. The direct costs per unit are Rs.1.50 for material, Re.1.00 for labour and Rs.2.50 for hire of machinery and other variable overheads. The set-up cost per batch would be Rs.600 and the stock-holding costs have been estimated at 20% per annum. Calculate the optimum quantity to be manufactured in each batch and the total inventory cost for the year.
- 4) Suppose you are reviewing the production lot-size decision associated with a production operation where the production rate is 8000 units per year, annual demand is 2000 units, set-up cost is Rs.300 per production run and holding cost is Rs.1.60 per unit per year. The production run is 500 units every 3 months. Would you recommend a change in production lot size? If so why? How much could be saved, if any, by adopting the new production lot size?
- 5) A dealer supplies you the following information with regard to an item of inventory.

Annual demand:	5000 units.
Ordering costs:	Rs.250 per order
Inventory-holding costs.	30% of the value of inventory per year
Inventory stock-out costs:	Rs.10 per unit per year
Price:	Rs.100 per unit.

Find

 - a) Economic order quantity.
 - b) What quantity should be allowed to be backordered?
 - c) What will be the maximum inventory at any particular time of the year?
 - d) Optimum total relevant cost.
- 6) A furniture dealer sells special computer chairs. Each purchase order costs Rs.50 to the dealer and the holding costs amount to Rs.80 per chair per year. The dealer sells 90 chairs per month. He has estimated the shortage cost at Rs.20 per chair per year.
 - a) What is the EOQ for the chair?
 - b) When the supply of the chairs comes, how many chairs are expected to be delivered to the customers immediately?
 - c) What is the minimum total relevant cost?

- 7) A company has a demand of 12000 units/year for an item and it can produce 2000 such items per month. The cost of one setup is Rs.400 and the holding cost per unit per month. The shortage cost of one unit is Rs.20 per year. Find the optimum lot size and the total cost per year assuming Rs.4 as the cost per unit. Also find the maximum inventory level.

- 1) A magazine vendor buys magazines from its publisher for Rs.10 each and sells them for Rs.15 each. The magazine is a weekly magazine and the probability distribution of weekly demand is given as follows.

Demand:	30	31	32	33	34	35	36
Probability:	0.09	0.13	0.18	0.18	0.15	0.12	0.15

If the vendor cannot return unsold magazines how many magazines should the vendor order each week?

- 2) The purchase price of a unit of a certain product is Rs.1,000 and its holding cost per unit per period is Rs.100. If the order quantity is 4 units, find the permissible range of C_2 given the following probability distribution of demand.

Demand:	0	1	2	3	4	5	6	7	8
Probability:	0.05	0.10	0.10	0.20	0.25	0.15	0.05	0.05	0.05

- 3) A sweet-meat company sells its 'Mango Barfi' by weight. The barfi fetches the best price on the day it is prepared but there is an unlimited market for one day old barfi. The company makes a profit of Rs.20 per kg. on fresh barfi but a loss of Rs.4 for one day old barfi. The daily demand for the product is a random variable with p.d.f. given below.

$$f(x) = 0.1 - 0.005x ; 0 \leq x \leq 20$$

$$= 0 ; \text{otherwise}$$

How many kgs. of the barfi must the company make daily?

- 4) A wholesaler in flowers wants to decide how many baskets of flowers he must procure every morning. He estimates the demand for flowers to be rectangular between 30 and 40 baskets. The purchase price for one basket is Rs.50 and the selling price is Rs.80. One day old flowers are worthless. What would be your advice regarding the number of baskets the wholesaler must procure every morning?

The probability distribution of weekly sales of a certain item is as follows.

Weekly sales:	0	1	2	3	4	5	6
Probability:	0.03	0.06	0.15	0.34	0.23	0.12	0.07

The cost of carrying one unit for one week is Rs.40 and the cost of understock is Rs.60 per unit per week. If the demand occurs at a uniform rate over the week how many units of this item must be stocked at the beginning of each week?

The demand for 'integrated circuits' at an electronic goods dealer is a random variable with probability distribution as given below.

Weekly demand(in '00 units):	0	1	2	3	4	5	6
Probability:	0.08	0.10	0.22	0.25	0.15	0.15	0.05

Determine the optimum inventory policy for the dealer if he is to order the circuits from a manufacturer at the beginning of each week. From past experience he has found that the cost of keeping 100 circuits for a week is Rs.0.20 while the loss due to not having the 100 circuits in stock is Re.1 per week. Assume demand occurs at a uniform rate.

The daily demand in kgs. for a certain product is a continuous random variable with p.d.f.

$$f(x) = 0.05 ; 0 \leq x \leq 20$$

$$= 0 ; \text{otherwise}$$

The cost of making one kg. of this product is Rs.180 while the selling price is Rs.300 per kg. How many kgs. must be made daily if the demand occurs at a uniform rate?

The daily demand for a certain commodity is a random variable with a probability distribution which is rectangular between 500 and 1,000 units. The cost of manufacture of a unit is Rs.80. If the inventory policy is to stock 700 units every morning, compute the selling price. Assume that all unsold units at end of day are worthless.

The ABC company is engaged in the manufacture of personal computers. The XYZ company undertakes to supply 24000 chassis every year to the computer company. The XYZ company being a small-scale entrepreneur undertakes to supply the chassis required on a daily basis. It finds that the cost of holding a computer chassis in a stock is Re.1 per unit per month and there is a clause in the contract penalizing it Rs.2 per unit per month for missing the scheduled delivery date. The XYZ company produces the chassis at the rate of 3000 chassis per month and the setup cost is Rs.3500 per batch. Determine

- How frequency should batches be started and what should be the batch size?
 - What is the minimum total relevant cost?
- Annual demand for a particular item of inventory is 10000 units. Inventory carrying cost per unit per year is 20% and ordering cost is Rs.40 per order. The price quoted by the supplier is Rs.4 per unit. However the supplier is willing to give a discount of 5% for orders of 1500 or more. Is it worthwhile to avail of discount offer?

Find the optimal order quantity for a product for which price breaks are as follows:

Quantity ordered (units)	Unit cost (Rs.)
$0 \leq Q < 100$	20
$100 \leq Q < 200$	18
$200 \leq Q$	16

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost

of the product and the cost of ordering is Rs.25 per order.

The annual demand for a product is 64,000 units. The ordering cost is Rs.10 per order and the estimated cost of carrying one unit in stock for a year is 20%. The normal price of the product is Rs.10 per unit. However, the supplier gives a quantity discount of 2% on an order of at least 1,000 units at a time and a discount of 5% if the order is for at least 5,000 units. Suggest the most economic purchase quantity per order.

Company A wants to know what production cost its major competitor Company B has assigned to product item X. After a bit of investigation Company A has collected the following data about Company B's production process:

Production lot size.....	2,600 units
Set-up cost	Rs.135
Annual demand.....	30,000 units
Production rate.....	200 units per day
Inventory holding costs.....	28% of the average inventory value per year

Company A has further learnt that Company B produces according to the economic lot size model. What is the Company B's cost of producing product item X? (1 year = 300 days).

REPLACEMENT THEORY

- 1) A firm is considering replacement of a machine, whose cost is Rs.12, 200 and the scrap value Rs.200. The running (maintenance & operating) costs are found to be as follows.

Year	1	2	3	4	5	6	7	8
Running cost (Rs.):	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

- 2) The data collected in running a machine, the cost of which is Rs.60,000, are given below.

Year	1	2	3	4	5
Resale value(Rs.)	42,000	30,000	20,400	14,400	9,650
Cost of spares(Rs.)	4,000	4,270	4,880	5,700	6,800
Cost of labour(Rs.)	14,000	16,000	18,000	21,000	25,000

Determine the optimum period for replacement of the machine.

- 3) The Mini Transport Company owns three mini buses, two of which are two years old while the third one is only a year old. Each of these buses was purchased for Rs.8 lakhs. The company contemplates replacing the three buses by two full-sized buses, each such bus containing 50% more seating capacity than a mini bus. Cost of each is Rs.12 lakhs. Using the following data, state whether the mini buses be replaced by the Full- sized buses. If not, state why? If yes, state when?

FOR A MINI BUS

Year	1	2	3	4	5	6	7	8
Running cost(Rs.000's)	30	36	48	50	80	112	150	200
Resale value (Rs.000's)	700	610	550	490	320	200	100	50

FOR A FULL-SIZED BUS

Year	1	2	3	4	5	6	7	8
Running cost(Rs.000's)	34	39	47	58	72	90	120	160
Resale value (Rs.000's)	1000	920	860	810	760	660	540	400

- 4) The data on the running costs per year and resale price of equipment A whose purchase price is Rs.200, 000 are as follows.

Year	1	2	3	4	5	6	7
Running cost(Rs.)	30,000	38,000	46,000	58,000	72,000	90,000	110,000
Resale value (Rs.)	100,000	50,000	25,000	12,000	8000	8000	8000

- a) What is the optimum period of replacement?

- b) When equipment A is two years old, equipment B which is a new model for the same usage is available. The optimum period of replacement is 4 years with an average cost of Rs.72,000. Should equipment A be changed with equipment B? If so, when?

A two wheeler is costing Rs.50, 000 and operating & maintenance cost for its use over a specific distance are estimated to be Rs.8000 for each of first five years and increase by Rs.2000 per year from sixth year onwards. If money is worth 10% per year, when should the two wheeler be replaced optimally? (No scrap value)

A manufacturer is offered two machines A & B. The price of A is Rs.5000 and running costs are estimated at Rs.800 for the first five years each, increasing by Rs.200 every year after fifth year. Machine B, which has similar capacity as A, costs Rs.2500, but will have running costs of Rs.1200 for each of the first six years, increasing by Rs.200 per year thereafter. If money is worth 10% per year and there is no scrap value for the machines, determine which machine should be purchased.

- 7) ARUN company limited provides ferry service across the Ganges. One of their boats is in poor condition and requires immediate repairs costing Rs.20,000. The ferry however can also be disposed off right now for Rs.7000. If repaired and used it will have a life of ten years but will need minor repairs at the end of five years costing Rs.8000. Operating cost of the ferry will be Rs.16000 each year if all the repairs are carried out. At the end of ten years, the ferry will have scrap value of Rs.5000. As an alternative, the company may purchase a new ferry for Rs.40,000. It has an estimated life of ten years with a minor repairs required at the end of five years amounting to Rs.4000. The operating cost for the ferry will be Rs.12000 every year. At the end of ten years the ferry will have a scrap value of Rs.6000. What should the company do if money is worth 12% per year? (Assume operating cost incurred at the beginning of the year)
- 8) A system contains 10,000 resistors each of which is to be replaced on failure. The cost of replacing an individual resistor is Rs.1, while if all resistors are replaced at same time, the cost per resistor would be reduced to only 35 paise. The percentages of resistors surviving $S(t)$ at the end of month t are as follows.

T	0	1	2	3	4	5	6
S(t)	100	97	90	70	30	15	0

Design the optimum replacement policy.

- 9) A large computer installation contains 2000 components of identical nature which are subject to failure as follows.

Month end	1	2	3	4	5
% failure to date	10	25	50	80	100

The cost of individual replacement is Rs.3. If all the components are replaced in one lot, then the cost per component is Rs.1.

- Assess which policy of replacement would be economical.
- If group replacement is economical at current costs, then assess at what cost of individual replacement would be uneconomical.
- How high can the cost per unit in group replacement be to make a preference for individual replacement policy?

- 10) In a factory 1000 lamps are in use. The life distribution of lamps is as follows.

Life (in hours)	300 - 400	400 - 500	500 - 600	600 - 700	700 - 800
Probability	0.07	0.13	0.35	0.27	0.18

Cost of replacement on failure is Rs.6 per lamp. Cost group replacement is Rs.1.8 per lamp. Assuming that failures occur just before the end of an interval, whenever they occur during an interval. Determine optimum replacement policy.

DECISION THEORY

A dealer in second hand machinery is offered 5 machines by a company for Rs.5000/- only. He expects to sell each of the machines for Rs.2200/- at a fair, but he knows that any machines not sold would be a waste and not fetch anything later on. Obtain the payoff matrix under the assumption that the dealer has the option of buying as many machines as he chooses, at the rate of Rs.1000/- per machine. Using the various principles of choice, determine the optimal strategy for the dealer in each case.

From the following matrix, the elements of which indicate the costs, obtain the decisions using different principles of decision making.

	A ₁	A ₂	A ₃	A ₄	A ₅
S ₁	26	22	13	22	18
S ₂	26	22	34	30	20
S ₃	18	22	18	18	20
S ₄	22	22	18	18	18

A newspaper boy is thinking of selling a special one time edition of a sports magazine to his regular newspaper customers. Based on his knowledge of his customers, he believes that he can sell between 9 to 12 copies. The magazines can be purchased at Rs.8/- each and can be sold for Rs.12/- each. Magazines that are not sold can be returned to the publisher for a refund of 50%.

- Construct the payoff matrix for the above problem.
- If the boy further estimates that the probabilities associated with selling of 9 to 12 copies are 0.30, 0.40, 0.20, 0.10 respectively, determine the best decision using the E.M.V. approach.
- Find the E.P.P.I. and hence find the E.V.P.I.

A motor parts dealer finds that the cost of a particular item in stock for a week is Rs.10/- and the cost of a unit shortage is Rs.30/-. The probability distribution of weekly sales (in '000) is as follows.

Weekly Sales (in '000):	0	1	2	3	4	5	6
Probability:	0.10	0.10	0.20	0.25	0.15	0.10	0.10

How many units per week should the dealer order? Also find the E.P.P.I. (Expected payoff (cost) with perfect information) and hence the E.V.P.I.

For the past 200 days, the sales of bread from the Fresh Food Bakery have been as follows.

Daily Sales (loaves):	0	100	200	300	400
No. of Days:	10	60	60	50	20

- The bakery's production costs are Rs.2 per loaf, sale price is Rs.4 per loaf, and any bread unsold at the end of the day is contracted to a local farmer who pays Re.1. per loaf. Draw up a payoff table for each sales/production combination.
- Obtain the regret matrix.
- Determine the best action using the E.O.L. approach.
- What is the maximum amount the bakery owner should be willing to spend for perfect information about the number of loaves expected to be demanded in a day.

A farm owner is considering drilling a well. In the past only 70% of the wells had yielded water at a depth of 200 feet in the area. On finding no water at a depth of 200 feet some persons drilled further down upto 250 feet but only 20% struck water at that depth. The cost of drilling is Rs.50 per foot. The farmer estimates that if he does not have his own well he will have to pay Rs.15,000 over the next 10 years. (in Pv terms) to buy water from a neighbour.

The following decision could be optimal.

- Do not drill a well.
- Drill upto 200 feet.
- If no water is found at 200 feet, drill further upto 250 feet.

Draw a decision tree to represent the situation and determine the farm owner's strategy under the E.M.V. approach.

A manufacturer of a popular brand of refrigerator intends to market its new fuel economy model. It has three options as follows.

- To go ahead and launch.
- To run a test market.
- To abandon the model.

If the product is launched, the demand may be poor, good or excellent. The probabilities of poor, good or excellent demand are 20%, 40% and 40% respectively. If a test market is used at a cost of Rs.2 lakhs, it will give the indication of market being either favourable or unfavourable, there being 50 : 50 chance of market being favourable and unfavourable. If the test market response is favourable and the product is launched, the probability of poor, good and excellent market are 10%, 20% and 70% respectively. And if the test market response is unfavourable and the product is launched, the corresponding probabilities of poor, good and excellent market are 80%, 10% and 10%. The payoffs of the above categories of demand are Rs.4 lakhs, Rs.8 lakhs and Rs.14 lakhs respectively.

- Ignoring the expense already incurred, draw a decision tree for the above problem.
- Suggest an optimal strategy for the manufacturer.

A landowner has an option to sell rights for oil exploration. If he sells the rights, he can get Rs.5 lacs and a further Rs.60 lacs if oil is found. However, as an alternative he can himself conduct a test to check whether oil is there, the cost of which is Rs.12 lacs. If the test succeeds, he can expect a profit of Rs.120 lacs by selling the oil. Alternatively, the landowner can have soundings performed at a cost of Rs.3 lacs. The test of soundings is perfect 70% of the times if oil exists and 90% of the times, if oil does not exist. If it is expected that the oil exists with a probability of 0.3, what should the landowner do? Use a decision tree.

THEORY OF GAMES

Identify the saddle point in the following payoff matrices and the pure strategies for each player:

(i)

$$\begin{array}{c} X \\ Y \end{array} \begin{bmatrix} 1 & -1 & -3 & -5 & 2 \\ 6 & 3 & 2 & 9 & 1 \\ -4 & 5 & 1 & 2 & 7 \\ 5 & 4 & 6 & 9 & 5 \\ 2 & 2 & 10 & 2 & 2 \end{bmatrix}$$

(ii)

$$\begin{array}{c} A \\ B \end{array} \begin{bmatrix} 7 & 3 & 4 & 10 \\ 3 & 0 & -2 & -4 \\ 6 & 6 & 6 & 7 \\ -3 & 4 & 2 & 3 \end{bmatrix}$$

What is the value of the game in each case?

A payoff table of a two person zero sum game is given below.

A

$$\begin{array}{c} B \\ \begin{bmatrix} 10 & 7 \\ 8 & 9 \end{bmatrix} \end{array}$$

- Calculate maximin strategy of player A.
- Calculate minimax strategy of player B.
- Is there a saddle point?
- What proportion of the time should player A adopt his first and his second strategy?
- What percentage of the time should player B adopt his first and his second strategy?
- What will be the average value 'v' of the game?

For the following games-

- Reduce them to 2×2 matrices using the principle of dominance.
- Hence determine the optimal strategies and the value of the game in each case.

(i)

$$\begin{array}{c} A \\ B \end{array} \begin{bmatrix} 4 & 3 & 0 \\ -3 & -1 & -4 \\ 2 & 7 & 4 \end{bmatrix}$$

(ii)

$$\begin{array}{c} A \\ B \end{array} \begin{bmatrix} 5 & 4 & -11 \\ -1 & -10 & 0 \\ 2 & 3 & -11 \end{bmatrix}$$

Solve the following game after reducing it to a 2×2 game using dominance.

A

$$\begin{array}{c} B \\ \begin{bmatrix} 30 & 2 \\ 4 & 14 \\ 6 & 9 \end{bmatrix} \end{array}$$

Solve the following game using the graphical method.

A

$$\begin{array}{c} B \\ \begin{bmatrix} -1 & 4 \\ 6 & 3 \\ 3 & 2 \\ 8 & 1 \end{bmatrix} \end{array}$$

Solve the following game using the graphical method.

$$X \begin{matrix} & Y \\ \begin{bmatrix} 0 & 2 & 8 \\ 7 & 4 & 1 \end{bmatrix} \end{matrix}$$

Two players A and B play a game of matching coins in which each has 3 coins; 10 paise, 25 paise and 50 paise coins. Each player selects a coin without the knowledge of the other player's choice. If the sum of the coins is even, A wins B's coin and if the sum of coins is odd, B wins A's coin.

a) Prepare a payoff matrix for Player A.
b) Find the optimal strategy for each player and the value of the game.

A and B each take out either one or two matches and simultaneously guess how many matches his opponent has taken. If only one of the players guesses correctly then the loser has to pay the winner as many rupees as is the correct answer, otherwise the game is a draw and the payout is zero. Obtain the payoff matrix.

Formulate the following game as an LPP and obtain its solution.

$$A \begin{matrix} & B \\ \begin{bmatrix} 3 & 4 & -2 \\ -3 & 0 & 1 \\ -1 & -4 & 2 \end{bmatrix} \end{matrix}$$

Formulate the following game as an LPP and obtain its solution.

$$X \begin{matrix} & Y \\ \begin{bmatrix} 6 & 1 & 7 \\ 1 & 9 & 3 \end{bmatrix} \end{matrix}$$

VITAL STATISTICS

Calculate the crude and standardized death rates for the local Population from the following data and compare them with crude death rate of the standard population.

Age Group	Standard		Local	
	Population	Deaths	Population	Deaths
0 - 10	600	18	400	16
10 - 20	1000	5	1500	6
20 - 60	3000	24	2400	24
60 - 100	400	20	700	21

Calculate S.T.D.R. for Country A on the basis of the following data.

Age (in yrs)	Specific death rates for country A (per '000)		Life table stationary population of continent (standard)	
	Males	Females	Males	Females
0 - 4	20.0	5.0	600	700
5 - 14	1.0	0.5	2100	2000
15 - 24	1.4	1.0	5500	5150
25 - 34	2.0	1.0	4800	4650
35 - 44	3.3	2.0	4300	4100
45 - 54	7.0	5.0	4500	4200
55 - 64	15.0	12.0	4000	4300
65 - 74	40.0	35.0	2600	3050
75 & above	120.0	110.0	1600	1850

The number of births occurring in a country in a particular year is shown here classified according to age of mother, together with the female population in each age group of the reproductive period.

Age Group	Female population	Number of births to mothers in the age-group
15 - 19	84796	2349
20 - 24	70018	14547
25 - 29	72660	16746
30 - 34	75924	10229
35 - 39	75109	5257
40 - 44	71625	1432
45 - 49	66660	93

The total population of the country during the year was 2, 285,800. With the above data, determine

- the crude birth rate.
- the general fertility rate.
- the age-specific fertility rates.
- the total fertility rate.

The fertility rates (computed on the basis of females alone) are shown in the following table along with the mortality rates.

Age Group	Fertility rate (per thousand)	Mortality rate (per thousand)
15 - 20	13.5	0.40175
20 - 25	55.968	0.42418
25 - 30	70.2864	0.45215
30 - 35	51.6912	0.49320
35 - 40	37.9968	0.54330
40 - 45	15.2736	0.59699
45 - 50	1.92	0.70473

Compute GRR and NRR on the basis of above data.

Calculate Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR) from the following data.

Age Group	Female Population (in '000s)	Female births	Female Mortality rate
15 - 19	1478	19423	0.189
20 - 24	1223	70965	0.105
25 - 29	1604	95324	0.119
30 - 34	1778	62178	0.127
35 - 39	1687	33289	0.146
40 - 44	1459	8713	0.162
45 - 49	1371	608	0.189

From the following data, calculate

- GFR
- IFR
- GRR
- NRR

Age Group	Female Population (in '000s)	No. of births	Years lived by 1000 Female births
15 - 19	152.7	2971	4813
20 - 24	164.2	22773	4790
25 - 29	176.1	28245	4761
30 - 34	177.6	19548	4726
35 - 39	185.2	11462	4685
40 - 44	167.1	3235	4632
45 - 49	150.3	217	4552

Assume that sex ratio at birth was 13 males to 12 females.

LIFE TABLES-I

Complete the following table.

Age(x)	ℓ_x	d_x	$1000q_x$	L_x	T_x	e_x^0
10	93102	-	0.62	-	-	-
11	-	-	0.66	-	-	-
12	-	-	0.72	-	-	-
13	-	-	0.80	-	-	-
14	-	-	0.90	-	-	-
15	-	-	1.00	-	-	-
16	-	-	1.12	-	-	-
17	-	-	1.23	-	-	-
18	-	-	1.33	-	-	-
19	-	-	1.40	-	4,842,446	-

Complete the following table.

Age (x)	ℓ_x	d_x	q_x	L_x	T_x	e_x^0
0	1273	*	*	*	*	*
1	1129	*	*	*	*	*
2	1090	*	*	*	58510	*
3	1066	*	*	*	57432	*
4	1049	-	-	-	56374	-

Given that the probability that a life aged 30 will survive 10 years is 0.85 and the probability that a life aged 40 will survive 10 years is 0.6, find the probability that, of three persons all aged 30, at least one will die between ages 40 and 50 and at least one will die after age 50.

Three persons are aged 30, 35, and 40 respectively. Find the probability that

a) one of them dies before age 45 while the others survive to age 55.

b) at least one of them attains age 65.

[use the LIC (1970-73) ultimate table]

If a mortality table is represented by the function.

$$\ell_x = 1000 \sqrt{100 - x}$$

find

a) the probability of a life surviving from birth to age 19.

b) the probability of a life aged 36 dying before age 51.

c) the probability of a life aged 64 surviving to age 91.

A life aged 50 is subject to a constant force of mortality of 0.048790. Find the probability that he will die between the ages of 70 and 80.

LIFE TABLES-II

- 1) If $\ell_{30} = 937753$, $\ell_{60} = 859916$ & $\ell_{70} = 682859$; fit Gompertz law of mortality.
- 2) Given $\ell_{30} = 33839.37$, $\ell_{40} = 33542.311$, $\ell_{50} = 32669.855$, $\ell_{60} = 30039.787$; fit Makeham's law of mortality $\mu_x = A + Bc^x$.
- 3) The total population of a city is 50 lakhs. On the assumption that it is stationary population experiencing H^M mortality, find
- the number of persons who are aged 55 years or more.
 - the number of persons who are aged 25 years or more but less than 45 years.
 - the number of persons who are less than 18 years of age.
 - the number of persons who are aged 70 years or more.
- 4) The population of a city is 95 lakhs. Assuming that it is a stationary population experiencing H^M mortality, find the number of schools and college the city needs on the basis of one school for every 1500 students and one college for every 8000 students. Assume that 75% of the residents in the city who are aged 5 years or more but less than 15 years qualify for study at school and 65% of the residents, who are aged 15 years or more but less than 20 years qualify for study at college.
- 5) A select mortality table (period of selection 2 years) is given below find

- ${}_2P_{[30]} + 1$
- ${}_2q_{[31]} + 1$
- ${}_1q_{[32]}$

Age at entry [x]	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	Age attained x + 2
30	486024	485094	484096	32
31	485028	484080	483059	33
32	484012	483042	481994	34

In a certain select mortality table the period during which selection is assumed to have effect on the mortality experienced is four years. Given the following table and that $\ell_{[30]} = 494, 985$, find the values of

- $\ell_{[20]+2}$
- ℓ_{24}
- $\ell_{[21]}$
- $\ell_{[24]}$

Age at entry [x]	$d_{[x]}$	$d_{[x]+1}$	$d_{[x]+2}$	$d_{[x]+3}$	$d_{[x]+4}$	Age attained x + 4
20	641	778	864	907	927	24
21	640	781	866	913	933	25
22	640	782	871	919	942	26
23	639	785	876	928	950	27
24	639	789	884	936	963	28

ANNUITIES-I

- 1) A sum of Rs.50 is contributed by a 25 years old person to his provident fund during the current month, what will this particular contribution amount to by the time he retires at the age of 60? Assuming a rate of interest of 8% p.a.?
- 2) The compound interest on Rs.1000 in four years at a certain rate is Rs.200 and in 5 years it is Rs.256 at the same rate. Find the rate of interest.
- 3) A sum of Rs.100 is lent at interest of 12% p.a. convertible monthly. What is the total interest earned in 12 years and 5 months? Calculate effective rate of interest.
- 4) Find the accumulated value at the end of 12 years and one and a half month of an amount of Rs.750 invested at 9% p.a. for the first six years and thereafter at 10% p.a. convertible quarterly.
- 5) A has a right to receive an amount of Rs.500 at the end of 6 years from now. The right has been sold to B for a present value calculated at the rate of 7% p.a. The money thus received was invested by A in a deposit account at 9% p.a. payable half yearly. After 4 years the account had to be closed and A then invested the amount available at 5% p.a. How much has A gained or lost in the transaction, at the end of 5 years?
- 6) Three loans of Rs.800 each were given to Mr. Shah four years, three years and two years ago, respectively. The interest charged by the lender was 12% p.a. Mr. Shah could make a repayment of Rs.600 only at present with the promise to clear the dues at the end of three and a half years from now. How much will he have to pay then?
- 7) Three years ago A bought industrial share for Rs.500 and has received on it three yearly dividends of Rs.50/- each the first one having been paid one year after purchase. He wants to sell the share now and realize a rate of interest of 8% p.a. on the transaction. For how much should he sell?
- 8) A person has purchased a bond of the face value of Rs.1000 on which interest is payable 7% p.a. He received in all three-interest payments, the first one falling due one year after purchase. At the end of three years if the bond is redeemable at no extra premium (i.e. on par) and if the person has realized an interest yield of 8% p.a. in the transaction , what is the purchase price?

ANNUITIES-II

- The cash price of a scooter is Rs.40000/-. A company however offers installment plan where an immediate payment of Rs.5000 is to be made and a series of 5 equal half yearly payments made thereafter, the first installment being payable at the end of six months. If the company wishes to realize a rate of interest of 10% convertible half yearly in the transaction, calculate half yearly installment?
- Mr. X is entitled to a payment of Rs.200 p.a. for 10 years, the first payment being made one year from now and after ten years. The annual payment is increased to Rs.400 p.a. then the payment is ceased at the end of 15th year. Calculate the present value and accumulated value of payments to Mr. X at rate of interest 7%p.a.?
- Calculate the present and accumulated value at an effective rate of 9% p.a. of an annuity due of Rs.120 p.a. payable quarterly for twelve years?
- Calculate the present value and accumulated value at the rate of interest of 7% p.a. of a deferred annuity under which 10 payments of Rs.600 each are made at the interval of every three years, the first payment being due 20 years hence.
- Obtain the present value of an increasing annuity for ten years under which the first payment of Rs.200 is paid one year from now and the successive payments increase by Rs.150 per year. Assume 8% rate of interest.
- A person takes a loan of Rs.2 lacs at 10% rate of interest for house repair which he has to repay in 20 equal installments. Find
- annual installment
 - the interest contained in the eighth payment
 - principal outstanding after the 8th payment is made.
- Assume that first payment is made at the end of one year from now.
- Calculate the present value of an
- immediate perpetuity and
 - perpetuity due
- if the payment is done at an effective rate of interest of 8% p.a. of Rs.180 p.a. payable twice a year.
- Calculate the present value of an immediate perpetuity of Rs.200 p.a. payable half yearly at a rate of interest of 6% p.a. convertible half yearly. The first payment being received at the end of six years from now.

P.Y. B.Sc. Practical No. 9.1.6 PREMIUMS AND ASSURANCES

Consider the following data.

x	40	41	42	43	44	45
l_x	80935	80480	79999	79488	78942	78357
d_x	455	481	511	546	585	626

Calculate, assuming rate of interest of 10% p.a. the present value of

a) temporary assurance of Rs.5000/- for 3 years for a person aged 40 years.

b) endowment assurance of Rs.3000/- for a person aged 42 years receivable on attaining age 45 years.

c) double endowment assurance of Rs.3000/- for 3 years for a person aged 42 years.

d) increasing temporary assurance of Rs.10000/- for 3 years for a person aged 40 years.

e) deferred temporary assurance of Rs.10000/- on a person aged 40 years for 3 years with

deferment period of 2 years.

Using commutation functions based on LIC (1970-73) ultimate mortality @ 6% interest, calculate

for a person aged 30, the present value of

a) endowment assurance of Rs.15000/- for a period of 25 years.

b) whole life assurance of Rs.50000/-

c) increasing whole life assurances of Rs.5000/-

d) temporary assurance of Rs.10000/- for a period of 10 years, provided the death occurs after

attaining age 35 years.

e) special endowment assurance providing both increasing death benefits and increasing survival

benefits of Rs.5000/- for a period of 10 years.

An endowment assurance of Rs.10000/- is given to a person aged 28 for a terms of 10 years with

the following benefits.

a) one fifth of the sum assured at the end of every alternate year in case the person survives till that

period

b) full sum assured at the end of year of death, whenever it occurs during the term of 10 years.

Find the value of this assurance..

[Basis: LIC (1970-73) table at 6 %.]

- 4) On the basis of the LIC (1970-73) ultimate table at 6 %, calculate the net level annual premiums for a sum assured of Rs.10000/- for the following assurance on (30).
- whole life assurance.
 - whole life assurance, premiums limited to 20 years.
 - endowment assurance for 25 years, premiums limited to 15 years.
 - Deferred temporary assurance -the assurance to commence at age 35 years and then to continue for 10 years.
 - double endowment assurance for 25 years.
- 5) Calculate the net level annual premium under a special Endowment assurance of Rs. 30000/- on (30) for 25 years. In the event of death during the first 10 years total premiums paid are returnable and in the event of death during the last 15 years or on survival to the end of 25 years, the basic sum assured becomes payable.[Basis: LIC (1970-73) and 6 % interest.]
- 6) A special policy provides for the following benefits.
- an initial sum of Rs.1000/- with guaranteed annual additions of Rs.400/- for each year's,
 - premium paid after the first, if death occurs within the term of assurance,
 - Rs.10000/- payable on survival to the end of term of assurance and
 - free paid up assurance of Rs.20000/- at death after expiry of the term of assurance.
- Calculate net annual premium under the policy on the life aged 30 years for a term of 25 years. [Basis: LIC (1970-73) table & 6% rate of interest.]
- 7) A life aged 30 effected an endowment assurance policy of Rs.10000/- payable at the end of 25 years or at previous death, with the condition that the premiums payable during the first 5 years shall be ordinary whole life premium, the premium payable thereafter being correspondingly increased. Calculate the annual premium during the last 20 years using LIC (1970-73) tables at 6% rate of interest.
- 8) A life aged 40 effected an endowment assurance policy of Rs.5000/- payable at the end of 15 years or at previous death with the condition that the annual premium payable during the first 5 years is three times the annual premium payable thereafter. Calculate the annual premium payable during the first 5 years using LIC (1970-73) ultimate table at 6% rate of interest.
- 9) Calculate net annual premium under a special endowment assurance of Rs. 18000/- on (35) for 25 years, the premium payment limited to 20 years. In the event of death during the term of assurance, total premiums paid are returnable and on survival to the end of 25 years, the basic sum assured becomes payable. [Basis: LIC (1970-73) table.]

LIFE ANNUITIES AND TEMPORARY ANNUITIES

- 1) Consider the following data.

Age (x)	50	51	52	53	54	55	56	57
No. of persons living (l_x)	91972	91261	90484	89632	88703	87689	86585	85387

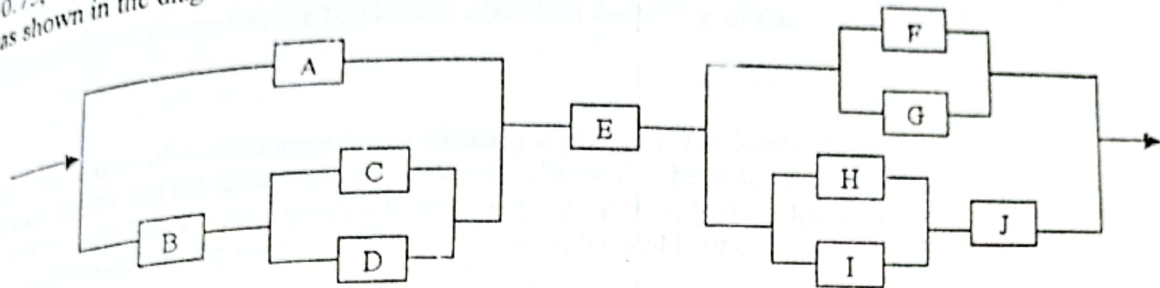
Assuming 6% p.a. rate of interest, calculate, using above data, the present value of

- temporary life annuity of Rs.10000/- p.a. for a person aged 51 years for a period of 4 years, the first payment being made immediately.
 - deferred temporary life annuity of Rs.5000/- p.a. to a person aged 50 years for a period of 3 years, the first payment being made on attaining age 53 years.
 - deferred temporary life annuity due of Rs.2000/- p.a. to a person aged 50 years for a period of 4 years with a deferment period of 2 years.
- 2) Using commutation functions based on LIC (1970-73) ultimate mortality tables at 6% p.a. rate of interest compute the present value of
- an immediate life annuity of Rs.10000/- p.a. for a person aged 60 years.
 - a temporary immediate life annuity of Rs.10000/- p.a. for 10 years to a person aged 50 years.
 - a life annuity due of Rs.10000/- p.a. to a person aged 50 years.
 - a deferred temporary immediate life annuity of Rs.10000/- p.a. for 20 years to a person aged 40 years; deferment period being 5 years.
 - an increasing temporary life annuity due of Rs.1000/- p.a. for 25 years to a person aged 30 years.
- 3) A person received a bonus of Rs.60000/- at the time of his retirement (i.e. at age 60 years). He wishes to buy an increasing life annuity with Rs.5000/- as the first payment payable at the age 62 years and the amount increasing by Rs.A every year. Find the value of A based on the commutation functions of LIC (1970-73) ultimate mortality tables.
- 4) A person wishes to keep some amount with a trustee firm so as to provide a life annuity of Rs.2500/- p.a. to his grand son, now aged exactly 18 years, the first payment to be made on his 25th birthday. How much amount of money should he deposit today with the firm so as to provide the desired annuity to his grand son? [Use LIC (1970-73) ultimate mortality tables.]
- 5) An annuity due of Rs.200/- p.a. is payable to a life aged 60 years. If the annuitant wishes to alter the annuity into an annuity commencing Rs.100/- and increasing by a level amount each year, determine the amount of annual increase. [Basis: LIC (1970-73) ultimate table using 6% p.a. rate of interest].
- 6) Find the numbers living at ages 25 to 28 years if at 6%
 $a_{25} = 15.05$, $a_{26} = 14.974$, $a_{27} = 14.892$, $a_{28} = 14.806$, $a_{29} = 14.715$, $l_{29} = 98206$.
- 7) Find the value of a pure endowment benefit of Rs.6000/- for a person aged 40 years payable on his surviving to age 60 at 6% p.a. given that $a_{40:20} = 13.060$ and $a_{40} = 12.691$ @ 3% p.a.

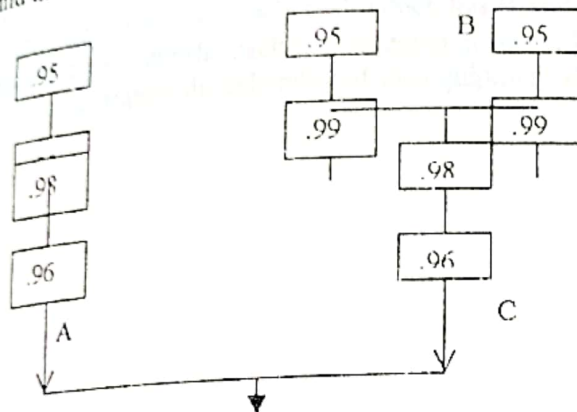
T.Y.B.Sc. Practical No.9.2.1

RELIABILITY

- 1) A system consists of 10 components A,B,C,D,E,F,G,H,I and J with reliabilities 0.8, 0.75, 0.99, 0.95, 0.85, 0.98, 0.7, 0.88, 0.96 and 0.95 respectively which are connected as shown in the diagram given below. Find the reliability of the system.



- 2) Find the reliability of the configuration given in the following figure:



(Note: The figures indicate reliabilities)

- 3) A set of tests can be performed to detect a disease before it develops fully and the condition goes beyond cure. But none of these tests are confirmatory in nature. The sets have series of stages to be performed sequentially. The tests are denoted by A_i ($i = 1, 2, \dots, 9$) and B_j ($j = 1, 2, \dots, 9$). Stages and tests along with their probabilities of failure are listed below.

Set A

- (i) A_1 (0.05)
- (ii) either (A_2 (0.10) and A_3 (0.10) together) or A_4 (0.15)
- (iii) A_5 (0.12)
- (iv) either A_6 (0.15) or A_7 (0.12) or A_8 (0.05) and A_9 (0.05) together.

Set B

- (i) either B_1 (0.05) or B_2 (0.07)
- (ii) B_3 (0.12)
- (iii) either B_4 (0.08) and B_5 (0.15) together or B_6 (0.09) and B_7 (0.20) together.

Determine the reliability of each of the two sets of tests.

- 4) An electronic circuit consists of 15 valves, 20 resistors and 10 capacitors, all connected in series. The components in each category are identical and their failure times are found to follow exponential distribution with the following mean failure time:

Components	Mean Failure Time (in hours)
Valves	10,000
Resistors	20,000
Capacitors	20,000.

What is the reliability of the system for 100 hours? Also, find the MTBF of the system.

- 5) Suppose the failure time of a component is exponentially distributed. Ten identical components are tested and the failure times in hours are observed to be as follows: $t_1 = 480$, $t_2 = 630$, $t_3 = 560$, $t_4 = 500$, $t_5 = 495$, $t_6 = 475$, $t_7 = 562$, $t_8 = 478$, $t_9 = 614$, $t_{10} = 566$. Estimate the hazard rate and the reliability of the component for 500 hours.

- 6) An equipment consists of 100 parts distributed in 3 branches A (20), B (20) and C (60). A is connected in series with a branch which has B and C connected in parallel. In each branch, the parts are themselves connected in series. Each part has constant hazard rate 0.005. Compute the reliability of the equipment at 10 hours.

TIME SERIES

- 1) The following data show the number of thefts (in thousands) in the United States for the years 1985-1995. Draw a trend line using the method of semi-averages for the data. Also, plot the graph of the original data.

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
No. of thefts	19.0	20.6	20.1	20.7	21.5	23.4	24.7	23.8	24.5	23.3	21.6

- Hence predict the number of thefts for the year 1996.
 2) Assuming a parabolic trend in the sales (in thousand Rs.), determine the trend values for the following data.

Year	1991	1992	1993	1994	1995	1996	1997
Sales	7	9	13	20	19	17	15

- Also, estimate sales for the year 1998.
 3) Calculate the trend values for the following data by the method of least squares assuming a linear trend.

Year	Quarter			
	I	II	III	IV
1995	25	30	21	32
1996	27	28	25	34
1997	22	27	21	30
1998	24	25	20	33

- 4) Calculate seasonal indices by the ratio to moving average method for the following data:

Year	Quarter			
	I	II	III	IV
1981	68	62	61	63
1982	65	58	66	61
1983	68	63	63	67

- 5) Using ratio to trend method, compute seasonal indices for the following data.

Quarter	Year			
	1980	1981	1982	1983
I	75	86	90	100
II	60	65	72	78
III	54	63	66	72
IV	59	80	85	93

Calculate seasonal indices using link relatives method:

Year	Quarter			
	I	II	III	IV
1991	6.0	6.5	7.8	8.7
1992	5.4	7.9	8.4	7.3
1993	6.8	6.5	9.3	6.4
1994	7.2	5.8	7.5	8.5

SIMULATION

1) Trucks arrive at a certain toll post according to the following frequency distribution.

Time between arrivals (in min)	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Frequency	4	12	27	23	16	12	6

The service time required by truck drivers at the post has following frequency distribution.

Service time (in min)	0.5	1.0	1.5	2.0
Frequency	25	35	27	13

Estimate average waiting time of the truck drivers and proportion of idle time of the server by simulating for 10 arrivals.

for arrival distribution : 85, 96, 28, 66, 68, 35, 19, 45, 69, 59

for service distribution : 74, 29, 92, 62, 31, 44, 24, 70, 52, 43

[Observation begins at time $t = 0$. First customer arrives later.]

2) In case of a dealer for a certain product, the probability distributions of the daily demand and the lead time are as follows.

Daily Demand Distribution

Demand	3	4	5	6	7	8	9	10	11	12
Probability	0.02	0.08	0.11	0.16	0.19	0.13	0.10	0.08	0.07	0.06

Lead Time Distribution

Lead Time (days)	2	3	4	5
Probability	0.20	0.30	0.35	0.15

Shortage cost (C_2) is Rs.20 per unit per day, ordering cost (C_s) is Rs.80 per order, holding cost (C_1) is Rs.2 per unit per day, opening stock on 1st day is 30 units.

Consider a simulation plan which calls for a reorder quantity of 40 units and reorder level of 20 units. It is assumed that orders are placed / received before the end of the day. Using following random numbers generate demand for 15 days and by simulation arrive at average cost per day.

Random Numbers (Demand) : 68, 13, 09, 20, 73, 07, 92, 99, 93, 18, 24, 22, 07, 29, 57.

Random Numbers (Lead time) : 47, 74, 25, 21, 47

3) A Company manufactures 30 units per day. The sale of these items depends upon demand which has the following distribution.

Demand (units)	27	28	29	30	31	32
Probability	0.10	0.15	0.20	0.35	0.15	0.05

The production cost and sale price of each unit are Rs.40 and Rs.50, respectively.

Any unsold product is to be disposed off at a loss of Rs.15 per unit. There is a penalty of Rs.5 per unit if demand is not met. Using the following random numbers estimate the total profit/loss for the company for the next ten days.

Random Numbers 10, 99, 65, 99, 95, 01, 79, 11, 16, 20.

4) Generate Random observations from each of the following (5 in a, b and 2 in c).

a) Uniform distribution : $f(x) = \frac{1}{18}$; $6 < x < 24$

= 0 ; otherwise

b) Exponential distribution with mean = 2.

c) Gamma distribution with $m = 3$, $\lambda = 0.4$

- 9) The daily production of a factory is normally distributed with mean $\mu_1 = 200$ kg and s.d $\sigma_1 = 20$ kg. The no. of production hours is also normally distributed with mean $\mu_2 = 5$ hours and s.d $\sigma_2 = 1$ hour. Simulate the daily production and daily production hours for ten days to estimate the daily production rate. Use the following random numbers.
- For production: 71, 50, 22, 01, 80, 19, 37, 13, 55, 34
- For production hours: 62, 65, 20, 95, 59, 21, 84, 87, 39, 87

A dentist schedules all her patients for 30-minute appointments. The following summary shows the various categories of work and the probability distribution of time required to complete the work.

Category of Dental Work	Time Required (minutes)	Probability
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Check-up	15	0.20

Assuming that all patients arrive at the clinic at their scheduled arrival time, starting at 8 a.m., simulate the dentist's clinic for 7 patients and determine average waiting time for the patients as well as idle time for the dentist. Use the following random numbers: 40, 82, 11, 34, 25, 66, and 17.

- 1) The demand function at point $X = 4$ and $P = 6$ is given by $X = 10 - P$. If the price increases by 5 %, determine the relative decrease in demand and hence an approximation to the elasticity of demand. Compare the result with the correct value of elasticity of demand corresponding to $X = 4$.
- 2) If X_1 and P_1 are demand and price of tea and X_2 and P_2 are demand and price of coffee and the demand functions are given by

$$X_1 = P_1^{-1.3} P_2^{0.5}$$

$$X_2 = P_1^{0.3} P_2^{-0.5}$$

- Show that two commodities are competitive. Also obtain four elasticities of demand.
- 3) Given the demand function
 $P = 100 - 0.5 X^2$ and the cost function
 $C = \frac{X^3}{3} - 6X^2 + 108X$,

determine

- a) the equilibrium output.
 b) the change in price, output and profit after the imposition of a tax of Re. 1 per unit of output.
 c) the price if the government grants subsidy of Re.1 per output.
- 4) A monopolist produces amounts X_1 and X_2 of two goods A and B at the total cost

$$C = X_1^2 + 4X_1X_2 + 6X_2^2.$$

The demand functions for these goods are $P_1 = 40 - X_1$ and $P_2 = 100 - 5X_2$ respectively, where P_1 and P_2 are the prices. Determine the quantities and prices, which maximize the total profit. Also find maximum profit.

- 5) The demand functions of discriminating monopolist are $P_1 = 12 - q_1$ and $P_2 = 20 - 3q_2$ and the total cost function is $C = 3 + 2(q_1 + q_2)$. Determine the prices set and quantities sold in the two markets that will maximize the total profit. Show that discriminating monopolist charges more price in the less elastic market.

- 6) Assume that the demand and cost function of duopolists are

$$P_1 = 100 - 2q_1 - q_2, C_1 = 30q_1 \text{ and}$$

$$P_2 = 95 - q_1 - 3q_2, C_2 = 25q_2 \text{ respectively.}$$

Find the optimum prices, quantities and profits when

- a) duopolists recognize their dependence.
 b) duopolists do not recognize their dependence.
- 7) The cost function of a firm is given by $C = 2Q - 2Q^2 + Q^3$. Find

- a) AC function.
 b) MC function
 c) at what level of output AC is minimum?
 d) Verify that at minimum average cost $AC = MC$.
- 8) Given the demand function $P = 86 - 25X$, verify the relationship between marginal revenue, average revenue and elasticity of demand (η_d).

T.Y.B.Sc. Practical No.9.2.5

MATHEMATICAL ECONOMICS- II

- 1) Given the production function $Q = f(L, K) = LK - 0.2L^2 - 0.8K^2$. Construct the average and marginal productivities for labour (L). Assuming that $K = 10$, examine the three stages of the production function.
- 2) The production function is given by $Q = 5L^{0.3}K^{0.7}$, where L and K represent labour and capital respectively. The prices per unit of labour and capital are Rs.20/- and Rs.40/- respectively. If the cost is fixed at Rs.500/-, determine the amounts of labour and capital in order to maximise the output.
- 3) A firm's production function is $Q = 5L^{0.7}K^{0.3}$. The price of labour is Re.1 per unit and the price of capital Rs.2 per unit. Find the minimum cost combination of capital and labour for an output rate of 20.
- 4) The firm's production function is

$$q = 12 - \frac{L + K}{L \cdot K}$$

The prices of labour, capital and output are Rs.1, 4 and 9 respectively. Find the maximum profit combination of capital, labour and output.

E.Y.B.Sc. Practical No.9.2.6

INPUT-OUTPUT MODELS

1) Assume that the economy is divided into two sectors and input output table in physical units is as follows.

Producing Sectors	Purchasing Sectors		Final Demand
	I	II	
I	200	250	50
II	100	500	400

a) Obtain the technology matrix.

b) If the final demands of the two sectors are desired to be 180 and 440 units for sectors I and II respectively, then find the total output in accordance.

Find the demand vector $\underline{F}$ which is consistent with the output vector $\underline{X}$ and technology matrix A .

$$\underline{X} = \begin{bmatrix} 25 \\ 21 \\ 18 \end{bmatrix} ; \quad A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}$$

Compute the gross value added in each sector and show that the total value added in each sector and total final demand is equal.

A three sector input-output matrix $[I - A]$ is given as

$$\begin{bmatrix} 1 & -0.5 & 0 \\ -0.2 & 1 & -0.5 \\ -0.4 & 0 & 1 \end{bmatrix}$$

With labour coefficients (0.4, 0.7, 1.2) and the household demand for the outputs of three sectors as 1000, 5000 and 4000 respectively, determine the level of output. If the wage rate is Rs.10/- per labour day, find the equilibrium prices of the commodities in the three sectors.

Assume that the economy is divided into three sectors and input-output table in physical units is as follows.

Producing Sectors	Purchasing Sectors			Final Demand
	I	II	III	
I	100	150	225	75
II	135	150	300	115
III	270	200	300	130
Labour (in lacs)	12	10	16	

If the final demands for the products of the sectors I, II and III rise to 100, 200 and 250 respectively, find the corresponding rise in the employment.
