

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B: 1. All question are compulsory.
2. Figures to the right indicate full marks.

Q.1 Attempt any four of the following: 20

- A Define increasing and decreasing functions with an example.
- B Show that $|x|$ is continuous everywhere.
- C Write a note on graphing of polynomials.
- D Find the interval in which $f(x) = 3 - 2x + x^4$ is increasing or decreasing
- E Using newton's method find the approximate root for the equation $f(x) = x^2 - 4x - 10$ (only four iterations)
- F Find the relative extrema of $f(x) = 4x^4 - 16x^2 + 17$.

Q.2 Attempt any four of the following: 20

- A Evaluate the integral:
 i) $\int_1^{\frac{1}{\sqrt{2x-1}}} dx$ ii) $\int_0^{\pi/2} \frac{1}{(1 + \cot x)} dx$
- B Solve the differential equation: $(2 + 2y^2)y' = e^x y$.
- C Find the area enclosed between the curves: $y = x^2$, $y = x + 6$.
- D Evaluate $\int_1^2 \frac{1}{x^2} dx$ using Simpson's Rule with $n = 10$
- E Use Euler's Method to estimate solution of: $\frac{dy}{dx} = y - x$ with $y(x) = 2$, step size = 0.2 over $0 \leq x \leq 1$.
- F Define Indefinite integral and give formula for standard functions(any four)

Q.3 Attempt any four of the following: 20

- A Consider function $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$.
Find limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along (i) $x = \text{axis}$ (ii) $\text{line } y = x$.
- B $f(x, y, z) = 2x^3 y^2 + y^3 z + 3z^2 xy$. Find all first order derivatives at $(1, 1, -1)$.

- C Find directional derivative of $f(x, y) = x^2 + 3xy$ along $(1, -1)$ in direction of $\vec{i} - \vec{j}$.
- D Find equation of tangent plane and normal to the surface $z = x^2y$ at $(2, 1, 4)$.
- E Find relative extrema: $f(x, y) = 4xy - y^4 - x^4$.
- F Find linearization: $f(x, y) = x^3 - xy + y^2$ at $(1, 1)$.

Q.4 Attempt any three of the following:

15

- A Solve the differential equation: $x \frac{dy}{dx} - y = x$
- B Show that $y = x e^{-x}$ satisfies the equation $xy' = (1 - x)y$
- C Find the area of the region under the curve $y = x - x^2 + 1$ and above the x-axis.
- D Find length of curve: $y = x^{2/3}$ from $x = 1$ to $x = 8$.
- E Define derivative. Hence find the same for $y = x^n$.
- F Define gradient and del operator. Give an example.
