

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B: 1. All question are compulsory.
2. Figures to the right indicate full marks.

Q.1 Attempt any four of the following: 20

- A Find the interval at which the function is increasing $f(x) = x^3 - 27x + 5$
 B Draw the graph of $y = 4 - 3a^2 + a^3$
 C Explain types of function
 D If $f(x) = 4x + 4$ $x < 2$
 $= x^2 + 3x + 2$ $x \geq 2$ at $x = 2$ then find f is differentiable or not ?
 E Find for what value of x the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards and downwards
 F Find the points of maxima and minima of a function: $f(x) = 2x^3 - 3x^2 + 6$

Q.2 Attempt any four of the following: 20

- A Evaluate the following integral: $\int \frac{x^2}{\sqrt{1-x^3}}$
 B Evaluate the following integral: $\int x \cdot \log x \, dx$.
 C Evaluate: $\int_{-1}^4 (5 - 2x)^8 dx$
 D Use Simpson's rule with $n = 6$ to approximate the integral $\int_0^6 \frac{1}{1+x} dx$
 E Find the area of the curve enclosed by the curve $y = x^2$ and $y = \sqrt{x}$
 F Find the length of the curve $x = \cos 2t$, $y = \sin 2t$ and $0 \leq t \leq \pi/2$

Q.3 Attempt any four of the following: 20

- A Show that the function $f(x, y) = 2x^2 - 4y$ is differentiable at points $(1, 1)$
 B Find all second order partial derivative of f . also verify whether $f_{xy} = f_{yx}$ given that $f(x, y) = x^3y^4 + x^2y$?
 C Find an equation of plane tangent to $2x^2 + 3y^2 - z^2 - 4$ at $(1, 1, -1)$
 D Find direction derivative of $f(x, y) = 3x + 4y$, $u = (2, 3)$, $\bar{v} = 4\bar{i} + 5\bar{j}$
 E Determine the critical points and locate any relative maxima, minima and saddle points of the function of defined by $f(x, y) = 2x^2 + 2xy + 2y^2 - 6x$
 F Find $\frac{dz}{dt}$, where $z = x^2y^3 + y \cos x$ and x and y are $x = \log t^2$ and $y = \sin 4t$.

Q.4 Attempt any three of the following:

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- A The equation $x^3 - 2x - 2 = 0$ has one real solution. Approximate it by Newton's Method take initial value $x_0 = 0.1$
- B Divide 100 into two parts such that sum of their square is minimum
- C Use Euler's method to solve initial value problem $\frac{dy}{dx} = x - y^2$, $y(0) = 1$, $0 \leq t \leq 1$ with step size $\Delta x = 0.25$
- D Solve $\frac{dy}{dx} = \frac{x+y}{x-y}$ using separable form of differential equation.
- E Find the first order differential equation of the following function:
a) $f(x, y) = 4x - 5y + 23$ b) $f(x, y) = e^{x+y}$
- F Find the linearization of $f(x, y) = x^3 + xy + y^2$ at $(1, 2)$
