University of Mumbai

No. AAMS(UG)/98 of 2021-22

CIRCULAR:-

Attention of the Principals of the Affiliated Colleges, the Head of the University Departments and Directors of the Recognized Institutions in Faculties of Humanities and Science & Technology.

They are hereby informed that the recommendations made by the Board of Studies in Mathematics at its online meeting held on 23rd April, 2021 vide Item No. 1 (ii) and subsequently passed by the Board of Deans at its online meeting held on 11th June, 2021 vide item No. 6.18 (R) have been accepted by the Academic Council at its meeting held on 29th June, 2021 vide item No. 6.18(R) and that in accordance therewith, Finalize the proposed syllabus of M.Sc./M.A. (Sem-III & IV) in Mathematics under (CBCS) in Evaluation Scheme A, B & C has been brought into force with effect from the academic year 2021-22 accordingly. (The same is available on the University's website www.mu.ac.in).

MUMBAI – 400 032
8th October, 2021

(Dr. B.N.Gaikwad)
I/c REGISTRAR

To

The Principals of the Affiliated Colleges, the Head of the University Departments and Directors of the Recognized Institutions in Faculties of Humanities and Science & Technology.

A.C/6.18 (R) 29/06/2021

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No. AAMS(UG)/ 98 -A of 2021-22 MUMBAI-400 032 8th October, 2021

Copy forwarded with Compliments for information to:-

1) The Dean, Faculties of Humanities and Science & Technology,
2) The Chairman, Board of Studies in Mathematics,
3) The Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development,
5) The Co-ordinator, University Computerization Centre,

(Dr. B.N.Gaikwad)
I/c REGISTRAR

Copy to :-

1. **The Deputy Registrar, Academic Authorities Meetings and Services (AAMS),**
2. **The Deputy Registrar, College Affiliations & Development Department (CAD),**
3. **The Deputy Registrar, (Admissions, Enrolment, Eligibility and Migration Department (AEM),**
4. **The Deputy Registrar, Research Administration & Promotion Cell (RAPC),**
5. **The Deputy Registrar, Executive Authorities Section (EA),**
6. **The Deputy Registrar, PRO, Fort, (Publication Section),**
7. **The Deputy Registrar, (Special Cell),**
8. **The Deputy Registrar, Fort/ Vidyanagari Administration Department (FAD) (VAD), Record Section,**
9. **The Director, Institute of Distance and Open Learning (IDOL Admin), Vidyanagari,**

They are requested to treat this as action taken report on the concerned resolution adopted by the Academic Council referred to in the above circular and that on separate Action Taken Report will be sent in this connection.

1. **P.A to Hon'ble Vice-Chancellor,**
2. **P.A Pro-Vice-Chancellor,**
3. **P.A to Registrar,**
4. **All Deans of all Faculties,**
5. **P.A to Finance & Account Officers, (F.& A.O),**
6. **P.A to Director, Board of Examinations and Evaluation,**
7. **P.A to Director, Innovation, Incubation and Linkages,**
8. **P.A to Director, Board of Lifelong Learning and Extension (BLLE),**
9. **The Director, Dept. of Information and Communication Technology (DICT) (CCF & UCC), Vidyanagari,**
10. **The Director of Board of Student Development,**
11. **The Director, Department of Students Walfare (DSD),**
12. **All Deputy Registrar, Examination House,**
13. **The Deputy Registrars, Finance & Accounts Section,**
14. **The Assistant Registrar, Administrative sub-Campus Thane,**
15. **The Assistant Registrar, School of Engg. & Applied Sciences, Kalyan,**
16. **The Assistant Registrar, Ratnagiri sub-centre, Ratnagiri,**
17. **The Assistant Registrar, Constituent Colleges Unit,**
18. **BUCTU,**
19. **The Receptionist,**
20. **The Telephone Operator,**
21. **The Secretary MUASA**

for information.

AC - 29/6/2021
Item No. 6.18

UNIVERSITY OF MUMBAI

Syllabus
for the
Program : M.Sc./M.A. Sem.-III & IV
CBCS
Course : Mathematics

(Choice Based and Credit System with effect from the academic year 2021-22)

AC 29/6/2021
Item No. 6.18

UNIVERSITY OF MUMBAI

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of the Course	M. Sc. / M. A. Mathematics, Sem III& IV
2	Eligibility for Admission	As per University regulations
3	Passing Marks	40% (Internal 16/40 and External 24/60 in each Theory course and 40 marks in Project).
4	Ordinances / Regulations (if any)	-
5	No. of Years / Semesters	Two Semesters (III &IV)
6	Level	PG
7	Pattern	Semester
8	Status	Revised
9	To be implemented from Academic Year	From Academic Year : 2021-2022

Date: 19.05.2021 Signature:

19.05.2021

Name Prof. R. P. Deore Chairman of BoS of Mathematics.

Dr. Anuradha Majumdar (Dean, Science and Technology)
Prof. Shivram Garje (Associate Dean, Science)
Prof. R. P. Deore , Chairman (BoS) Member(BoS)
Prof. P. Veeramani, Member
Prof. S. R. Ghorpade , Member
Prof. Ajit Diwan, Member
Dr. Sushil Kulkarni, Member
Dr. S. A. Shende, Member
Prof. V. S. Kulkarni
Dr. Sanjeevani Gharge, Member
Dr. Mittu Bhattacharya, Member
Dr. Abhaya Chitre, Member
Dr. S. Aggarwal, Member
Dr. Amul Desai, Member

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1. Preamble
2. Programme Outcomes
3. Course structure with minimum credits and Lectures/ Week
4. Teaching Pattern for semester III & IV
5. Consolidated Syllabus with Course outcomes for semester III& IV
6. Scheme of Evaluation

1.Preamble

The Board of Studies in Mathematics has prepared the syllabus of M.A./M.Sc. Semester III & IV (w. e. f. 2021-22) in the subject of Mathematics under the Choice Based Credit System (CBCS). While revising the Syllabus for semester III & IV, the Board of Studies in Mathematics has not changed the syllabus of core courses but tired to maintain the flow of each course. Some of the optional courses are revised as per need of an hour.

The Board of Studies in Mathematics has introduced five skill courses in semester IV and a candidate has to complete one skill course out of these five skill courses. Looking towards the applicability of free software like SAGE MATH and GAP, the Board of Studies in Mathematics has newly introduced a course like Computational Algebra as one of the skill courses in semester IV.

The Board of studies in mathematics has introduced a project based course at semester IV. The projects will be guided by the faculty of the affiliated center and it will be evaluated by one of the external experts possibly from outside the department. A learner should learn new things ,generate and propagate new ideas in projects.

The board of studies in mathematics has taken care of learning outcomes and student's progression while revising the curriculum of semester III & IV.

2. Programme Outcomes:

Students of M.A./M.Sc. (Mathematics) degree Programme of Mumbai University at the time of graduation will be able to

1. Applications of Mathematics: Take a job in Teaching, financial institutes, Insurance sectors, engendering sector as Students are given courses in Algebra, Real and Complex Analysis, Differential Geometry, Probability, Statistics, Partial differential equations, Numerical Analysis, Optimization which has application in these industries.

2. Problem Analysis: Identify, formulate problem encountering while working and in life and apply mathematical knowledge to solve the problem.

3. Formulate the solution: Obtained Solution of complex mathematical problem.

4. Independent working and team work: Work independently as well as in team for a solution of a problem.

5. Communication: Communicate effectively over a problem and explain solution.

6. Life-long learning: Independently learn new topics, acquire knowledge and apply for betterment of individual and society.

7. Independent Thinking: Should able to independently think over a problem and make efforts towards solution.

8. Students will be well prepared for NET examination conducted by UGC for Lectureship and JRF.

9. Students are learning Elementary probability Theory and statistics which is base for career in financial, insurance sector.

3. Course Structure with Credits and Lecture/ Week

Semester III

Sr. No.	Subject code	Units	Subject	Credits	L/W
			Algebra III		
01	PSMT/ PAMT 301	Unit I	Groups	06	04
		Unit II	Rings and Ideals		
		Unit III	Modules		
		Unit IV	Modules over PID		
			Functional Analysis		
02	PSMT/ PAMT 302	Unit I	Baire Spaces and Hilbert Spaces	06	04
		Unit II	Normed Linear Spaces		
		Unit III	Bounded Linear Transformations		
		Unit IV	Basic Theorems		
			Differential Geometry		
03	PSMT/ PAMT 303	Unit I	Isometries of R^n	06	04
		Unit II	Curves		
		Unit III	Regular Surface		
		Unit IV	Curvature		
			Elective Courses		
04	PSMT/ PAMT 304	Elective I (Any one from 1 to 10)		03	04
05	PSMT/ PAMT 305	Elective II (Any one from 1 to 10)		03	04

Note:

- PSMT301/PAMT301, PSMT302/PAMT302, PSMT303/PAMT303 are compulsory courses for Semester III.
- 2. PSMT 304/PAMT 304 and PSMT 305/PAMT305 are Elective Courses for Semester III.
- 3. Elective course Courses I and II will be any TWO of the following list of ten courses:

1. Algebraic Topology

2. Advanced Complex Analysis
3. Commutative Algebra
4. Algebraic Number Theory
5. Advanced Partial Differential Equations
6. Integral Transforms
7. Numerical Analysis
8. Graph Theory
9. Coding Theory
10. Design Theory

Semester IV

Sr. No.	Subject code	Units	Subject	Credits	L/W
			Algebra IV		
01	PSMT/ PAMT 401	Unit I	Algebraic Extensions	06	04
		Unit II	Normal and Separable Extensions		
		Unit III	Galois Theory		
		Unit IV	Applications		
			Fourier Analysis		
02	PSMT/ PAMT 402	Unit I	Fourier Series	05	04
		Unit II	Dirichlet's Theorem		
		Unit III	Fejer's Theorem and Applications		
		Unit IV	Dirichlet Problem in the Unit Disc		
			Calculus on Manifolds		
03	PSMT/ PAMT 403	Unit I	Multivariable Algebra	05	04
		Unit II	Differential Forms		
		Unit III	Basics of Submanifolds of R^n		
		Unit IV	Stoke's theorem		
04	PSMT/ PAMT 404	**Skill Based Course** (Any one from I to V)		04	04
05	PSMT/ PAMT 405	**Project**		04	04

Note:

- PSMT401/PAMT401, PSMT402/PAMT402, PSMT403/PAMT403 are compulsory courses for Semester IV.
- PSMT 404/PAMT 404 is Skill Based Course for Semester IV.
- Skill Based Course will be any ONE of the following list of **FIVE** courses:

1 Business Statistics

2 Statistical Methods

3 Computer Science

4 Linear and Non Linear Programming

5 Computational Algebra

4. Teaching Pattern for Semester III & IV

1. Four lectures per week for each of the courses: PSMT301/PAMT301, PSMT302/ PAMT302, PSMT303/PAMT303, PSMT304/PAMT304 and PSMT305/PAMT305.
2. Four lectures per week for each of the courses: PSMT401/PAMT401, PSMT402/ PAMT402, PSMT403/PAMT403, PSMT404/PAMT404.
3. Each lecture is of 60 minutes duration.
4. In addition, there shall be tutorials, seminars as necessary for each of the five courses.
5. PSMT 405/PAMT 405 is a project based Course for Semester IV.

The projects for this course are to be guided by the Faculty members of the Department of Mathematics of the concerned college. Each project shall have maximum of 08 (eight) students. The workload for each project is 2 L/W.

5. Consolidated Syllabus for Semester III and IV

Teaching Pattern for Semester III and IV
1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

Semester-III

All Results have to be done with proof unless and otherwise stated.

PSMT 301 /PAMT 301 Algebra III

Course Outcomes:

1. Students will learn about classical groups like Simple groups, Solvable groups and Nilpotent groups and applications of these classical groups.
2. Motive of an introduction of the Zariski topology is to take a glance at algebraic geometry which is like ocean in its own right. This topic provides geometric point of view of algebra. It gives students a wider perspective to rethink the nature of a prime ideal, maximal ideal and very precise the geometric position of prime and maximal ideals. It also helps to visualize radical of an ideal in geometric setting and local nature of a prime ideal.
3. Students will learn Finitely generated modules, Free modules, Free module of rank n.
4. Students will understand the Structure theorem for finitely generated modules over a PID and Applications to the Structure theorem for finitely generated Abelian groups and linear operators.

Unit I. Groups (15 Lectures)
Simple groups, A_5 is simple. Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus Lemma, Jordan-Holder theorem,
Direct and Semi-direct products, Examples such as
(i) The group of affine transformations $x \mapsto ax + b$ as semi-direct product of the group of linear transformations acting on the group of translations.
(ii) Dihedral group D_{2n} as semi-direct product of $\mathbb{Z}_2$ and $\mathbb{Z}_n$.
Classification of groups of order 12.

Unit II. Rings and Ideals (15 Lectures)
Nilradical and relation to prime ideals, Jacobson radical and maximal ideals, Radical of an ideal, Annihilator ideal, Local rings and equivalent conditions for a local ring, Prime spectrum of a ring and Zariski topology, idempotents and connectedness, ring homomorphisms and induced map on Spec. Hilbert Nullstellensatz (only statement) and its corollaries.

Unit III. Modules (15 Lectures)
Modules over rings, Submodules. Module homomorphisms, kernels. Quotient modules. Isomorphism theorems. Generation of modules, finitely generated modules, (internal) direct sums and equivalent conditions. Free modules, free module of rank n. For a commutative ring R, R^n is isomorphic to R^m if and only if $n = m$. Matrix representations of homomorphisms between free modules of finite rank. Dimension of a free module over a P.I.D.

Unit IV. Modules over PID (15 Lectures)
Noetherian modules and equivalent conditions. Rank of an R-module. Torsion submodule of an R-module M, torsion free modules, annihilator ideal of a submodule. Finitely generated modules over a PID: If N is a submodule of free module M (over a P.I.D.) of finite rank n, then N is free of rank $m \leq n$. Any submodule of a finitely generated module over a P.I.D. is finitely generated. Structure theorem for finitely generated modules over a PID: Fundamental theorem, Existence (Invariant Factor Form and Elementary Divisor Form), Fundamental theorem, Uniqueness. Applications to the Structure theorem for finitely generated Abelian groups and linear operators.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, Indian Edition 2007.
3. D. S. Dummit and R. M. Foote, Abstract Algebra.
4. N. Jacobson, Basic Algebra, Volume 1, Dover, 1985.
5. S. Lang, Algebra, Springer Verlag, 2004

PSMT 302 / PAMT 302 Functional Analysis

Course Outcomes:

1. Students will learn Hilbert spaces and Banach spaces.
2. Students will be able to understand the concept of dimension of a Hilbert space, bounded linear transformations, norms, inner products, dual spaces and their difference from the finite dimensional cases.
3. Students should know about ℓ^p, L^p spaces, dual spaces and their properties;
4. Students should understand the fundamental theorems as mentioned in the syllabus.

Unit I: Baire spaces and Hilbert spaces (15 Lectures)
Baire spaces. Open subspace of a Baire space is a Baire space. Complete metric spaces are Baire spaces and application to a sequence of continuous real valued functions converging point-wise to a limit function on a complete metric space. Hilbert spaces, examples of Hilbert spaces such as ℓ^2, $L^2[-\pi, \pi]$, $L^2(\mathbb{R}^n)$ (with no proofs). Inner product induced by norm, Bessel's inequality. Equivalence of complete orthonormal set and maximal orthonormal set, orthogonal decomposition, Parseval's identity. Existence of a maximal orthonormal set, separability of Hilbert space.

Unit II: Normed Linear Spaces (15 Lectures)
Normed Linear spaces, Banach spaces. Examples of Normed linear spaces, Arzela-Ascoli theorem, ℓ^p $(1 \le p \le \infty)$ spaces are Banach spaces, $L^p(\mu)(1 \le p \le \infty)$ spaces: Holder's inequality, Minkowski's inequality, $L^p(\mu)(1 \le p \le \infty)$ are Banach spaces. Quotient space of a normed linear space. Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to infinite dimension of a normed linear spaces.

Unit III: Bounded Linear Transformations (15 Lectures)
Bounded linear transformations, Equivalent characterizations. The space $\mathcal{B}(X, Y)$. Completeness of $\mathcal{B}(X, Y)$ when Y is complete. Dual space of a normed linear space, Dual space of ℓ^p $(1 \le p < \infty)$, Riesz Representation theorem for Hilbert spaces. Dual of $L^p(\mu)(1 \le p < \infty)$ spaces: Riesz-Representation theorem for $L^p(\mu)(1 \le p < \infty)$ spaces. Separable spaces, examples of separable spaces such as ℓ^p $(1 \le p < \infty)$. If the dual space X' of X is separable, then X is separable.

Unit IV: Basic Theorems (15 Lectures)
Hahn-Banach theorem (Extension and Separation), applications of Hahn-Banach theorem. Open mapping theorem, Closed graph theorem, Uniform boundedness principle and application.

Recommended Text Books:

1. Andrew Browder, Mathematical Analysis, An Introduction, Springer International Edition, 1996.
2. E. Keryszig, Introductory Functional Analysis with Applications, Wiely India, 1978.
3. B. V. Limaye, Functional Analysis, New Age International, 1996.
4. J. R. Munkres, Topology, Prentice Hall, 2000.
5. M. T. Nair, Functional Analysis, Prentice Hall, India
6. H. L. Royden, Real Analysis, Pearson, 4th edition, 2017.
7. G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.

PSMT303/PAMT303: Differential Geometry

Course Outcomes:

1. Students will be able to grasp parametrization of curves and surfaces.
2. Students will be able to understand the various geometrical aspects like tangent, arc length, curvature, torsion etc of plane and space curves.
3. Students will be able to understand the role of first fundamental theorem and second fundamental theorem in the computation of Gaussian curvature, mean curvature and principal curvature.
4. Students will aware about properties of various special types of curves and surfaces.

Unit I: Isometries of $\mathbb{R}^n$ (15 Lectures)
(Pre-requisite: Lines and planes) Orthogonal transformations of $\mathbb{R}^n$ and Orthogonal matrices. Reflection, Rotations and Translations of $\mathbb{R}^2$ and $\mathbb{R}^3$, Euler's theorem, Hyperplanes, Refection map about a hyperplane W of $\mathbb{R}^n$ through the origin, Isometry of $\mathbb{R}^n$, Isometries of the plane, Orientation preserving and reversing isometries of $\mathbb{R}^n$, Glide reflection.

Unit II: Curves (15 Lectures)
Parametrized curves, Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Curvature and torsion of curves in $\mathbb{R}^3$, Plane curves, Signed curvature for plane curves, Fundamental theorem for plane curves, Space curves, Serret-Frenet equations. Fundamental theorem for space curves.

Unit III: Regular Surfaces (15 Lectures)
Regular surfaces in $\mathbb{R}^3$, Examples. Surfaces as graphs, Surfaces as level sets, Surfaces of revolution. Tangent space to a surface at a point, Equivalent definitions. Smooth functions on a surface, Differential of a smooth function defined on a surface. Orientable surfaces. Mobius band is not orientable.

Unit IV: Curvature (15 Lectures)
The first fundamental form. Isometries of surfaces, Surface area, The Gauss map, The shape operator of a surface at a point, Self-adjointness of the shape operator, The second fundamental form,Normal curvature, Principle curvatures and directions, Euler's formula, Meusnier's Theorem, Gaussian curvature and mean curvature, Computation of Gaussian curvature. Geodesics.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. C. Bar, Elementary Differential geometry, Cambridge University Press, 2010.
3. M. DoCarmo, Differential geometry of curves and surfaces, Prentice Hall Inc., 1976.
4. S. Kumaresan, Linear Algebra, A Geometric Approach, 2000.
5. A. Pressley, Elementary Differential Geometry, Springer UTM.

PSMT304/PAMT304 & PSMT305/PAMT305 ELECTIVE COURSES I & II
The Elective Courses I and II will be any TWO of the following list of ten courses:

1. Algebraic Topology

Course Outcomes:

1. Students will learn about homotopy of maps, homotopy of paths and the fundamental group and its applications.
2. Students will learn about universal covering spaces.
3. Students will understand the Singular homology and Excision theorem.

Unit I. Fundamental Group (15 Lectures)
Homotopy. Path homotopy. The fundamental group. Simply connected spaces. Coveringspaces. Path lifting and homotopy lifting lemma. Fundamental group of the circle.

Unit II. Fundamental group, Applications (15 Lectures)
Deformation retracts and homotopy types. Fundamental group of S^n. Fundamental group of the projective space. Brower fixed point theorem. Fundamental theorem of algebra. Borsuk-Ulam theorem. Seifert-Van Kampen Theorem (without proof). Fundamental group of wedge of circles. Fundamental group of the torus.

Unit III. Universal Covering Spaces (15 Lectures)
Equivalence of covering maps and Equivalences of covering spaces. The general lifting lemma. The necessary and sufficient condition of equivalence of two covering maps. Universal covering space. Conditional existence of Universal covering space. Example of a space with no universal covering space.

Unit IV. Singular Homology (15 lectures)
Singular p-simplex, singular chain complex, singular homology group, reduced singular homology group, acyclic space, zero dimensional singular homology group, induced homomorphism. Axioms for singular theory: Identity axiom, composition axiom, homotopy axiom, exactness axiom and commutativity axiom. Excision Theorem.

Recommended Text Books:

1. Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. John Lee, Introduction to Topological Manifolds, Springer GTM, 2000.
3. James Munkres, Topology, Prentice Hall of India, 1992.
4. James Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.2.

2. Advanced Complex Analysis

Course Outcomes:

1. Students will learn about monodromy and the monodromy theorem.
2. Students will gain knowledge of Elliptic Functions and Zeta functions.
3. Students will understand Uniform convergence, Ascoli's theorem, Riemann mapping theorem.

Unit I. Monodromy (15 Lectures)
Holomorphic functions of one variable. Germs of holomorphic functions. Analytic continuation along a path. Examples including $z^{1/n}$ and $\log(z)$, Homotopy between paths. The monodromy theorem.

Unit II. Riemann Mapping Theorem (15 Lectures)
Uniform convergence, Ascoli's theorem, Riemann mapping theorem.

Unit III. Elliptic Functions (15 Lectures)
Lattices in $\mathbb{C}$. Elliptic functions (doubly periodic meromorphic functions) with respect to a lattice. Sum of residues in a fundamental parallelogram is zero and the sum of zeros and poles (counting multiplicities) in a fundamental parallelogram is zero, Weierstrass $\mathcal{P}$-function, Relation between $\mathcal{P}$ and $\mathcal{P}'$, Theorem that $\mathcal{P}$ and $\mathcal{P}'$ generate the field of elliptic functions.

Unit IV. Zeta Function (15 Lectures)
Gamma and Riemann Zeta functions, Analytic continuation, Functional equation for the Zeta function.

Recommended Text Books:

1. L. V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.
2. John Conway, Functions of one complex variable, Narosa India, 1978.
3. S. Lang, Complex Analysis, Springer, 1999 .
4. Narasimhan, Complex Analysis in one Variable, Birkhauser, 1985.
5. E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press, 2003.

3. Commutative Algebra

Course Outcomes:

1. Students will learn about basics of rings and modules, primary decomposition and associated primes.
2. Students will gain knowledge of integral extensions, valuation rings, discrete valuation rings, Dedekind domains.
3. Students will understand the Going up theorem and Going down theorem.

Unit I. Basics of rings and modules (15 Lectures)
Basic operations with commutative rings and modules, Polynomial and power series rings, Prime and maximal ideals, Extension and contractions, Nil and Jacobson radicals, Chain conditions, Hilbert basis theorem, Local rings, Localization, Nakayama's lemma, Tensor products.

Unit II. Primary decomposition (15 Lectures)
Primary ideal, primary decomposition of an ideal, examples, example of an ideal for which primary decomposition does not exist. Minimal decomposition, First uniqueness theorem for minimal primary decomposition of a decomposible ideals. Associated prime ideals, minimal prime ideals, containment of minimal primes in associated prime ideals, Behavior of primary ideals under localization. Second uniqueness theorem for minimal primary decomposition of a decomposible ideals.

Unit III. Integral Extensions (15 Lectures)
Integral extensions, Going up and going down theorems, The ring of integers in a quadratic extension of rationals, Noether normalization, Hilbert's nullstellensatz.

Unit IV. Dedekind Domains(15 Lectures)
Artinian rings, Discrete valuation rings, Alternative characterizations of discrete valuation rings, Dedekind domains, Fractional ideals, Factorization of ideals in a Dedekind domain, Examples.

Recommended Text Books:

1. M. F. Atiyah and I. G. McDonald, Introduction to Commutative Algebra, Addison Wesley, 2007.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons, 20011.
3. 1. S. Lang, Algebra, Springer, 2004.
4. O. Zariski and P. Samuel, Commutative Algebra, Volume 1, Princeton , NJ, 1965.

4. Algebraic Number Theory

Course Outcomes:

1. Students will learn about algebraic numbers, algebraic integers and further properties of rings of integers.
2. Students will understand the class group .
3. Students will gain knowledge of Ramification theory and Diophantine equations

Unit I. Algebraic Numbers and algebraic integers (15 Lectures)
Number fields, Algebraic numbers, Integral extensions, Ring of integers in a number field, Quadratic fields, Real and imaginary quadratic fields, Ring of integers in a quadratic field, Examples like the ring of Gaussian integers.

Unit II. Rings of integers (15 Lectures)
Norms and traces, basis for the ring of algebraic integers, Norm of an ideal, prime factorization of ideals, Norm of a principal ideal, Definition of Dedekind domain, Fractional ideals, Existence and uniqueness of factorization

Unit III. Class group (15 Lectures)
Principal fractional ideals, Norm map is multiplicative on integral ideals, Minkowski lemma, Finiteness of the class group, Explicit example of factorization in quadratic number fields, Legendre symbol, Jacobi symbol and quadratic reciprocity.

Unit IV. Ramification theory and Diophantine equations (15 Lectures)
Ramification, residue degree, transitivity of ramification and residue degrees, Proof of ramification theorem, Examples, Group of units, Applications to Diophantine equations.

Recommended Text Books:

1. M. Artin, Algebra, Prentice-Hall, India, 2000.
2. S. R. Ghorpade, Lectures on Field Theory and Ramification Theory, IITB.
3. Marcus, Number Fields, Springer.
4. Niven and Zuckermann, An Introduction to the Theory of Numbers, 1980.
5. Algebraic Number Theory, T.I.F.R. Lecture Notes, 1966

5. Advanced Partial Differential Equations

Course Outcomes:

1. Students will be able to grasp nature of the differential operator viz parabolic, hyperbolic and elliptic.

2. Students will be able to understand the solution and various properties of the Laplacian operator, heat operator and wave operator.

3. Students will aware about applications of the Laplacian operator, heat operator and wave operator.

Unit-I: Local existence theory (15 Lectures)
The differential operator, Real first order equations, the general Cauchy problem, Cauchy-Kowalevsky theorem, Local solvability: the Lewy example, the fundamental solution.

Unit-II: Laplace operator (15 Lectures)
Symmetry properties of the Laplacian, basic properties of the Harmonic functions, Green's identities, The mean value theorem, Liouville's theorem, the Fundamental solution, the Dirichlet and Neumann boundary value problems, the Green's function. Applications to the Dirichlet problem in a ball in $\mathbb{R}^n$ and in a half space of $\mathbb{R}^n$.

Unit-III: Heat operator (15 Lectures)
The properties of the Gaussian kernel, solution of initial value problem for heat equation: homogeneous and non-homogeneous, The fundamental solution for heat operator, Heat equation in a bounded domains, Maximum principle for the heat equation and applications.

Unit-IV: Wave operator (15 Lectures)
Wave operator in dimensions 1, 2 & 3; Cauchy problem for the wave equation. D'Alemberts solution,of the one dimensional wave equation, Poisson formula of spherical means, Hadamards method of descent, Inhomogeneous Wave equation, Wave equation in a bounded domain.

Recommended Text Books:

1. G. B. Folland, Introduction to partial differential equations, Overseas Press, 1995.

2. F. John, Partial Differential Equations, Springer publications, 1964.

6. Integral Transforms

Course Outcomes:

1. Students will be able to grasp the concept of integral transforms and development of corresponding kernels.

2. Students will be able to understand various properties of the Laplace transform, Fourier transform, Mellin transform and $Z-$transform.

3. Students will aware about applications of the integral transform in the solution of initial and boundary value problems.

Unit I: The Laplace Transform (15 Lectures)
Definition of Laplace Transform, Laplace transforms of some elementary functions, Existence theorem, Properties of Laplace transform, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Convolution Theorem, Inversion theorem, Laplace transform of special functions: Heaviside unit step function, Dirac-delta function and Periodic function, Application of Laplace transform to evaluation of integrals and solutions of ODEs & PDEs: One dimensional heat equation & wave equation.

Unit II: The Fourier Transform (15 Lectures)
Fourier integral representation, Fourier integral theorem, Fourier Sine & Cosine integral representation, Riemann-Lebesgue lemma, Fourier transform pairs, Fourier Sine & Cosine transform pairs, Fourier transform of elementary functions, Properties of Fourier Transform, Convolution Theorem, Convolution integrals of Fourier, Parseval Identity, Cosine & Sine convolution integrals, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems, Heat conduction in solids (one dimensional problems in infinite & semi infinite domain).

Unit III: The Mellin Transform (15 Lectures)
Derivation for Mellin transform & its inversion by Fourier integral theorem, Properties and evaluation of Mellin transforms, Complex variable method, Convolution theorem for Mellin transform, Applications of Mellin transform: Summation of series, Products of random variables, Application to boundary value problems.

Unit IV: The Z-Transform (15 Lectures)
Definition of Z-transform, Z-transform of some elementary sequences, Properties of Z-transform Convolution Theorem, Inversion of the Z-transform, Applications of Z-transform to solutions of difference equations and summation of series.

Recommended Text Books:

1. L. Andrews and B. Shivamogg, Integral Transforms for Engineers, Prentice Hall of India, 1999.

2. R. Bracemell, Fourier Transform and its Applications, MacGraw hill, 1963.

3. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their Applications, CRC Press Taylor & Francis, 2014.

4. Brian Davies, Integral transforms and their Applications, Springer, 1985.

5. I. N. Sneddon, Use of Integral Transforms, Tata-McGraw Hill, 1972.

7. Numerical Analysis

Course Outcomes:

1. Students will be able to grasp the concept of numerical solution of various mathematical problems and corresponding erorrs.
2. Students will be able to understand the approximation of functions by least square method.
3. Students will aware about applications of various numerical techniques in the solution of difference equations, ordinary and partial differential equations.

Note: Numerical methods be taught with error estimate, convergence and stability

Unit I: Approximation of functions (15 Lectures)
Least squares approximation, Weighted least squares method, Gram-Schmidt orthogonalizing process, Least squares approximation by Chebyshev polynomials. Discrete Fourier Transform and Fast Fourier Transform.

Unit II: Differential and Difference Equations (15 Lectures)
Differential equations: Solutions of linear differential equations with constant coefficients by Predictor corrector methods and Milne's method. Galerkin's method for two point linear boundary value problems. Difference Equations: Linear difference equation with constant coefficients and methods of solving them.

Unit III: Numerical Integration (15 Lectures)
Derivation of a formula for numerical integration in terms of finite difference and its special cases viz Trapezoidal, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule. Boole's and Weddle's rule, Guass Legendre numerical integration, Gauss-Chebyshev numerical integration, Gauss-Hermite numerical integration, Gauss-Laguree numerical integration with the derivation of all methods using the method of undetermined coefficients. Romberg's method. Gaussian quadratures, Multiple integrals.

Unit IV: Numerical solutions of Partial Differential Equations (15 Lectures)
Classifications of Partial Differential Equations, Finite Difference approximations to derivatives. Numerical methods of solving elliptic, parabolic and hyperbolic equations.

Recommended Text Books

1. H. M. Antia, Numerical Analysis, Hindustan Publictions.
2. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and sons, 2008.

3. Jain, Iyengar, Numerical Methods for Scientific and Engineering Problems, New Age International, 2009.

4. Erwin Kreyszig, Advanced Engineering Mathematics, Wiley John Wiley & Sons, 1999.

5. S.S. Sastry, Introductory Methods of Numerical Analysis, Prentice-Hall, India, 2012.

8. Graph Theory

Course Outcomes: Students should know the following:

1. Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem. Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa' \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

2. Cut vertices, Cut edges, Bond.

3. Trees, Characterizations of Trees, Spanning trees, Fundamental cycles.

4. Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

5. Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness.

6. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-Hamiltonian graph, Traveling salesman problem.

7. Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Hall's theorem (Necessary and sufficient condition for complete Matching), Konig's theorem (Maximum matching is same as minimum vettex cover), Tutte's theorem, Personal assignment problem,

8. Independent sets and covering- $\alpha + \beta = p$, Gallai's theorem.

9. Ramsey theorem-Existence of $r(k, l)$, Upper bounds of $r(k, l)$, Lower bound for $r(k, l) \geq 2m/2$ where $m = \min\{k, l\}$, Generalize Ramsey numbers-$r(k_1, k_2, \ldots, k_n)$, Graph Ramsey Theorem, Evaluation of $r(G, H)$ when for simple graphs $G = P_3, H = C_4$.

Unit I. Connectivity(15 Lectures)
Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa' \leq \delta$; Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

Unit II. Trees (15 Lectures)
Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula, Connector problem-Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)
Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese post- man problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisa- tion, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

Unit IV. Matching and Ramsey Theory (15 Lectures)
Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Halls theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering-$\alpha + \beta = p$; Gallai's theorem, Ramsey theorem-Existence of $r(k;l)$; Upper and Lower bounds of $r(k;l)$. $r(k;l) \geq 2^{m/2}$ where $m = \min\{k,l\}$; Generalize Ramsey numbers- $r(k_1, k_2, \ldots, k_n)$; Graph Ramsey theorem, Evaluation of $r(G;H)$ when for simple graphs $G = P_3; H = C_4$:

Recommended Text Books:

1. M. Behzad and A. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.

2. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier.

3. J. A. Bondy and U.S. R. Murty, Graph Theory, GTM 244 Springer, 2008.

4. Reinhard Diestel, Graph Theory GTM 173, 5th edition, Springer.

5. K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.

6. D.B.West, Introduction to Graph Theory, PHI, 2009.

9. Coding Theory

Course Outcomes: Students should know the following:

1. Error detection, Correction and Decoding. Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

2. Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbor decoding for linear codes, Syndrome decoding.

3. Definition of cyclic codes, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

4. Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

Unit I. Error detection, Correction and Decoding (15 Lectures)
Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbour/minimum distance decoding. Relation between minimum distance of a code and the error-detecting and error-correcting capabilities of the code.

Unit II. Linear codes (15 Lectures)
Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbour decoding for linear codes, Syndrome decoding.

Unit III. Bonds in coding theory (15 Lectures)
Sphere-covering bound, Gilbert–Varshamov bound, Hamming bound and perfect codes, Binary Hamming codes, q-ary Hamming codes, Golay codes, Singleton bound and MDS codes, Plotkin bound, Nonlinear codes, Hadamard matrix codes, Nordstrom–Robinson code, Preparata codes, Kerdock codes, Griesmer bound, Linear programming bound.

Unit IV. Constructions of codes (15 Lectures)
Propagation rules, Reed–Muller codes, Subfield codes, Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes. Alternant codes Goppa codes, Reed-Solomon codes. Quadratic-residue codes, Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Hamming code and simplex code.

Recommended Text Books:

1. S. R. Ghorpade, Aspects of coding theory, Lecture Notes IITB.
2. San Ling and Chaoing xing, Coding Theory- A First Course, Cambridge University Press, 2004.
3. Lid and Pilz, Applied Abstract Algebra, 2nd Edition, Springer, 1984.

10. Design Theory

Course Outcomes: Students should know the following:

1. Balanced Incomplete Block Designs, Basic Definitions and Properties, I incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.

2. Symmetric BIBDs An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

3. Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

4. Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Matrices and Hadamard Matrices, A Product Construction, Williamsons Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

Unit I. Introduction to Balanced Incomplete Block Designs (15 Lectures)
What Is Design Theory? Basic Definitions and Properties, Incidence Matrices, Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.

Unit II. Symmetric BIBDs (15 Lectures)
An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

Unit III. Difference Sets and Automorphisms (15 Lectures)
Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

Unit IV. Hadamard Matrices and Designs (15 Lectures)
Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Ma- trices and Hadamard Matrices, A Product Construction, Williamson's Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Ma- trices, Bent Functions.

Recommended Text Books:

1. T. Beth, D. Jungnickel and H. Lenz, Design Theory, Volume 1 (Second Edition), Cambridge University Press, Cambridge, 1999.

2. D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cambridge, 1985.

3. D. R. Stinson, Combinatorial Designs: Constructions and Analysis, Springer,2004.

4. W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall.

Semester-IV

PSMT401/PAMT401: Algebra IV

Course Outcomes:

1. Students will learn about algebraic extensions and their properties.
2. Splitting fields and their degrees can be computed. The notion of normal extension is introduced and its equivalent properties are discussed.
3. Finite fields as splitting fields are visualized and notion of algebraic closure is discussed in detail.
4. Galois extensions are studied and the fundamental theorem of Galois theory is established.
5. Cyclotomic extensions are studied in detail and order of its Galois group is computed.
6. Examples of fixed fields, field automorphisms and fundamental theorem are studied in special cases.

Unit I. Algebraic Extensions (15 lectures)
Prime subfield of a field, definition of field extension K/F, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element. Algebraic extensions, Finite extensions, degree of an algebraic element, degree of a field extension. If α is algebraic over the filed F and $m_\alpha(x)$ is the minimum polynomial of α over F, then $F(\alpha)$ is isomorphic to $F[X]/(m_\alpha(x))$. If $F \subseteq K \subseteq L$ are fields, then $[L : F] = [L : K][K : F]$. If K/F is a field extension, then the collection of all elements of K which are algebraic over F is a subfield of K. If L/K, K/F are algebraic extensions, then so is L/F. Composite filed K_1K_2 of two subfields of a field and examples. Classical Straight-edge and Compass constructions: definition of Constructible points, lines, circles by Straight-edge and Compass starting with $(0, 0)$ and $(1, 0)$, definition of constructible real numbers. If $a \in \mathbb{R}$ is constructible, then a is an algebraic number and its degree over $\mathbb{Q}$ is a power of 2. $\cos 20^0$ is not a constructible number. The regular 7-gon is not constructible. The regular 17-gon is constructible. The Constructible numbers form a subfield of $\mathbb{R}$. If $a > 0$ is constructible, then so is $\sqrt{a}$. Impossibility of the classical Greek problems: 1) Doubling a Cube, 2) Trisecting an Angle, 3) Squaring the Circle is possible.

Unit II. Normal and Separable Extensions (15 lectures)
Splitting field for a set of polynomials, normal extension, examples such of splitting fields of $x^p - 1$ (p prime), uniqueness of splitting fields, existence and uniqueness of finite fields. Algebraic closure of a field, existence of algebraic closure. Separable elements, Separable extensions. Perfect Fields. Frobenius automorphism of a finite field. Every irreducible polynomial over a finite field is separable. Primitive element theorem.

Unit III. Galois Theory(15 Lectures)
Galois group $G(K/F)$ of a field extension K/F, Galois extensions, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory.

Unit IV. Applications (15 Lectures)
Cyclotomic field $Q(\zeta_n)$ (splitting field of $x^n - 1$ over $\mathbb{Q}$), Cyclotomic polynomial, degree of Cyclotomic field $Q(\zeta_n)$. Galois group for an irreducible cubic polynomial, Galois group for an irreducible quartic polynomial. Solvability by radicals in terms of Galois group and Abel's theorem on the insolvability of a general quintic polynomial.

Rcommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons, 2011.
3. C. R. Handlock, Field Theory and its classical problems, Cambridge University Press, 2000.
4. Ireland K. & Rosen M., A Classical Introduction to Modern Number Theory, Springer, 1990.
5. N. Jacobson, Basic Algebra, Dover, 1985.
6. S. Lang, Algebra, Springer Verlag, 2004

PSMT402/PAMT402: Fourier Analysis

Course Outcomes:

1. Students will be able to understand the Fourier series expansion of a periodic function and their convergence.
2. Students will be able to grasp properties of the Dirichlet kernel, Fejer kernel, Poisson kernel and the concept of a good kernel.
3. Students will aware about application of a Fourier series in the solution of the Dirichlet problem and heat equation.

Unit I: Fourier series (15 Lectures)
The Fourier series of a periodic function, Bessel's inequality for a 2π periodic Riemann integrable function, Dirichlet kernel, Convergence theorem for the Fourier series of a 2π periodic and piecewise smooth function, Uniqueness theorem. Derivatives, Integrals and Uniform Convergence properties, Fourier series on intervals, Even and odd extensions, Fourier series of a periodic function of an arbitrary period.

Unit II: Dirichlet's theorem (15 Lectures)
Review: Lebesgue measure of $\mathbb{R}$; Lebesgue integrable functions, Dominated Convergence theorem, Bounded linear maps (no questions be asked). Fourier coefficients of integrable and square integrable periodic functions, The Riemann-Lebesgue lemma and its converse, Bessel's inequality for a L^2 periodic functions, Dirichlet's theorem, Concept of Good kernels, Dirichlet's kernel is not good kernel.

Unit III: Fejer's Theorem and applications (15 Lectures)
Cesaro summability, Cesaro mean and Cesaro sum of the Fourier series, Fejer's Kernel, Fejer's kernel is a good kernel, Fejer's Theorem, Parseval's identity. Convergence of Fourier series of an L^2 periodic function w.r.t the L^2-norm, Riesz-Fischer theorem on Unitary isomorphism from $L^2(-\pi, \pi)$ onto the sequence space l^2 of square summable complex sequences.

Unit IV: Dirichlet Problem in the unit disc(15 Lectures)
Abel summability, Abel sum of the Fourier series, The Poisson kernel, The Poisson kernel is a good kernel, Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The solution of Dirichlet problem for the unit disc.The Poisson integral, Applications of Fourier series to heat equation on the circle.

Recommended Text Books:

1. R. Beals, Analysis An Introduction,Cambridge University Press, 2004
2. R. Bhatia, Fourier Series, MAA Press AMS, 2005.

3. G.B. Folland, Fourier Analysis and its Applications, American Mathematical Society, Indian Edition 2010.
4. E. M. Stein and R. Shakarchi, Fourier Analysis an Introduction, Princeton University Press, 2003.
5. E. M. Stein and R. Shakarchi, Real Analysis an Introduction, New age International.

PSMT 403/PAMT 403: Calculus on Manifolds

Course Outcomes:

1. Students will be able to grasp the concept of tensor, alternating tensor, wedge product and differential forms.

2. Students will be able to understand fields and forms on manifolds.

3. Students will be able to understand the application of Classical theorems: Stoke's theorem, Green's theorem, Gauss divergence theorem.

Unit I: Multilinear Algebra (15 Lectures)
Multilinear map on a finite dimensional vector space V over $\mathbb{R}$ and $k-$ tensors on V, the collection $\tau^k(V)$ (or $\otimes^k(V^*)$) of all $k-$ tensors on V, tensor product $S \otimes T$ of $S \in \tau^k(V)$ and $T \in \tau^k(V)$. Alternating tensor and the collection $\wedge^k V^*$ of $k-$tensors on V. The exterior product (or wedge product), basis of $\wedge^k V^*$, orientation of a finite dimensional vector space V over $\mathbb{R}$.

Unit II: Differential Forms (15 Lectures)
Differential forms: $k-$forms on $\mathbb{R}^n$, wedge product $\omega \wedge \eta$ of a $k-$ form ω and $l-$ forms η, the exterior derivative and its properties, Pull back forms and its properties, closed and exact forms, Poincare's lemma.

Unit III: Basics of Submanifolds of $\mathbb{R}^n$ (15 Lectures)
Submanifolds of $\mathbb{R}^n$, submanifolds of $\mathbb{R}^n$ with boundary, Smooth functions defined on Submanifolds of $\mathbb{R}^n$, Tangent vector and Tangent space of Submanifolds of $\mathbb{R}^n$. $p-$forms and differential $p-$forms on a submanifolds of $\mathbb{R}^n$, exterior derivative $d\omega$ of any differential $p-$forms on a submanifolds of $\mathbb{R}^n$, Orientable submanifolds of $\mathbb{R}^n$ and Oriented submanifolds of $\mathbb{R}^n$, Orientation preserving map, Vector fields on submanifolds of $\mathbb{R}^n$, outward unit normal on the boundary of a submanifolds of $\mathbb{R}^n$ with non-empty boundary, induced orientation of the boundary of an oriented submanifolds of $\mathbb{R}^n$ with non-empty boundary.

Unit IV: Stoke's Theorem (15 Lectures)
Integral $\int_{[0,1]^k} \omega$ of a $k-$form on cube $[0,1]^k$, Integral $\int_c \omega$ of a $k-$ form on an open subset A of $\mathbb{R}^k$ where c is a singular $k-$ cube in A, Theorem (Stoke's Theorem for $k-$ cube): If ω is $k-1$ form on an open subset A of $\mathbb{R}^k$ and c is a singular $k-$ cube in A then $\int_c d\omega = \int \partial c\omega$.

Integration of a differentiable $k-$ form on oriented k dimensional submanifolds M of $\mathbb{R}^n$: Change of variables theorem: If $c_1, c_2 : [0,1]^k \longrightarrow M$ are two Orientation preserving maps in M and ω is any $k-$ form on M such that $\omega = 0$ outside of $c_1([0,1]^k) \cap c_2([0,1]^k)$ then $\int_{c_1} \omega = \int_{c_2} \omega$, Stokes' theorem for submanifolds of $\mathbb{R}^k$, Volume element, Integration of functions on a submanifold of $\mathbb{R}^k$, Classical theorems: Green's theorem, Divergence theorem of Gauss, Green's identities.

Recommended Text Books:

1. A. Browder, Mathematical Analysis, Springer International Edition, 1996.
2. V. Guillemin and A. Pollack, Differential Topology, AMS Chelsea Publishing, 2010.
3. J. Munkers, Analysis on Manifolds, Addision Wesley, 1997.
4. M. Spivak, Calculus on Manifolds, W.A. Benjamin Inc., 1965.

PSMT404/PAMT404 ELECTIVE COURSE
Elective Course will be any ONE of the following Skill Courses:

Skill Course
Skill Course I: Business Statistics

Course Outcomes: Students should know the following:

1. Classification of Data: Requisites of Ideal Classification, Basis of Classification.
2. Organizing Data Using Data Array: Frequency Distribution, Methods of Data Classification, Bivariate Frequency Distribution, Types of Frequency Distributions.
3. Tabulation of Data: Objectives of Tabulation, Parts of a Table, Types of Tables, General and Summary Tables, Original and Derived Tables.
4. Graphical Presentation of Data: Functions of a Graph, Advantages and Limitations of Diagrams(Graph), General Rules for Drawing Diagrams.
5. Types of Diagrams: One-Dimensional Diagrams, Two-Dimensional Diagrams, Three-Dimensional Diagrams, Pictograms or Ideographs, Cartograms or Statistical Maps.
6. Exploratory Data Analysis: Stem-and-Leaf Displays.
7. Objectives of Averaging, Requisites of a Measure of Central Tendency, Measures of Central Tendency,
8. Mathematical Averages: Arithmetic Mean of Ungrouped Data, Arithmetic Mean of Grouped (Or Classified) Data, Some Special Types of Problems and Their Solutions, Advantages and Disadvantages of Arithmetic Mean, Weighted Arithmetic Mean.
9. Geometric Mean: Combined Geometric Mean, Weighted Geometric Mean, Advantages, Disadvantages and Applications of G.m.
10. Harmonic Mean: Advantages, Disadvantages and Applications of H.M. Relationship Between A.M., G.M. and H.M.
11. Averages of Position: Median, Advantages, Disadvantages and Applications of Median.
12. Partition Values quartiles, Deciles and Percentiles: Graphical Method for Calculating Partition Values.
13. Mode: Graphical Method for Calculating Mode Value. Advantages and Disadvantages of Mode Value.
14. Relationship Between Mean, Median and Mode, Comparison Between Measures of Central Tendency.

15. Significance of Measuring Dispersion (Variation):Essential Requisites for a Measure of Variation.

16. Classification of Measures of Dispersion.

17. Distance Measures: Range, Interquartile Range or Deviation.

18. Average Deviation Measures: Mean Absolute Deviation, Variance and Standard Deviation,

19. Mathematical Properties of Standard Deviation, Chebyshev's Theorem, Coefficient of Variation.

20. Measures of Skewness: Relative Measures of Skewness.

21. Moments: Moments About Mean, Moments About Arbitrary Point, Moments About Zero or Origin, Relationship Between Central Moments and Moments About Any Arbitrary Point, Moments in Standard Units, Sheppard's Corrections for Moments.

22. Kurtosis: Measures of Kurtosis.

Unit I. Data Classification, Tabulation and Presentation (15 Lectures)
Classification of Data: Requisites of Ideal Classification, Basis of Classification. Organizing Data Using Data Array: Frequency Distribution, Methods of Data Classification, Bi-variate Frequency Distribution, Types of Frequency Distributions. Tabulation of Data: Objectives of Tabulation, Parts of a Table, Types of Tables, General and Summary Tables, Original and Derived Tables. Graphical Presentation of Data: Functions of a Graph, Advantages and Limitations of Diagrams (Graph), General Rules for Drawing Diagrams. Types of Diagrams: One-Dimensional Diagrams, Two-Dimensional Diagrams, Three-Dimensional Diagrams, Pictograms or Ideographs, Cartograms or Statistical Maps. Exploratory Data Analysis: Stem-and-Leaf Displays.

Unit II. Measures of Central Tendency (15 Lectures)
Objectives of Averaging, Requisites of a Measure of Central Tendency, Measures of Central Tendency. Mathematical Averages: Arithmetic Mean of Ungrouped Data, Arithmetic Mean of Grouped (or classified) Data, Some Special Types of Problems and Their Solutions, Advantages and Disadvantages of Arithmetic Mean, Weighted Arithmetic Mean. Geometric Mean: Combined Geometric Mean, Weighted Geometric Mean, Advantages, Disadvantages and Applications of G.m. Harmonic Mean: Advantages, Disadvantages and Applications of H.M. Relationship Between A.M., G.M. and H.M. Averages of Position: Median, Advantages, Disadvantages and Applications of Median. Partition Values quartiles, Deciles and Percentiles: Graphical Method for Calculating Partition Values. Mode: Graphical Method for Calculating Mode Value. Advantages and Disadvantages of Mode Value. Relationship Between Mean, Median and Mode, Comparison Between Measures of Central Tendency.

Unit III. Measures of Dispersion (15 Lectures)
Significance of Measuring Dispersion (Variation):Essential Requisites for a Measure of Variation. Classification of Measures of Dispersion. Distance Measures: Range, Interquartile Range or Deviation. Average Deviation Measures: Mean Absolute Deviation, Variance and Standard Deviation, Mathematical Properties of Standard Deviation, Chebyshev's Theorem, Coefficient of Variation.

Unit IV. Skewness, Moments and Kurtosis (15 Lectures)
Measures of Skewness: Relative Measures of Skewness. Moments: Moments About Mean, Moments About Arbitrary Point, Moments About Zero or Origin, Relationship Between Central Moments and Moments About Any Arbitrary Point, Moments in Standard Units, Sheppard's Corrections for Moments. Kurtosis: Measures of Kurtosis.

Practicals

Four practicals, one on each unit on realistic data and its analysis should be conducted with the help of R-statistical package.

Reference Book: J. K. Sharma , Business Statistics, Pearson Education India, 2012.

Skill Course II: Statistical Methods

Course Outcomes: Students should know the following:

1. Measures of central tendencies: Mean, Median, Mode.
2. Measures of Dispersion: Range, Mean deviation, Standard deviation. Measures of skewness. Measures of relationship: Covariance, Karl Pearson's coefficient of Correlation, Rank Correlation. Basics of Probability.
3. Sampling Distribution, Student's t-Distribution, Chi-square (χ^2) Distribution, Snedecor's F-Distribution. Standard Error. Central Limit theorem. Type I and Type II Errors, Critical Regions. F-test, t-test, χ^2 test, goodness of Fit test.
4. The Anova Technique. The basic Principle of Anova. One Way ANOVA, Two Way ANOVA. Latin square design. Analysis of Co-variance.
5. R as Statistical software and language, methods of Data input, Data accessing, usefull built-in functions, Graphics with R, Saving, storing and retrieving work.

Unit I. Basic notions of Statistics (15 Lectures)
Measures of central tendencies: Mean, Median, Mode. Measures of Dispersion: Range, Mean deviation, Standard deviation. Measures of skewness. Measures of relationship: Covariance, Karl Pearson's coefficient of Correlation, Rank Correlation. Basics of Probability.

Unit II. Sampling and Testing of Hypothesis (15 Lectures)
Sampling Distribution, Student's t-Distribution, Chi-square (χ^2) Distribution, Snedecor's F- Distribution. Standard Error. Central Limit theorem. Type I and Type II Errors, Critical Regions. F-test, t-test, χ^2 test, goodness of Fit test. Testing of Hypothesis.

Unit III. Analysis of Variance (15 Lectures)
The Anova Technique. The basic Principle of Anova. One Way ANOVA, Two Way ANOVA. Latin square design. Analysis of Co-variance.

Unit IV. Use of package R (15 Lectures)
R as Statistical software and language, methods of Data input, Data accessing, useful built-in functions, Graphics with R, Saving, storing and retrieving work.

Practicals

Four practicals, one on each unit on realistic data and its analysis should be conducted with the help of R-statistical package.

Recommended Text Books:

1. S. C. Gupta And V. K. Kapoor, Fundamentals Of Mathematical Statistics, Sultan Chand & Sons, 1994.
2. C. R. Kothari and G. Garg, Research Methodology Methods and Techniques, New Age International, 2019.
3. S. G. Purohit, S.D. Gore and S.R. Deshmukh, Statistics using R, Alpha Science Int., 2008.

Skill Course III: Computer Science

Aim: Mathematics students are well versed in logic. This Skill course aims at giving input of necessary skills of algorithms and data structures and relational database background so that the students are found suitable to be absorbed as trainee software professional in industry. Prerequisite for this course: Good knowledge of C, C++ or java or python.

Course Outcomes: Students should know the following:

1. Basics of object oriented programming principles, templates, reference operators NEW and delete in C++, the java innovation which avoids use of delete, classes polymorphism friend functions, inheritance, multiple inheritance operator overloading basics
2. Basic algorithms, selection sort, quick sort, heap sort, priory queses, radix sort, merge sort, dynamic programming, app pairs, shortest paths, image compression, topological sorting, single source shortest paths reference, hashing intuitive evaluation of running time.
3. Stacks queues, linked lists implementation and simple applications, trees implementation and tree traversal (stress on binary trees),
4. Concept of relational databases , normal forms BCNF and third normal forms. Armstrongs axioms. Relational algebra and operations in it.

Unit I. OOPS Concepts (15 lectures)
Basics of object oriented programming principles, templates, reference operators NEW and delete in C++, the java innovation which avoids use of delete, classes polymorphism friend functions, inheritance, multiple inheritance operator overloading basics only.

Unit II. Basic Algorithms (15 lectures)
Basic algorithms, selection sort, quick sort, heap sort, priory queues, radix sort, merge sort, dynamic programming, app pairs, shortest paths, image compression, topological sorting, single source shortest paths reference, hashing intuitive evaluation of running time.

Unit III. Data Structures (15 lectures)
Stacks queues, linked lists implementation and simple applications, trees implementation and tree traversal (stress on binary trees),

Unit IV. Relational Databases (15 lectures)
concept of relational databases, normal forms BCNF and third normal forms. Armstrong's axioms. Relational algebra and operations in it.

Recommended Text Books:

1. T. Aron and others, Data structure using C, Pratibha Publications, 2020.
2. Balaguruswamy, Programming in C++, Tata McGraw Hill, 2008
3. S. Sahani, Data Structures and applications, TMH.
4. Programming in Java, Schaum Series.
5. J.D. Ullam, Principles of Database systems, Computer Science Press, 1982.

Skill Course IV: Linear and Non-linear Programming

Course Outcomes:

1. Understand the concept of an objective function, a feasible region, and a solution set of an optimization problem.

2. Understand the broad classification of optimization problems, and where they arise in simple applications.

3. Use the simplex method to find an optimal vector for the standard linear programming problem and the corresponding dual problem.

4. Use Lagrange multipliers to solve nonlinear optimization problems.

5. Write down and apply Kuhn-Tucker conditions for constrained nonlinear optimization problems.

6. Apply approximate methods for constraint problems.

7. Understand the importance of convexity in nonlinear optimization prob- lems.

8. Apply basic line search methods to one-dimensional optimization problems, gradient methods to optimization problems, ,conjugate gradient methods to optimization problems

Unit I. Linear Programming (15 Lectures)
Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

Unit II.Transportation Problems(15 Lectures)
Transportation and Assignment problems of linear programming, Sequencing theory and Travelling salesperson problem.

Unit III. Unconstrained Optimization(15 Lectures)
First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newtons Method, Secant Method, Gradient Methods, Steep-est Descent Methods.

Unit IV. Constrained Optimization Problems(15 Lectures)
Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

Recommended Text Books:

1. E. K. P. Chong and S. H. Zak, Introduction to Optimization, Wiley-Int., 1996.

2. F. S. Hillier and G.J. Lieberman, Introduction to Operations Research (Sixth Edition), McGraw Hill, 1990.

3. G. Hadley, Linear Programming, Narosa Publishing House, 1995.

4. S. S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd, New Delhi, 1984.

5. Rangarajan and K. Sundaram, A First Course in Optimization Theory, Cambridge University Press, 1996.

6. K. Swarup, P. K. Gupta and Man Mohan, Operations Research, S. Chand andsons, New Delhi, 2010.

7. H. A. Taha, Operations Research-An introduction, Macmillan Publishing Co. Inc., 1997.

Skill Course V: Computational Algebra

Course Outcomes:

1. Students will learn to balance between theory and practicals via the use of computers.
2. Previously learnt concepts can be strengthened by allowing them to explore topics using the mathematical softwares.

Unit I. Representation Theory (15 lectures)
Linear representations of a finite group on a finite dimensional vector space over $\mathbb{C}$. If ρ is a representation of a finite group G on a complex vector space V, then there exits a G-invariant positive definite Hermitian inner product on V. Complete reducibility (Maschke's theorem). The space of class functions, Characters and Orthogonality relations. For a finite group G, there are finitely many isomorphism classes of irreducible representations, the same number as the number of conjugacy classes in G. Two representations having same character are isomorphic. Regular representation. Schur's lemma and proof of the Orthogonality relations. Every irreducible representation over $\mathbb{C}$ of a finite Abelian group is one dimensional. Character tables with emphasis on examples of groups of small order.

Unit II. Group theory software (15 lectures)
Introduction to Sage Math and GAP softwares. Permutation groups, examples, Groups with generators, center of a group, derived series examples, Character tables, Matrices over finite fields.

Unit III. Ideals, Varieties and Algorithms (15 lectures)
Polynomials in one variable, Affine spaces, Parameterizations of Affine varieties, Polynomial rings in more variables, Dickson's lemma, Hilbert basis theorem, Basics of invariant theory, Groebner basis, Buchberger Algorithm and Applications.

Unit IV. Commutative Algebra software (15 lectures)
Introduction to Singular and Macaulay. Polynomials in more than two variables over fields, quotient rings, localizations, Groebner bases.

Recommended Text Books:

1. M. Artin, Algebra, Prentice Hall of India, 2011.
2. David A. Cox, John Little and Donal O'Shea, Ideals, Varieties, and Algorithms, Springe, 2015.
3. S. Sternberg, Group theory and Physics, Cambridge University Press, 1994.
4. Bernd Sturmfeles, Algorithms in Invariant Theory, Springer, 2008.

PSMT405/PAMT405: PROJECT

Evaluation of Project work
The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the respective college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee.

Each project output shall be displayed on the website of the University.
The Marks for the project are detailed below:

1. Contents of the project: 50 Marks
2. Attendance: 10 Marks
3. Presentation of the project: 20 Marks
4. Viva of the project: 20 Marks
5. Total Marks= 100 Marks per project per student

6. Scheme of Examination
The scheme of examination for the syllabus of Semesters III & IV of M.A./M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows.

Scheme of Evaluation R8435 for M. Sc /M. A.–

1. A) 80: 20 for distance education (external evaluation of 80 marks and internal evaluation of 20 marks) under the choice based credit system (CBCS).
2. B) 60:40 for university affiliated PG centers (external evaluation of 60 marks and internal evaluation of 40 marks).
3. C) 100 percent internal evaluation scheme for University department of mathematics (One mid semester test of 30 marks, 05 marks for attendance, 05 marks for active participation and one end semester test of 60 marks, both tests will be conducted by the department and answer book will be shown to the students).

Duration:- Examination shall be of 60 Marks and $2\frac{1}{2}$ Hours duration.
Theory Question Paper Pattern for above Schemes:-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.

3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, ·· and the allocation of marks depend on the weight-age of the topic.
5. Each question will be of maximum 18 marks when marks of all the sub-questions are added (including the options) in that question.
6. For scheme A: 60 marks will be converted into 80 marks.

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