(1)

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1. 67058

Q1

(a) (2 marks each

(i) False

$0 \le f(x) \le 1 \quad \forall x$.

(ii) $\boxed{K = 4}$

Hence statement is false.

(iii) False.

$P(x \le a) = 1 - P(x \ge a)$

(iv) mean of $x = \frac{a+b}{2} = 8$

(v) True

(vi) False, $H_1: \mu > 10$ is a one Tailed test.

(vii) False

Power of the test = 1 − size of type-II error.

$P(X = x) = 0$

$F(X \le m) = 1/2$

$F(0) = 1/2$

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(iv) $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ $\quad -\infty < z < \infty$

$\mu = 0$
$\sigma = 1$

$= 0$, o.w.

(v) Definition — 2 marks.

(vi) Point estimation — (1)
Interval estimation — (1)

(vii) Type - I error — (2)

Q 2(a)

(i) Raw moments — (2)
Central moments — (2)
μ_3 in terms of raw moments — (1)

(ii) $F(x) = \int_0^x \frac{1}{360} dt$

— (2)

$= \frac{x}{360}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{360} & 0 \leq x < 360 \\ 1 & x \geq 360 \end{cases}$$ (1)

$$P(90 < x < 180) = \int_{90}^{180} \frac{1}{360}\, dx$$

$$= \frac{1}{4} \quad \text{— (1)}$$

Also $P(90 < x < 180) = F(180) - F(90)$

$$= \frac{1}{4} \quad \text{— (1)}$$

(b)

(i) formulae $\left(\left(1\tfrac{1}{2}\right) + \left(1\tfrac{1}{2}\right) \right)$

(ii) $k\int_{1}^{3} (x-1)^3\, dx = 1$

$$k\left[\frac{(x-1)^4}{4}\right]_1^3 = 1 \quad \text{— (02)}$$

~~(02)~~

$\Rightarrow \quad k = 1/4$

$$\text{mean} = \frac{1}{4}\int_{1}^{3} x(x-1)^3\, dx$$

(02)

$$= 2.6$$

$$E(x^2) = \frac{1}{4}\int_{1}^{3} x^2 (x-1)^3\, dx$$

(02)

$$= 6.867$$

$V(x) = 0.1067$ (01)

(c) Let $0 \le x < 1$.

$$F(x) = \frac{2}{3}\int_0^x t\,dx = \frac{x^2}{3}$$

Let $1 \le x < 2$.

$$F(x) = \frac{2}{3}\int_0^1 t\,dt + \frac{2}{3}\int_0^x (2-t)\,dt$$

$$= \frac{2}{3}\left(2x - \frac{x^2}{2} - 1\right)$$

Let $2 \le x < 3$.

$$F(x) = \frac{2}{3}\int_0^1 t\,dt + \frac{2}{3}\int_1^2 (2-t)\,dt + \frac{2}{3}\int_2^x t(3-t)\,dt$$

$$= \frac{1}{21}\left(9x^2 - 2x^3 - 6\right)$$

$$F(x) = 0 \qquad x < 0$$

$$= \frac{x^2}{3} \qquad 0 \le x < 1$$

$$= \frac{2}{3}\left(2x - \frac{x^2}{2} - 1\right) \qquad 1 \le x < 2$$

$$= \frac{1}{21}\left(9x^2 - 2x^3 - 6\right) \qquad 2 \le x < 3$$

$$= 1 \qquad x \geq 3$$

To find median.

$F(m) = 1/2$

x	0	1	2
$F(x)$	0	1/3	2/3

$\therefore F(1) \leq 1/2 \leq F(2)$

$\therefore$ median lies in 1 & 2

$\therefore \frac{2}{3}\left(2m - \frac{m^2}{2} - 1\right) = \frac{1}{2}$

$\Rightarrow 2m^2 - 8m + 7 = 0.$

$\therefore m = 2 \pm \frac{1}{\sqrt{2}}.$

Permissable value of m is $2 - \frac{1}{\sqrt{2}} = 1.293$

To find Q_1

$F(Q_1) = 1/4$

$\therefore F(0) < \frac{1}{4} < F(1)$

Q_1 lies in 0 & 1

$\therefore \frac{Q_1^2}{3} = \frac{1}{4}$

$Q_1 = \pm 0.866.$

Permissable value is $Q_1 = 0.866.$

Q3

(a) Definition — (2)
Mean — (3)
Variance — (5)

(b) (i) 6 Properties — (6M.

(ii) $f(x) = \frac{1}{2a} \quad -a \le x \le a$
$= 0, \quad$ o.w. — (1)

$P(x \le 0.5) = 0.7$
$\Rightarrow \int_{-a}^{0.5} \frac{1}{2a} dx = 0.7$ — (2)

$\Rightarrow a = 1.25$

$P(0.3 < x < 0.7) = \int_{0.3}^{0.7} \frac{1}{2 \times 1.25} dx$

$= 0.16$ — (1)

(c)(i) Normal approx to Binomial —(c

(ii) $X \sim Bin(50, P = 0.4)$. $np = 20$, npq
$\because n > 30$ use Normal approx.
(i) $P(X > 25) = P(X > 25.5)$
$= P\left(Z > \frac{25.5 - 20}{\sqrt{12}}\right)$

$\therefore P(X > 25.5) = P(Z > 1.5877)$

$= 0.05618$

(ii) $P(X < 18) = P(X < 17.5)$

$= P(Z < -0.7217)$

$= 0.23524$

Q4(a) Each Definition — (1)

Each Example — (1)

(b) (i) Sampling distribution for Sample proportion —(05)

(ii) $H_0: \mu = 50 \quad H_1: \mu > 50.$

$\alpha = 0.02$

$n = 49 \quad \bar{x} = Rs\ 62$

$s.d = \sigma = 10$

$$Z_{cal} = \frac{62-50}{10/\sqrt{49}} = 8.4$$

$Z_{0.02} = 2.05$

$\therefore Z_{cal} > Z_{0.02}$ Hence

reject Ho and accept H_1.

— (05)

(c)

(i) Procedure for Testing giving all six steps — (08)

(ii) $H_0 : P = \frac{1}{2}$ $H_1 : P = \frac{1}{4}$

$X \sim Bin(10, P)$

under H_0

$P(x / H_0) = (10_{C_x}) (\frac{1}{2})^x (\frac{1}{2})^{10-x}$

under H_1

$P(x / H_1) = (10_{C_x}) (\frac{1}{4})^x (\frac{3}{4})^{10-x}$

$C.R = \{x / x \leq 3\}$

$\alpha = P(Rej\ H_0 / H_0 \text{ is true})$ $\because H_0$ is a simple hypo

$= P(x \leq 3 / P = 1/2)$

$= 0.171875$

Power $= 1 - \beta$

$= P(Rej\ H_0 / H_1 \text{ is true})$

$= P(x \leq 3 / P = 1/4)$

$= 0.77587501$

Q5

(a)

(i) c.d.f defn — (2)
properties — (3)

(ii) $\int_0^1 (a+bx^2)\,dx = 1$

$\Rightarrow \dfrac{a+b}{3} = 1$ — (1)

$E(x) = \frac{3}{5}$

$\Rightarrow \int_0^1 x(a+bx^2)\,dx = \frac{3}{5}$

$\Rightarrow \frac{a}{2} + \frac{b}{4} = \frac{3}{5}$

$\Rightarrow 10a + 5b = 12$ — (2)

Solving (1) & (2) we get
$a = 0.6$ $b = 1.2$

(b)

(i) Definition — (2)
Derivation of c.d.f — (2)

(ii) x = time to connect a call
$X \sim N(43, 8^2)$

(i) $P(X < 60) = P(Z < 2.125) = 0.983$

(ii) $P(X > 35) \Rightarrow P(Z > -1)$
$= 0.8413.$

(c)
(i) Derivation — (05).

(ii) $H_0 : \mu_1 = \mu_2$ $\quad H_1 : \mu_1 > \mu_2$
Define μ_1 & μ_2.
$\alpha = 0.05$.

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1 = 61.2$
$\bar{x}_2 = 59.4$
$\sigma_1 = 7.9$
$\sigma_2 = 7.9$
$n_1 = 84$
$n_2 = 34$

$Z_{cal} = 1.121$
$Z_{0.05} = 1.65$
$Z_{cal} < Z_{0.05}$ Hence do not reject H_0.
(05)