00067056 (1)

(1)

set 1

Q 1. (a) 2 marks each

(i) False, $P(a<X<b) = P(a \leq X \leq b)$

(ii) $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$

(iii) $X \sim U(-2,2)$, $E(X) = 0$

False

(iv) $X \sim Exp(0.5)$,

$Var(X) = \frac{1}{\lambda^2} = 4$

(v) False mean, median, mode coincide

(vi) estimator

(vii) False, assumption about population characteristic.

(b) — 2 marks each.

(i) Definition - continuous random variable

(ii) $E(X-a) = E(X) - a$

(iii) One application of Exponential distribution

(iv) $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

(v) 95% confidence interval for population mean

$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$

(vi) One sided and two sided test - one difference

(vii) Definition - Null hypothesis

Q-2. (a)

(i) Define p.d.f. — (2)

properties — (3)

(ii) X : life expectancy

1. $P(2 < X < 10) = \int_2^{10} f(x)\, dx$

$= 8 \int_2^{10} \frac{1}{x^3}\, dx$

$= 8 \left(-\frac{x^{-2}}{2}\right)_2^{10}$

$= 0.96$ — (3)

2. $P(0 < X < 5) = \int_2^5 f(x) \cdot dx$

$= 8\left(-\frac{x^{-2}}{2}\right)_2^5$

$= 0.84$ — (2)

Q. 2. (b)

(i) Def^n - central moments — ②

formula for measure of skewness — $1\frac{1}{2}$

measure of kurtosis — $1\frac{1}{2}$

(ii) $E(X) = \int x\, f(x) \cdot dx$

$= \int_0^1 12\, x \cdot x^2 (1-x)\, dx$

$= 0.6$ — ②

$E(X^2) = \int x^2 f(x) \cdot dx$

$= 0.4$ — ②

$V(X) = E(X^2) - [E(X)]^2$

$= 0.04$ — ①

(c) (1) $f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} 0 = 0$ $\quad x < 0$

$f(x) = \frac{d}{dx} \frac{3}{2}\left(x - \frac{x^3}{3}\right)$

$= \frac{3}{2}(1-x^2)$ $\quad 0 \le x \le 1$

$f(x) = \frac{d}{dx} 1 = 0$ $\quad x > 1$

$\therefore f(x) = \frac{3}{2}(1-x^2)$ $\quad 0 \le x \le 1$

$= 0$ $\quad$ otherwise — ④

(2) $P(X < 0.25) = \frac{3}{2} \int_0^{0.25} (1-x^2)\, dx$

$= \frac{3}{2} \left(x - \frac{x^3}{3} \right)_0^{0.25}$

$= 0.3672$ — ②

(3) $E(X) = \int x\, f(x)\, dx$

$= 0.375$ — ②

(4) mode -

$f'(x) = \frac{3}{2}(-2x)$

$= -3x$

$f'(x) = 0 \Rightarrow x = 0$

$f''(x) = -3 < 0$

$\therefore f(x)$ is maximum at $x = 0$

$\therefore$ mode $= 0$ — ②

Q. 3. (a)

Exponential distⁿ def — 3
mean — 3
variance — 4

(b)

(i) Uniform random variable over an interval (a,b) — 2
c.d.f — $1\frac{1}{2}$
median — $1\frac{1}{2}$

(ii) $X \sim U(0,4)$

$f(x) = \frac{1}{4}$ $\quad 0 \le x \le 4$

$= 0$ $\quad$ otherwise — 2

(1). $P(X \geq 3) = \int_3^4 f(x)\,dx = \frac{1}{4}$ — $1\frac{1}{2}$

(2) $P(2 \le X < 3.5) = \int_2^{3.5} f(x)\,dx$

$= 0.375$ — $1\frac{1}{2}$

(c)

(i) p.d.f. of normal variate with $\mu = 40$, $\sigma^2 = 16$ — 2
important properties — 3

(ii) X : Life of torch batteries
$X \sim N(\mu = 50, \sigma^2 = 3^2)$

(1) $P(X > 53) = P\left(Z > \frac{53-50}{3}\right)$

$= P(Z > 1)$

$= 1 - 0.8413$

$= 0.1587$ — 2

no. of batteries $\simeq 794$ — $\frac{1}{2}$

(2) $P(X < 45) = P\left(Z < \frac{45-50}{3}\right)$

$= P(Z < -1.67)$

$= 0.0475$ — 2

no. of batteries $\simeq 238$ — $\frac{1}{2}$

9

Q 4. (a)

(i) Point estimation — 2

internal estimation — 3

(ii) $\mu = 150$ cm

$\sigma = 20$ cm.

$n = 100$ $\qquad \bar{x} \sim N\left(\mu = 150, \frac{\sigma^2}{\sqrt{n}} = \frac{20^2}{100}\right)$ — 1

(1) $P(\bar{x} > 151) = P\left(z > \frac{151-150}{20/10}\right)$

$= P(z > 0.5)$

$= 0.3085$ — 2

(2) $P(148 < \bar{x} < 155) = P(-1 < z < 2.5)$

$= 0.8351$ — 2

(b)

(i)

(1) Simple and composite hypothesis — 2

(2) Type I and Type II error — 3

(ii) $H_0 : \theta = 2$

$H_1 : \theta = 3$

$CR = \{x \mid x > 0.6\}$ — 1

$P(\text{Type I error}) = P(\text{reject } H_0 / H_0 \text{ true})$

$= P(x > 0.6 / \theta = 2)$

$= \int_{0.6}^{1} \theta\, x^{\theta-1} \cdot dx$

$= 2 \int_{0.6}^{1} x \cdot dx$

$= 2 \cdot \left(\frac{x^2}{2}\right)_{0.6}^{1}$

$= 0.64$ — 2

$P(\text{Type II error}) = P(\text{accept } H_0 / H_1 \text{ true})$

$= P(x \le 0.6 / \theta = 3)$

$= 3\int_{0}^{0.6} x^2 \cdot dx$

$= 0.216$ — 2

Q.5. (a)

$$\int f(x)\,dx = 1$$

$$a\int_0^1 x\,dx + a\int_1^2 (2-x)\,dx + \frac{3}{7}a\int_2^3 x(3-x)\,dx = 1$$

$$\Rightarrow a\left(\frac{3}{2}\right) = 1$$

$$a = \frac{2}{3}$$ — 3

$$F(x) = 0 \qquad x < 0$$
$$= \frac{x^2}{3} \qquad 0 \le x < 1$$
$$= \frac{2}{3}\left(2x - \frac{x^2}{2} - 1\right) \qquad 1 \le x < 2$$
$$= \frac{1}{21}\left(9x^2 - 2x^3 - 6\right) \qquad 2 \le x < 3$$
$$= 1 \qquad x \geq 3$$ — 4

Median -

$$\int_0^M f(x)\,dx = \frac{1}{2}$$

x	0	1	2
$F(x)$	0	1/3	2/3

$$\Rightarrow F(1) < \frac{1}{2} < F(2) \qquad \therefore F(1) < F(M) < F(2)$$

$$\therefore \int_0^1 f(x)\,dx + \int_1^M f(x)\,dx = \frac{1}{2}$$

$$\int_0^1 \frac{2}{3}x\,dx + \int_1^M \frac{2}{3}(2-x)\,dx = \frac{1}{2}$$

$$\Rightarrow M = 2 \pm \frac{1}{\sqrt{2}}$$

permissible value of M is $2 - \frac{1}{\sqrt{2}} = 1.2929$ — 3

(b)

(i) $X \sim \exp(\lambda)$

mean $= \frac{1}{\lambda} = 5 \qquad \therefore \lambda = 0.2$

$V(x) = \frac{1}{\lambda^2} = 25$ — 2

$$P(X < 3) = \int_0^3 f(x)\,dx$$
$$= 0.2\int_0^3 e^{-0.2x}\,dx$$
$$= \left(-e^{-0.2x}\right)_0^3$$
$$= -e^{-0.6} + 1 = 0.4512$$ — 3

Q 5. b.

(ii)

1. normal app. to Binomial — 2½
2. normal app. to Poisson — 2½

(c)

(i)

1. Critical region — 2½
2. Level of significance — 2½

(ii) $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$n_1 = 100 \quad \bar{x}_1 = 1120 \quad s_1 = 75$

$n_2 = 100 \quad \bar{x}_2 = 1062 \quad s_2 = 82$ — 2

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ — 1

$= 5.2193$ — 1

$z_{table} = 1.96$

$\because z_{cal} > z_{table}$ reject H_0. — 1