Q1 (a) ② marks each

(i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

(ii) $P(A \cap B) = P(A) \cdot P(B)$ - A & B indep.

$P(A) = \frac{2}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{3} + \frac{2}{5} - \frac{4}{15}$

$= \frac{4}{5}$

(iii) Two mutually exclusive events never happen together

(iv) Defn - discrete r.v.

(v) mean $= \mu_1' = 2$

std. dev. $= 3$

$\mu_2' = ?$

$\mu_2 = \mu_2' - \mu_1'^2$

$9 = \mu_2' - 4$

$\mu_2' = 13$

(vi) False, mean > variance for Binomial distn.

(vii) Poisson distn mean = Variance.

(b) ② marks each

(i) Defn - random experiment

(ii) Prob. of an event lies between 0 and 1.

(iii) 3 coins are tossed.

$P(\text{exactly one head}) = \frac{3}{8}$

(iv) $V(X) = 4$, $V(Y) = 16$

$V(3X + 2Y) = 100$

(v) $\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$

(vi) Discrete uniform distribution

$P(x) = \frac{1}{10} \quad x = 0, 1, 2, \cdots 9$

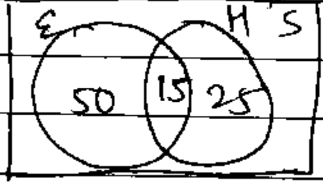
Q.2 (a)

1. certain event (2)
2. impossible event (2)
~~3. mutually exclusive events~~ ~~(2)~~
3. exhaustive events (2)
4. equally likely events (2)
5. complementary events (2)

(b)

(i) Conditional probability (4)

(ii)



1. P(student watches English movies only) $= \frac{50}{100}$ $= \frac{1}{2}$

2. P(student watches Hindi movies only) $= \frac{25}{100} = \frac{1}{4}$

3. P(student watches only one type) $= \frac{75}{100} = \frac{3}{4}$

4. P(atleast one type) $= \frac{90}{100} = \frac{9}{10}$

(1½) marks each.

(C)

(i) Probability of an event (2)
assumptions (3)

(ii) Bag 1 – 4 white 2 black
Bag 2 – 3 white 5 black

1. P(both are white) $= \frac{4}{6} \times \frac{3}{8} = \frac{12}{48} = \frac{1}{4}$ (1½)

2. P(both are black) $= \frac{2}{6} \times \frac{5}{8} = \frac{10}{48} = \frac{5}{24}$ (1½)

3. P(one white & one black) $= \left(\frac{4}{6} \times \frac{5}{8}\right) + \left(\frac{2}{6} \times \frac{3}{8}\right)$

$= \frac{26}{}$

Q.3. (a)

(I) p.m.f. of a discrete random variable (2)

properties (3)

(II)

1. $a + 4a + 3a + 7a + 8a + 10a + 6a + 9a = 1$

$48a = 1$

$a = 1/48$ (2)

2. $P(X < 3) = \frac{1}{48} + \frac{4}{48} + \frac{3}{48} = \frac{8}{48} = \frac{1}{6}$ (1)

3. $P(X \geq 4) = \frac{8}{48} + \frac{10}{48} + \frac{6}{48} + \frac{9}{48} = \frac{33}{48}$

$= \frac{11}{16}$ (1)

4. $P(0 < X < 5) = \frac{4}{48} + \frac{3}{48} + \frac{7}{48} + \frac{8}{48}$

$= \frac{22}{48} = \frac{11}{24}$ (1)

(b)

(i) Def^n raw moments and central moments (3)

relationship between first ~~four~~ three raw and central moments (3)

(ii) $E(X) = 6$

$E(Y) = 8$

$V(X) = 36$

$V(Y) = 64$

$E(XY) = 64$

1. $Cov(X, Y) = E(XY) - E(X).E(Y)$

$= 64 - (6)(8)$

$= 16$

2. $V(2X + 4Y) = (4)(36) + (16)(64) + 2 \cdot 2 \cdot 4 \cdot 16$

$= 1424$

3. $V(5X - 2Y) = (25)(36) + (4)(64) - 2 \cdot 5 \cdot 2 \cdot 16$

$= 836$

Q 4. (a) Discrete uniform distn — (10)
mean — (4m) var — (6m)

(b)
(i) Recurrence relation – Binomial (5)

(ii) $p = \frac{20}{30}$, $n = 6$

$X \sim \text{Binomial}(n=6, p=\frac{2}{3})$ (1)

1. $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$= {}^6C_4 (\frac{2}{3})^4 (\frac{1}{3})^2 + {}^6C_5 (\frac{2}{3})^5 (\frac{1}{3})^1 + {}^6C_6 (\frac{2}{3})^6 (\frac{1}{3})^6$

$= 0.6804$ (3)

2. $P(X=4) = 0.3292$ (1)

(c)
(i) Defn – ~~Hypergeometric~~ Binomial distn (3)
conditions (2)

(ii) X : no. of defective optical lenses
$n = 200$
$p = 2\% = 0.02$
$X \sim \text{Binomial}(n=200, p=0.02)$
using Poisson approximation to binomial,
$X \sim \text{Poisson}(m = np = 4)$ — (2)

1. P(3 or more defectives) $= P(X \geq 3)$

$= 1 - P(X < 3)$

$= 1 - \left(\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!}\right)$

$= 1 - 0.238103$

$= 0.761897$ (2)

2. P(at the most 2 lenses are defective)

$= P(X \leq 2)$

$= 0.238103$ (1)

Q.5 (a) (I) statement — (1 mark) Proof — ~~(1m)~~
~~Derivation~~ ~~$P(A \cap B) = P(A) \cdot P(B|A)$~~ 5

(II) Two cards are removed from a deck of 48 cards

1. $P(\text{Two diamonds}) = \frac{^{12}C_2}{^{48}C_2} = \frac{66}{1128}$

$= 0.0585$ 2

2. $P(\text{at least one diamond}) = \frac{^{12}C_1 \, ^{36}C_1}{^{48}C_2} + \frac{^{12}C_2}{^{48}C_2}$

$= 0.3829 + 0.0585$

$= 0.4414$ 3

(b)

(I)

1. $E(aX+b) = a\,E(X) + b$ 2

2. $V(aX+b) = a^2 V(X)$ 3

(II)

X	$P(X)$	$X \cdot P(X)$	$X^2 P(X)$
1	3/5	3/5	3/5
3	3/10	9/10	27/10
5	1/10	5/10	25/10
		20/10	58/10

$E(X) = 20/10 = 2$ 2

$V(X) = 1.8$ 3

(c)

(i) $P(X=2) = P(X=3)$

$\frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^3}{3!}$

$\frac{m^2}{2} = \frac{m^3}{6}$

$m = 3$ 3

$P(X=0) = \frac{e^{-3} 3^0}{0!}$

$= e^{-3}$ 2

6

Q5. (c)

(ii) X : no. of female voters in a sample of 10

1. Dist^n of X is hypergeometric with N=196 no. of females M=101 and n=10

$$p(X=x) = p(X=7) = \frac{\binom{101}{7}\binom{95}{3}}{\binom{196}{10}}$$

$$= 0.1304$$ (3)

2. Distribution of X is Binomial

$n=10$

$p = \frac{M}{N} = 0.5153$

$$p(X=7) = \binom{10}{7}(0.5153)^7(0.4847)^3$$

$$= 0.1318$$ (2)