1 67329

Ans Key SE I I

Time: 3Hrs Marks:-100

Q1. A) Select correct answer (12)

1 c) Total energy operator

2 a) Ae^{ax}

3 a)Time independent potential

4 a)(n+1) nodes

5 b) dual nature of matter

6 a)Tunneling effect

B) Answer in one sentence (3)

1 Mathematical representation of quantum mechanical observables

2 Different energy states with same energy value is called degenerate energy states.

3 **(any one of the following)**
A simple pendulum, An atom in a crystal lattice, and A diatomic molecule, etc

C) Fill in the Blanks (5)

1 independent

2 orthogonal

3 Unity or one

4 Zero

5 Reflection coefficient

Q2. A) Attempt any one (8)

1 Expectation value definition 2M
Expectation value of Position 2M
Use of operators in Expectation value 2M
Writing operator between $\psi *$ and ψ 2M

2 Equation of continuity $\frac{\partial \rho}{\partial t} + \frac{\partial S}{\partial x}$ =0, S= $-\frac{i\hbar}{2m}(\psi * \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi *}{\partial x})$

B) Attempt any one (8)

1 Postulates

1. Ψ defines state of system
2. Ψ related to probability of finding a particle
3. Ψ must be well behaved and normalisable
4. For every physical observable there iss quantum mechanical operator
5. If particle moving in conservative field then its wave functions are stationary state wave functions
6. Solutions of S.E obey principle off superposition

7.Different eigenvalues corresponding to different eigenfunctins yield nondegerate states

2 Thus $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$
Separation of variables
$\Phi(t) = e^{-iwt}$
$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\,\psi(x) = 0$

2

C) Attempt any one (4)

1 $E_n = \pi^2 \hbar^2 n^2 / 8mL^2$

2 Find the expectation value of x for a wave function

$$\Psi(x) = \sqrt{\frac{2}{l}}\ \sin\frac{\pi x}{l}\ ;\ 0 < x < l$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi * x\,\psi\, dx\ = l/2$$

Q3. A) Attempt any one (8)

1 Diagram - 1 mark
Wave functions in both region – 2 Marks
Obtaining amplitudes B and C in terms of A – 3 Marks
Obtaining Coefficients – 2 Marks

2 Labelled diagram – 1Mark
Wave equation – 1 Mark
Solution – 5 Marks
Energy level diagram – 1 Mark

B) Attempt any one (8)

1 Wave equation – 2 Marks
Solution-4 Marks
Wave function – 2 Marks

2 Set up – 2 Marks
Solution – 3 Marks
Proof of linear momentum – 3 Marks

C) Attempt any one (4)

1 Stating Wavefunction – 1 Marks
Integrating – 3 marks
Answer = 19.8%

2 113 eV and 226 eV

Q4. A) Attempt any one (8)

1 Time-independent Schrodinger wave equations for three regions and their general solutions...**(3 marks)**
Calculation of constants....**(3 marks)**
Calculation of reflection coefficient...**(1 mark)**
Calculation of transmission coefficient...**(1 mark)**

2 Tunneling effect:- The phenomenon of transmission of a particle through a potential barrier of finite width and height when its energy is less than the barrier height.**(2 marks)**
Write expression for transmission coefficient

$$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)}\sinh^2 k_2 a} \quad \text{where } k_2 = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)} \quad \textbf{(2 marks)}$$

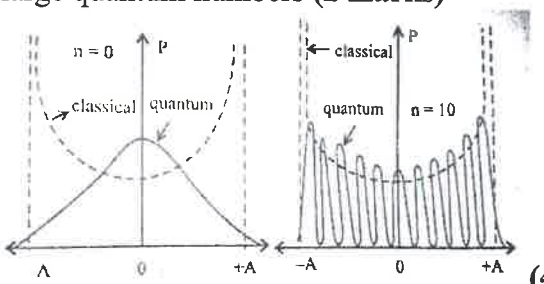
derivation of approximate transmission coefficient.**(4 marks)**

B) Attempt any one (8)

1 Correspondence principle:- The behavior of systems described by the

theory of quantum mechanics reproduces classical physics in the limit of large quantum numbers **(2 marks)**

2



(4marks)

C) Attempt any one (4)

1 E= 2 eV, V_0= 2 eV, a=1AU,

$T = 16\frac{E}{V_0}(1-\frac{E}{V_0})\exp[-2\sqrt{\frac{2m(V_0-E)}{\hbar^2}}]a$ **(2 marks)**

T= 0.109

T% = 10.9% **(2 marks)**

2 According to classical mechanics, $E = (1/2)m\omega^2A^2$ **(1 mark)**

According to quantum mechanics, $E = (n+\frac{1}{2})\hbar\omega$ (where n=.0,1,2,3,…)

(1 mark)

$n = \frac{m\omega^2A^2}{2\hbar} - \frac{1}{2} = 4.76 \times 10^{33}$. **(1 mark)**

Since this number is very large of order 10^{33}, so the system is indeed in the correspondence limit.**(1 mark)**

Q5. Attempt any Four (20)

1 Since $a_1^2 + a_2^2 \neq 1$, ψ is not normalized

Probability of finding the system in state 1 is $p_1 = a_1^2 / a_1^2 + a_2^2 = 0.64$

2 Superposition principle for wave function - 2 Marks

Probability density 1 Marks

Probability density not obeying Superposition principle

3 Writing p in terms of E – 2 Marks

Substitution – 3 Marks

4 Proof – 5 Marks

5 $E_n = [n+(1/2)]\hbar\omega$ and $E_{n+1} = [(n+1)+(1/2)]\hbar\omega$ **(1 mark)**

$\Delta E = E_{n+1} - E_n$

$\Delta E = \hbar\omega$ (1 mark)

$\Delta E/E_n = 2/(2n+1)$

If n is very large , then $2n+1 \cong 2n$ and $\Delta E/E_n = 1/n$ **(1 mark)**

If $n \to \infty$ then $\Delta E/E_n \to 0$.

6 The normalized eigenfunction for the first excited state of a simple harmonic oscillator is given by

$\Psi_1(x) = (\frac{\alpha^2}{\pi})^{(1/4)}\sqrt{2\alpha x \exp(-\alpha^2x^2/2})$ **(2 marks)**

$<x> = \int_{-\infty}^{+\infty}. \Psi_1^*(x) x \Psi_1(x) dx$

Solve to get the expectation value,

$<x> = 0$ **(2 marks)**

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