

Q1.	A)	Select correct answer	(12)
	1	b	2
	2	c	2
	3	c	2
	4	c	2
	5	a	2
	6	b	2
	B)	Answer in one sentence	(3)
	1	$\frac{\partial^2 \Psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} [E - V(x)] \Psi(x) = 0$ One dimensional Schrodinger's Time Independent Equation (STIE)	1
	2	If particle is restricted to a limited region by external forces so that it moves back and forth in that region only, then energy states of the particle are called bound states.	1
	3	The field emission of electrons from a cold metallic surface OR The electric breakdown of insulator OR The reverse breakdown of semiconductor diode OR The switching action of a tunnel diode OR The emission of α -particle from radioactive element	1
	C)	Fill in the Blanks	(5)
	1	Statistical	1
	2	Schrodinger's	1
	3	reflection coefficient	1
	4	decreases	1
	5	correspondence principle	1
Q2	A)	Attempt any one	(8)
	1	The function that represents the de Broglie waves is called the wave function , written as (Ψ) (psi). $\Psi = Ae^{-i(\omega t \pm kx)}$... (1) $\omega = 2\pi\nu = \frac{2\pi E}{h}$; since $E = h\nu$; $k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$ since $p = \frac{h}{\lambda}$ Substituting, $\omega = \frac{2\pi E}{h}$, $k = \frac{2\pi p}{h}$ we have $\Psi = Ae^{-\frac{i}{\hbar}(\omega t \pm kx)}$ ---- 2; where $\hbar = h/2\pi$ Differentiating Ψ [(in eq. (2)) partially with respect to x, we have $\frac{\partial \Psi}{\partial x} = \frac{ip}{\hbar} Ae^{-\frac{i}{\hbar}(Et - px)} = \frac{i}{\hbar} p \Psi$ ----- 3 Differentiating again w.r.t x, $\frac{\partial^2 \Psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)^2 Ae^{-\frac{i}{\hbar}(Et - px)} = -\frac{p^2}{\hbar^2} \Psi$ or $p^2 \Psi = -\hbar^2 \frac{\partial^2 \Psi}{\partial x^2}$ ---- 4 Differentiating eq. (2) partially w.r.t time,	

value $V = 0$ to a constant internal value $V = - 50 \text{ MeV}$. Estimate the probability that the neutron will be reflected at the nuclear surface.

- 2 An α particle having energy 10 MeV approaches a potential step of height 50 MeV and width 10^{-15} m . Determine the transition coefficient if mass of α particle is $6.68 \times 10^{-27} \text{ kg}$.

Q4. A) Attempt any one (8)

- 1 State correspondence principle. Show how quantum and classical probabilities of a one-dimensional oscillator leads to correspondence principle.
- 2 Discuss in detail the penetration of particle having energy E_0 across potential barrier of finite height V_0 and width (a) for the case $E_0 > V_0$.

B) Attempt any one (8)

- 1 Show that the STIE for a one-dimensional harmonic oscillator can be written in the form $(\frac{\partial^2}{\partial y^2} - y^2) \Psi = -2\epsilon\Psi$
- 2 Establish the Schrodinger's equation for linear harmonic oscillator and solve it to obtain its eigen value and eigen function.

C) Attempt any one (4)

- 1 An α -particle having energy 10 MeV approaches a potential barrier of height 30 MeV . Find the width of potential barrier if the transmission coefficient is 2×10^{-3} .
(Given: mass of α -particle = $6.68 \times 10^{-27} \text{ Kg}$).
- 2 A beam of electrons is incident on a potential barrier 5eV high and 5\AA wide. What should be their energy so that half of them tunnel through the barrier?

Q5. Attempt any Four (20)

- 1 Write a short on 'Operators'
- 2 'Wave functions add, not the probabilities', explain '.
- 3 Show that the eigen functions of a quantum mechanical operator with different eigen values are orthogonal.
- 4 A particle arrives at a step potential having height V_0 . Discuss the problem classically when energy of the particle is
(i) more than the step height
(ii) less than the step height
- 5 The wave function for the ground state of a harmonic oscillator of mass m and force constant k is proportional to $e^{-\frac{\alpha^2 x^2}{2}}$ where $\alpha^2 = \frac{m\omega}{\hbar}$ and $\omega^2 = \frac{k}{m}$. Show that this is a solution and find the corresponding eigen value
- 6 Find the expectation value $\langle x \rangle$ for the first excited state of a simple harmonic oscillator.

	$\frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} E A e^{-\frac{i}{\hbar}(Et - px)} = -\frac{iE}{\hbar} \Psi$ $E\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad \text{----- 5}$ <p>E is the total energy of particle</p> $E = \frac{p^2}{2m} + V(x, t) \quad \text{----- 6}$ <p>V is the potential energy of the particle and is a function of space and time coordinates in general</p> <p>Multiplying eqn (6) from the right by Ψ and, using eqn. (4) and eq. (5), we obtain</p> $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x, t) \Psi \quad \text{----- 7}$ <p>In general, in case of three dimensional motion, eq. (7) becomes.</p> $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) + V(x, y, z, t) \Psi \quad \text{----- 8}$ <p>Ψ and V are functions of x,y,z, t in general.</p> <p>Eq. (8) can be written as</p> $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(x, y, z, t) \Psi \quad \text{----- 9}$ <p>∇^2 is called Laplacian operator. In Cartesian coordinates, it is given by</p> $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{----- 10}$ <p>Eq. (9) is called three dimensional Schrodinger's time dependent equation. Schrodinger equation is valid only for non-relativistic problems because $E = \frac{p^2}{2m} + V$ is the classical energy equation. <i>(Statement 1 Derivation 7 marks)</i></p>	
2	<p>When we solve Schrodinger's equation, a number of solutions may be possible mathematically. But all these solutions are not acceptable solutions. In order that a wave function is acceptable, i.e., has any physical significance, it must be a well behaved function</p> <p>'Well' behaved" wave function must satisfy the following conditions :</p> <ol style="list-style-type: none"> I. Ψ must be normalizable, i.e., the integral of Ψ^2 over all space must be finite (since the particle is somewhere after all). II. Ψ must be finite at all points where the particle can be present. III. Since the probability of locating of the particle at a point at a given instant can have only one value, Ψ must be single valued everywhere in its permissible range. IV. Ψ must be continuous in all regions of its existence. V. Partial derivatives of Ψ i.e. $\frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial z}$ must be continuous and single valued everywhere VI. Ψ and its derivatives must go to zero at infinite distances. VII. In addition to these conditions, an acceptable wave function has to satisfy certain boundary conditions depending upon the problem. 	<p>2M</p> <p>1M Each cond</p>
B)	Attempt any one	(8)
1	<p>A one dimensional simple harmonic undamped wave propagating in x-direction is given by the equation.</p> $y = A \sin(\omega t + kx) \quad \text{---- 1}$ <p>where A- amplitude of the wave and is constant, ω- angular frequency and</p>	

- Q2. A) Attempt any one (8)
- 1 How does de Broglie postulate enter into Schrodinger's theory?
 - 2 Derive equation of continuity in quantum mechanics and discuss its significance.
- B) Attempt any one (8)
- 1 Discuss Max Born interpretation of wave mechanics. Hence explain 'Normalization of wave function'.
 - 2 Derive Schrodinger's Time Independent Equation (STIE).
- C) Attempt any one (4)
- 1 Find the expectation value of particle position if the eigen function describing the particle is given by

$$\Psi = ax \quad ; \quad 0 < x < 1$$

$$= 0 \quad \text{elsewhere.}$$
 - 2 Show that for stationary states, expectation of momentum is independent of time (consider 1-D motion).
- Q3. A) Attempt any one (8)
- 1 A particle is subjected to a three dimensional box and is subjected to a potential given by

$$V(x) = 0 \quad \text{inside the box}$$

$$V(x) = V_0 \quad \text{outside the box}$$
 Write down Schrodinger's time independent wave equation and obtain normalised solution.
 - 2 Consider an electron of energy E incident on the potential step defined by

$$V(x) = 0 \quad \text{for } x \leq 0$$

$$V(x) = V_0 \quad \text{for } x \geq 0$$
 Show that the particle can penetrate into the second region even if its energy is less than V_0 .
- B) Attempt any one (8)
- 1 Set up Schrodinger's equation for a free particle. Solve the equation to obtain the eigenfunction. Show that the expectation value of momentum of the particle is same as the momentum that a classical particle will have.
 - 2 Consider a particle confined to move in an infinite rectangular potential well. Show that expectation value of the position co-ordinate x of a particle in the well depends upon the length of the well.
- C) Attempt any one (4)
- 1 A neutron of kinetic energy 5 MeV tries to enter a nucleus and its potential energy drops at the nuclear surface very rapidly from a constant external

	<p>k - propagation constant.</p> $k = \frac{2\pi}{\lambda}, \omega = 2\pi\nu \quad \text{---- 2 (} \nu \text{ and } \lambda, \text{ both are constant)}$ <p>We differentiate eqn (1) twice, partially with respect to x</p> $\frac{\partial^2 y}{\partial x^2} = -(k)^2 y \quad \text{---- 4}$ <p>Similarly, we partially differentiate y twice with respect to t,</p> $\frac{\partial^2 y}{\partial t^2} = -(\omega)^2 y \quad \text{---- 5}$ <p>From eq. (4) and eq. (5)</p> $\frac{\partial^2 y}{\partial x^2} = \frac{k^2 \partial^2 y}{\omega^2 \partial t^2} = \frac{1}{(v\lambda)^2} \frac{\partial^2 y}{\partial t^2}$ <p>But $v\lambda = v_p$, the phase velocity of the wave</p> $\frac{\partial^2 y}{\partial x^2} = \frac{1}{(v_p)^2} \frac{\partial^2 y}{\partial t^2} \quad \text{---- 6}$ <p>Eq. (6) is a second order partial differential equation in space and time propagates in x-direction with speed v_p.</p> <p>Although we have derived eq. 6 for a simple harmonic progressive wave, it is perfectly general and holds for all waves in any medium, in which the wave speed up is independent of the precise character of the waves, i.e., when v_p is same regardless of frequency and wavelength of the waves. It applies equally well to waves which are progressive or stationary, 2 longitudinal or transverse, waves in a stretched string, sound waves in air, electromagnetic waves in vacuum. In all these cases, up depends only on</p> <p>the properties of the medium. Eq. (6) is called classical wave equation.</p>	<p>2M</p> <p>4M</p> <p>2M</p>
2	<p>Eigen Values and Eigen Functions For a particle of mass m and potential energy $V(r)$, the STIE can be written as : $\hat{H}\Psi = E\Psi$ where the Hamiltonian operator or energy operator is given by</p> $\hat{H} = \frac{p^2}{2m} + V(r) \quad \text{---- (1)}$ <p>Eq' (1) can have acceptable solutions (Ψ) for a number of values these of E. Let us call values E_k and the corresponding Ψ as Ψ_k. It means Ψ_k is a solution of eq. (1) when E is E_k. We can write eq. (1) as:</p> $\hat{H}\Psi_k = E_k\Psi_k \quad \text{----- 2}$ <p>k in eq' (2) can be a discrete or continuous variable</p> <p>Energies E_k are called the energy eigen values of Hamiltonian (energy) operator H and the corresponding Ψ_k are called energy eigen functions of operator H .</p> <p>Very often we come across an equation of quantum mechanics where an operator, L operating on a function Φ, produces same function Φ, i.e.,</p> $L \Phi = l\Phi, \quad \text{-----3}$ <p>Such an equation is called eigen value equation. Φ is called the eigen function l ; eigen value of operator L. Then eq. (3) is written as :</p> $L \Phi_n = l_n \Phi_n \quad \dots (4)$ <p>Let its eigen values l_n be discrete. It is fundamental postulate of quantum mechanics that any precise measurement of that observable can only yield one of eigen value l_1, l_2, l_3, \dots, etc.. If measurements are made on a number of identical systems all in the same state described by a particular eigen function Φ_k (say), then each measurement will yield a unique single value l_k.</p> <p>Coming back to eq. (4), if the eigen values l_1 and l_2 corresponding to eigen functions Φ_1 and Φ_2 are different, then the states represented by Φ_1 and Φ_2 are said to be non</p>	<p>1M</p> <p>2M,</p> <p>2M</p> <p>2M</p>

- N.B : (1) All questions are compulsory.
 (2) Figures to the right indicate maximum marks.
 (3) Use of non-programmable calculators is permitted.
 (4) Symbols used have their usual meaning

- Q1. A) Select correct answer (12)
- 1 The momentum operator in one dimension is
 a) $-i\hbar \frac{d}{dx}$ b) $i\hbar \frac{d}{dx}$ c) $i\hbar \frac{d}{dt}$ d) $-i\hbar \frac{d}{dt}$
 - 2 Which of the following is not a physical requirement for a wave function to be valid
 a) Single valued b) continuous in given region c) time independent
 d) None of these.
 - 3 A particle is confined in a cubical box. The degeneracy of the energy state E, if $E = 14 \frac{h^2}{8mL^2}$ is
 a) 6 b) 3 c) 9 d) 14
 - 4 A particle of energy E approaches a potential step of height V, greater than E. According to quantum mechanics the particle is
 a) always reflected b) always transmitted
 c) may be reflected or transmitted
 - 5 α -particles are emitted from the nucleus by _____.
 a) tunneling b) bombardment c) emission d) fission
 - 6 Diatomic molecule is an example of _____.
 a) harmonic oscillator b) simple oscillator
 c) damped oscillator d) multiple oscillator
- B) Answer in one sentence (3)
- 1 Give the statement of equation of continuity in classical mechanics with its usual meaning
 - 2 What is tunnel effect?
 - 3 What is energy of a simple harmonic oscillator in the lowest state known as?
- C) Fill in the Blanks (5)
- 1 $|\Psi|^2 = \dots\dots\dots$
 - 2 ----- is the normalized condition for three dimensional wave function Ψ
 - 3 The probability of finding the particle in classically forbidden region is called
 - 4 If particle is restricted to a limited region by external forces so that it moves back and forth in that region only, then energy states of the particle are called states.
 - 5 Scanning tunneling microscope (STM) type of microscope is based on the quantum mechanical phenomenon known as _____.

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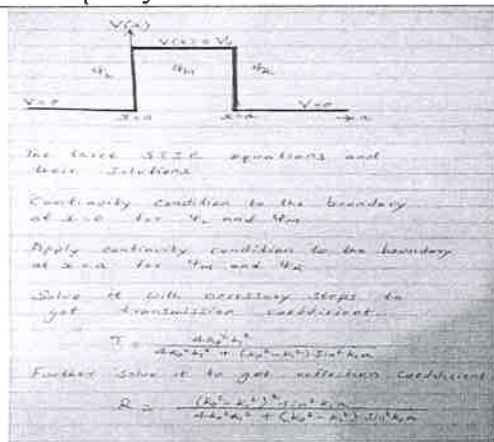
SE11

		degenerate and if they are same, the states are said to be degenerate states. (There can be more than two degenerate states for an eigen value)	1M
	C)	Attempt any one	(4)
	1	<p>∴ the normalised wave function of an oscillation is given by</p> $\psi = Ay e^{-y^2/2} \quad -\infty < y < \infty \text{ find } A$ <p>solⁿ: since ψ is normalised</p> $\int_{-\infty}^{\infty} \psi^* \psi dx = 1$ $\int_{-\infty}^{\infty} Ay e^{-y^2/2} \cdot Ay e^{-y^2/2} dy$ $A^2 \int_{-\infty}^{\infty} y^2 e^{-y^2} dy = 1$ $A^2 \sqrt{\frac{\pi}{2}} = 1 \quad \left[\int_{-\infty}^{\infty} y^2 e^{-y^2} dy = \sqrt{\frac{\pi}{2}} \right] \text{ Standard integral}$ $A^2 = \frac{2}{(\sqrt{\pi})^2}$ $A = \frac{\sqrt{2}}{\pi^{1/4}} \text{ or } \sqrt{\frac{2}{\sqrt{\pi}}}$	
	2	$\langle x \rangle = \int_0^1 \psi^*(x) x \psi(x) dx$ $= \frac{2}{1} \int_0^1 x \sin^2 \frac{\pi x}{1} dx$ $= \frac{1}{1} \int_0^1 x (1 - \cos \frac{2\pi x}{1}) dx$ $= \frac{1}{1} \int_0^1 x dx - \frac{1}{1} \int_0^1 x \cos \frac{2\pi x}{1} dx$ <p>Second integral is integrated by parts it is bound to be zero</p> $\therefore \langle x \rangle = \frac{1}{1} \frac{1^2}{2} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">$\langle x \rangle = 1/2$</div>	
Q3	A)	Attempt any one	(8)
	1	Diagram and description Schrodinger's time independent equation and Solutions coefficient of Reflection Coefficient of Transmission	2 4 1 1
	2	Schrodinger's time independent equation Energy eigen values $A = \sqrt{\frac{2}{L}}$ Normalised solution	1 3 3 1
	B)	Attempt any one	(8)
	1	Diagram Description Schrodinger's time independent equation	1 1 1

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		Separation of variable $D = \sqrt{\frac{8}{V}}$ Normalised solution	2 2 1
	2	Free particle Schrodinger's equation General solution Expectation value of momentum Comment	1 1 3 2 1
	C)	Attempt any one	(4)
	1	Formulae of k and T $k = 1.54 \times 10^{10} \text{ m}^{-1}$ $T = 2.05 \times 10^{-7}$	1 2 1
	2	Formula of R $R = 0.29$	1 3
Q4.	A)	Attempt any one	(8)
	1	 <p>The three薛定谔 equations and their solutions</p> <p>Continuity condition to the boundary at $x=a$ for ψ_1 and ψ_2</p> <p>Apply continuity condition to the boundary at $x=b$ for ψ_2 and ψ_3</p> <p>Solve it with necessary steps to get transmission coefficient</p> $T = \frac{4k_1 k_3}{(k_1 + k_3)^2 + (k_2^2 - k_1^2) \sin^2(k_2 b)}$ <p>Further solve it to get reflection coefficient</p> $R = \frac{(k_1^2 - k_3^2) \sin^2(k_2 b)}{(k_1 + k_3)^2 + (k_2^2 - k_1^2) \sin^2(k_2 b)}$	

10

2

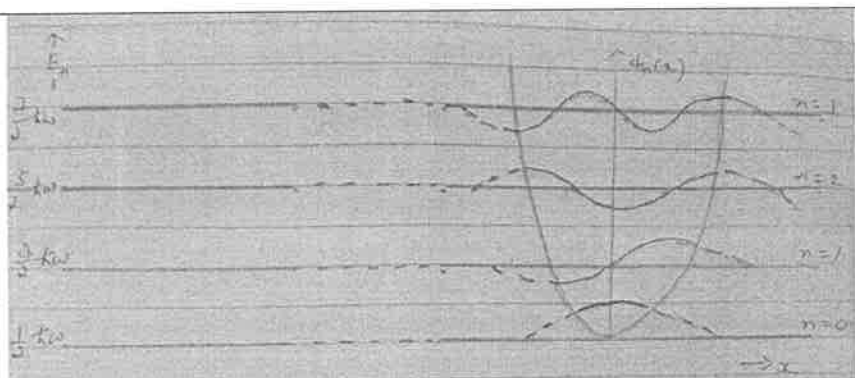


Fig. Energy level diagram of a quantum simple harmonic oscillator and first four eigenfunctions $\psi_0(x)$, $\psi_1(x)$, $\psi_2(x)$ and $\psi_3(x)$, plotted as a function of x within potential energy parabola. (2marks)

Explanation (2marks)

Definition of zero-point energy (2marks)

Zero-point vibrational energy for a diatomic

$$\frac{h\nu}{2} = \frac{h}{2\pi} \cdot \frac{\omega}{2} = \frac{h\nu}{2} \quad (2 \text{ marks})$$

$$= 2.2 \times 10^{-21} \text{ J}$$

$$= 2.4 \times 10^{-2} \text{ eV}$$

B) Attempt any one

(8)

1

Consider a simple harmonic oscillator, consisting of a mass m attached to a centre of force at the origin $(x=0)$ with a force $F = -kx$, where x is the displacement from the origin, k is the force constant.

P.E. of the particle at a point x is

$$V(x) = \frac{1}{2} kx^2 \quad \text{--- (1)}$$

Newton's equation of motion is

$$m \frac{d^2x}{dt^2} = -kx$$

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- (2) where } \omega = \sqrt{\frac{k}{m}}$$

Eq (2) is a one dimensional equation of harmonic oscillator having solution

$$x = A \cos \omega t \quad \text{--- (3)}$$

This is an oscillatory motion of amplitude A .

STZE for $S.H.O$ is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad \text{--- (4)}$$

Substituting (1) & (2) and multiply throughout by $\frac{2}{\hbar^2}$

$$\left(-\frac{d^2}{dx^2} + \frac{k}{m} x^2 \right) \psi(x) = \frac{2E}{\hbar^2} \psi(x)$$

We change over to a more convenient variable defined as

$$y = \left(\frac{m\omega}{\hbar} \right)^{1/2} x \quad \text{--- (5)}$$

$$\text{Let } E = \frac{\hbar\omega}{2} \epsilon \quad \text{--- (6)}$$

The STZE for the oscillator now takes the simple form

$$\left(\frac{d^2}{dy^2} - y^2 \right) \psi = -\epsilon \psi$$

2

Establishment of the 1-D Schrodinger equation of a linear harmonic oscillator.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} \left(E - \frac{1}{2} kx^2 \right) \psi = 0 \quad \text{(2 marks)}$$

Consider two dimensionless variables,

$$y = \left(\frac{\sqrt{mk}}{\hbar} \right)^{1/2} x$$

and

$$E = \frac{E}{\hbar\omega} = \frac{E}{\hbar\omega_0} \quad \text{(1 mark)}$$

Get the eigenvalue equation

$$\left(\frac{d^2}{dy^2} - y^2 \right) \psi = -2\epsilon \psi \quad \text{(3 marks)}$$

The wave equation for the oscillator is satisfied only for discrete values of total energy given by

$$\frac{2E_n}{\hbar\omega} = 2n+1$$

$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega \quad \text{(2 marks)}$$

5

	C)	Attempt any one	(4)
	1	$m = 9.1 \times 10^{-31} \text{ kg}$ $\hbar = 1.054 \times 10^{-34} \text{ Js}$ $V_0 = 10 \times 1.6 \times 10^{-19} \text{ joule}$ $E_0 = 1 \times 1.6 \times 10^{-19} \text{ joule}$ $a = 0.5 \times 10^{-7} \text{ m}$ $k = \sqrt{\frac{2m(V_0 - E_0)}{\hbar^2}} = 1.54 \times 10^{10} \text{ m}^{-1}$ $2ka = 15.4$ $T = e^{-2ka} = 2.05 \times 10^{-7} \quad (2 \text{ marks})$ If $E_0 = 2 \text{ eV}$, then by similar calculation $T = 5.05 \times 10^{-7} \quad (2 \text{ marks})$	
	2	$E_0 = 3 \text{ eV} = 4.8 \times 10^{-19} \text{ J}$ $V_0 = 4 \text{ eV} = 6.4 \times 10^{-19} \text{ J}$ $a = 2 \times 10^{-10} \text{ m}$ $2ka = 2 \times \sqrt{\frac{2m(V_0 - E_0)}{\hbar^2}} = 2.048$ $T = \frac{16E_0}{V_0} \left[\frac{1 - E_0}{V_0} \right] e^{-2ka} = 0.382 \quad (2 \text{ marks})$ Percentage transmission = 38.2% (2 mark)	
	Q5	Attempt any Four	(20)
	1	<p>Consider a particle moving in x direction. The probability density of finding the particle very close to position x and at time t is $\Psi ^2$. Therefore $\Psi(x, t) ^2$ is the probability of locating the particle between x and x + dx at time t</p> $\langle x \rangle = \frac{\int_{-\infty}^{\infty} x \Psi ^2 dx}{\int_{-\infty}^{\infty} \Psi ^2 dx} = \frac{\int_{-\infty}^{\infty} \Psi^* \Psi x dx}{\int_{-\infty}^{\infty} \Psi^* \Psi dx} \quad \text{---1}$ $\langle x \rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi dx \quad \text{-----2}$ <p>If the wave function Ψ is normalized, then the denominator is unity and we can use the same procedure to find the expectation value of any quantity e.g.</p> $\langle p \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{p} \Psi dx = -i\hbar \int_{-\infty}^{\infty} \Psi^* \frac{\partial \Psi}{\partial x} dx \quad \text{-----3}$	

	$\langle E \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{E} \Psi dx = i\hbar \int_{-\infty}^{\infty} \Psi^* \frac{\partial \Psi}{\partial t} dx \quad \text{--- 4}$ <p>In general, expectation value of any quantity Q is given by</p> $\langle Q \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{Q} \Psi dx \quad \text{---- 5}$ <p>Assuming that Ψ is normalized. If Ψ is not normalized all these equations will have denominator $\int_{-\infty}^{\infty} \Psi^* \Psi dx$ in equation 1.</p> <p>(Def 1m examples any two with individual 2marks)</p>	
2	<p>Operator :Every mathematical operation like adding, subtracting, multiplying, dividing, square root, differentiating etc can be represented by a characteristic symbol called an operator Hence Angular momentum is given by</p> $\vec{L} = \vec{r} \times \vec{p} = (ix + jy + kz)(ip_x + jp_y + kp_z)$ $= i(y p_z - z p_y) + j(z p_x - x p_z) + k(x p_y - y p_x)$ $= iL_x + jL_y + kL_z$ $L_x = (y p_z - z p_y) = -i\hbar \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$ $L_x = -i\hbar \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$ $L_y = -i\hbar \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) \quad \text{----- 1}$ $L_z = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$ <p>(Defn -1m, 1D expression - 3 m, 3 D representation - 1 m)</p>	
3	Condition of orthogonality Proof	1 4
4	$\Psi(x) = Ae^{ik_0x}$ $p = \hbar k_0$ $\lambda = \frac{h}{p}$	1 2 2
5	<p>Coefficient of transmission, T</p> $T = \frac{4k_1 k_2}{4k_1 k_2 + (k_1^2 - k_2^2) \sin^2 k_1 a} \quad \text{--- ①}$ <p>Coefficient of reflection, R</p> $R = \frac{(k_1^2 - k_2^2) \sin^2 k_1 a}{4k_1 k_2 + (k_1^2 - k_2^2) \sin^2 k_1 a} \quad \text{--- ②}$ <p>From ① and ②, $R + T = 1$</p> <p>Since, $k_1 \neq k_2$, $R \neq 0$.</p> <p>\therefore though classically total transmission is expected, quantum mechanically there is a finite probability of the particle being reflected. (3marks)</p> <p>But there is one exception, when $R=0$.</p> <p>i.e. $k_1 a = n\pi$, where a is barrier width</p> $\text{or } \frac{2\pi a}{\lambda} = n\pi$ $\text{or } a = \frac{n\lambda}{2} \quad \text{(2marks)}$	
6	Statement of correspondence principle Explanation	2 3

