(1) QP Code: 65053

S.Y. Sem IV Paper-I **Solution**

Q.1 (A) (i) False.
If $Y \sim N(2,9)$ then $\left(\frac{Y-2}{3}\right)^2$ has χ^2-distrn with $\underline{1}$ degree of freedom

(ii) False
Area under the Normal curve betn first and third quartile is $\underline{50\%}$

(iii) False
The median of exponential distrn is $\frac{\log_e 2}{\theta}$

(iv) False
p.d.f. of std. Cauchy distrn is $\frac{1}{\pi} \frac{1}{1+x^2}$.

(v) True.

Q.1 (B) (i) $X \sim U(a, b)$ where $a = 3$, $b = 7$

(ii) Mean of triangular distribution on (a, b) with peak at c is $\frac{a+b+\text{mode}}{3}$

(iii) $\beta_1(4, 3)$. Mean $= \frac{4}{7} = \frac{m}{m+n}$
$= \beta_1(m, n)$

(iv) $H_0 : \mu_d = 0$. $t = \frac{\bar{d}}{\sqrt{s^2/n}} \sim t_{n-1}$ under H_0.
t is the state statistic.

(v) $\frac{s_1^2/s_2^2}{F_{1-(\alpha/2)}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2/s_2^2}{F_{\alpha/2}}$ is $(1-\alpha)\%$ C.I. f

Q.2. (A) (i) Statement : (2 M) (Memoryless property)
Proof : (4 M).

(ii) p.d.f. $= f(x) = \frac{1}{\theta} e^{-x/\theta}$, $x > 0$, $\theta > 0$ —(1 M.)
$= 0$, elsewhere.

Distrn fn $= F_X(x) = \int_0^x \frac{1}{\theta} e^{-u/\theta}\, du = 1 - e^{-x/\theta}$, $x > 0$, $\theta > 0$
$= 0$, for $x \le 0$.

Q.2 (B) $\mu_r' = \int_0^1 x^r \frac{\Gamma(m+n)}{\Gamma m \, \Gamma n} x^{m-1} (1-x)^{n-1}\, dx$

$= \frac{\Gamma(m+n)\, \Gamma(r+m)}{\Gamma m \, \Gamma(r+m+n)}$

For $r = 1$, $\mu_1' = E(X) = \frac{\Gamma(m+n)\, \Gamma(m+1)}{\Gamma m \, \Gamma(m+n+1)} = \frac{m}{m+n} = \text{mean}.$

(2)

Q. 2 (C) $X \sim G(\alpha, \beta)$ — for p.d.f $\rightarrow$ 2 M

$E(X) = \int_0^\infty x \cdot \frac{1}{\Gamma\alpha\, \beta^\alpha} e^{-x/\beta} x^{\alpha-1}\, dx$ $\longrightarrow$ (3 M for mean)

$= \frac{1}{\Gamma\alpha\, \beta^\alpha} * \int_0^\infty e^{-x/\beta} x^{(\alpha+1)-1} dx = \frac{\Gamma\alpha+1\ \beta^{\alpha+1}}{\Gamma\alpha\ \beta^\alpha} = \alpha\beta$

$V(X) = \alpha\beta^2$ $\longrightarrow$ (3 M for $E(X^2)$ and 2 M for $V(X)$)

3 (A) (i) 1 M for each property

(ii) Mode = μ. — 5 M for Proof.

(B) MGF $= E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} dx$

$= e^{\mu t + \frac{\sigma^2 t^2}{2}}$ — (7 M)

Mean $= M'(t)\big|_{t=0}$ or By Direct method, Mean $= \mu$ — (3 M)

(c) (i) Proof:

$\mu'_{2r+1} = 0$ = odd ordered moment — (5 M)

(ii) Mean of log normal distribution =

$E(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\ln x - \mu}{\sqrt{2}\sigma}\right)^2} \frac{1}{x} dx = e^{\frac{\sigma^2}{2}+\mu}$ — (5 M)

4 (A) If $X \sim \chi^2_{(m)}$, $Y \sim \chi^2_{(n)}$ then $\frac{X/m}{Y/n} \sim F_{m,n}$.

joint p.d.f of X & Y $\rightarrow$ 2 M, $|J| \rightarrow$ 2 M, joint p.d.f of new r.v.s and marginal pdf of $\frac{X}{Y}$ — 6 M.

(B) Derivation of formula giving 2×2 contingency table $\rightarrow$ 6 M.

Appln of Yates' corrn (formula) $\longrightarrow$ 4 M.

Reason for applying Yates' Corrn $\longrightarrow$ 2 M.

(Cell freq. < 5). $\Rightarrow$ Test statistic does not have χ^2 distrn

(C) Derivation of Mean of t distrn: —

p.d.f $\longrightarrow$ 2 M.

$E(X) = 0$ for $\nu > 1$.

= undefined otherwise.

5 (A) (i) MGF of Gamma with single para. $\alpha = (1-t)^{-\alpha}$ — 5 M

: Find MGF of X+Y and identify it as Gamma M.G.F. with parameter $(\alpha_1+\alpha_2)$. — 5 M.

(B) $\Sigma a_i X_i$ also has Normal distrn (Proof). — 10 M

(C) (i) $H_0: \mu_x = \mu_y$ against $H_1: \mu_x < \mu_y$ (or $\mu_x > \mu_y$ or $\mu_x \neq \mu_y$)

Under H_0 $t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_X^2}{n_1} + \frac{S_Y^2}{n_2}}} \sim t_{n_1+n_2-2}$, Conclusion etc — (2+3 M)

(ii) Reciprocal of F_{n_1,n_2} has F_{n_2,n_1} distrn. (5 M)