Solution

QP Code: 65052

Solution
S.Y.B.Sc.(SEM-IV) Paper I –USST401

1

Q.1 **Attempt all sub-questions:** **(20)**

a. State TRUE or FALSE and correct if necessary.(2M each) (10)

i. False.
Harmonic mean of β_2(m = 5 ,n= 2)= $\frac{m-1}{n}$ = 2

ii. True

iii. True

iv. False.
Student's t variable is a ratio of Standard Normal Variable and square root of Chi square variable divided by corresponding d.f.

$$t = \frac{S.N.V.}{\sqrt{\frac{\chi_n^2}{n}}}$$

v. False.
μ_{11}=0.

b. **Answer the following : (2M each)** (10)

i. M.G.F. of Gamma ($\alpha = \frac{1}{5}$, λ = 6)
$M_X(t) = (1-5t)^{-6}$

ii. X ~ Standard Cauchy

$$f(x) = \frac{1}{\pi(1+x^2)} \qquad -\infty < x < \infty$$
$$= 0 \qquad \text{o.w.}$$

iii. $\frac{1}{X}$ ~ F(2,6)

iv. X ~ Normal (μ = 4 , σ^2 = 2)

v. Forgetfulness property of Exponential distribution
If X ~ Exponential ,then $P(X \ge s+t \mid X \ge s) = P(X \ge t)$

Q.2 **Attempt any TWO sub-questions:** **(20)**

a. i. $M_X(t) = (1-\frac{t}{\theta})^{-1}$ 3M (06)
X_1 and X_2 ~ Exp(θ) and independent
$M_{X_1+X_2}(t) = (1-\frac{t}{\theta})^{-2}$ 2M
$\therefore X_1 + X_2$ ~ Gamma(θ, λ = 2) 1M

ii. X ~ β_2 (m,n) (04)
Derivation of mode = $\frac{m-1}{n+1}$ m>1 ....4M

b. i. $M_X(t) = \frac{(e^t-1)^2}{t^2}$ 4M (06)
$= \frac{1}{t^2}[t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...]$ 1M
E(X) = coefficient of t = 1 ....1M

ii. Any four properties of Cauchy distribution. 4M (04)

c. (10)

$M_X(t) = (1-\frac{t}{\alpha})^{-\lambda}$ 3M

C.G.F. $= -\lambda \ln (1-\frac{t}{\alpha})$ 1M

$= \lambda\left[\frac{t}{\alpha}+\frac{t^2/\alpha^2}{2}+\frac{t^3/\alpha^3}{3}+\frac{t^4/\alpha^4}{4}+\ldots+\frac{t^r/\alpha^r}{r}+\ldots\right]$ 1M

$\therefore \quad K_1 = \mu_1' = \frac{\lambda}{\alpha}, \quad K_2 = \mu_2 = \frac{\lambda}{\alpha^2}, \quad K_3 = \mu_3 = \frac{2\lambda}{\alpha^3}$ 2M

Measures of Skewness $\beta_1 = \frac{4}{\lambda}$ $\quad \gamma_1 = \sqrt{\beta_1} = +\frac{2}{\sqrt{\lambda}} > 0$ 2M

Since $\gamma_1 > 0$, Gamma distribution is positively skewed. 1M

Q.3 **Attempt any TWO sub-questions:** **(20)**

a. i. $X \sim$ Normal (μ, σ^2) (05)

Derivation: Median of Normal distribution = μ 05M

ii. Odd order central moments of Normal distribution are zero (05)

Proof 5M

b. i. $X \sim$ Normal (μ, σ^2) (05)

M.G.F.= $M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2 t^2}$ **..........1M**

Distribution of $Z = \frac{X-\mu}{\sigma} = \frac{X}{\sigma} - \frac{\mu}{\sigma}$

M.G.F. of $Z = M_Z(t) = \exp(-\frac{\mu t}{\sigma}) M_X(\frac{t}{\sigma})$

$= e^{-\frac{\mu t}{\sigma}} e^{\frac{\mu t}{\sigma}+\frac{t^2}{2}}$

$= e^{\frac{1}{2}t^2}$ **............3M**

$\therefore Z \sim$ Normal $(\mu = 0, \sigma^2 = 1)$ **............1M**

ii. p.d.f. of Log Normal distribution with parameters (μ, σ^2) (05)

..................2M

$E(X) = e^{\mu+\frac{1}{2}\sigma^2}$ **..................3M**

c. $X \sim$ Normal (μ, σ^2) (10)

M.G.F.= $M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2 t^2}$ **..................4M**

C.G.F. $= \log_e M_X(t) = \mu t + \frac{1}{2}\sigma^2 t^2$ **..................1M**

$\mu_1 = \mu, \ \mu_2 = \sigma^2, \ \mu_3 = 0 \quad \mu_4 = 3\sigma^4$ **..................2M**

Measures of skewness

$\beta_1 = 0, \quad \gamma_1 = 0$ symmetric

Measures of kurtosis

$\beta_2 = 3 \quad \gamma_2 = 0$ mesokurtic **..................3M**

3

Q.4 **Attempt any TWO sub-questions:** **(20)**

a. i. Definition.......2M (05)

$(1-\alpha)$% confidence interval for population variance σ^2

When μ is known $\left(\frac{\sum_{1}^{n}(x_i-\mu)^2}{\chi^2(n,\alpha/2)}, \frac{\sum_{1}^{n}(x_i-\mu)^2}{\chi^2(n,1-\alpha/2)}\right)$ 3M

ii. X ~ F(m,n) (05)

p.d.f. 2M

Mean = n/n-2 .... 3M

b. i. **test statistic** = $\frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$ (07)

derivation **5M**

Test statistic after applying Yate's correction

test statistic = $\frac{N\left[|ad-bc|-\frac{N}{2}\right]^2}{(a+b)(c+d)(a+c)(b+d)}$ **........2M**

ii. Student's t with n degrees of freedom (03)

Mean = 0 **............3M**

c. M.G.F. = $(1-2t)^{-n/2}$ 3M (10)

C.G.F. = $-\frac{n}{2}\ln(1-2t)$

$= \frac{n}{2}\left[2t + \frac{(2t)^2}{2} + \frac{(2t)^3}{3} + \ldots\ldots\right]$ 2M

$K_1 = \mu_1' = n$, $K_2 = \mu_2 = 2n$, $K_3 = \mu_3 = 8n$ 2M

Measures of skewness

$\beta_1 = 8/n$ $\gamma_1 = \sqrt{\frac{8}{n}} > 0$ 2M

since $\gamma_1 > 0$, distribution is positively skewed................1M

Q.5 **Attempt any TWO sub-questions:** **(20)**

a. i. M.G.F. of Uniform distribution with parameters $(0,\theta)$ (05)

$M_X(t) = (e^{\theta t}-1)/\theta t$ 3M

Mean = $\theta/2$ 2M

ii. X ~ β_1 (m,n) (05)

$\mu_r' = \Gamma(m+n)\,\Gamma(m+r) / \Gamma(m)\,\Gamma(m+n+r)$ 3M

Mean m /m+n 2M

b. i. X ~ Normal (μ, σ^2) (07)

M.G.F.= $M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2 t^2}$

X ~ Normal (μ_1, σ_1^2)

Y ~ Normal(μ_2, σ_2^2)

Distribution of X+Y

$M_{X+Y}(t) = M_X(t) \times M_Y(t) = e^{\mu_1 t+\frac{1}{2}\sigma_1^2 t^2} \times e^{\mu_2 t+\frac{1}{2}\sigma_2^2 t^2}$ ....

$= e^{(\mu_1+\mu_2)t+\frac{1}{2}(\sigma_1^2+\sigma_2^2)t^2}$ 3M

X+Y ~ Normal ($\mu_1+\mu_2, \sigma_1^2+\sigma_2^2$) 1M

ii. **X ~ Normal (μ_1=5, σ_1^2=4)** (03)

Y ~ Normal (μ_2=10, σ_2^2= 9)

X and Y are independent

X+Y ~ Normal ($\mu_1+\mu_2$ = 15 , $\sigma_1^2+\sigma_2^2$ =13) 1M

P(X+Y<15) = 0.5 ....2M

Statement Central Limit Theorem. **3M**

c. i. Test procedure for testing $H_0 : \mu=\mu_o$ against all possible alternatives (06)

Test statistic $t_{cal} = \sqrt{n-1}\,(\bar{X} - \mu_0)\,/S$ OR $\sqrt{n}\,(\bar{X} - \mu_0)/s$ 2M

S: sample s.d. s = sample mean square

LOS = α

H_1: $\mu>\mu_o$

Decision Criterion :

Reject Ho if $t_{cal} > t_{n-1,\alpha}$

H_1: $\mu<\mu_o$

Decision Criterion :

Reject Ho if $t_{cal} < -t_{n-1,\alpha}$

H_1: $\mu\neq\mu_o$

Decision Criterion :

Reject Ho if $|t_{cal}| > t_{n-1,\alpha/2}$

........4M

ii. Mode of F(m,n) = $\frac{m(n-2)}{n(m+2)}$ n>2, derivation................4M (04)

****