Code No 54608

1

(3 Hours) [Total Marks: 100]

**Note:** (*i*) All questions are compulsory.
(*ii*)Figures to the right indicate marks for respective parts.

| | | | | |
|---|---|---|---|---|
| Q.1 | Choose correct alternative in each of the following (20) | | | |
| *i.* | Which of the following has precise and definite syntax. | | | |
| | (a) | pseudocode | (b) | algorithm description |
| | (c) | program in language like $C^{++}$ | (d) | None of these |
| | Ans | (c) program in language like $C^{++}$ | | |
| *ii.* | If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = l \neq 0$ then | | | |
| | (a) | f(x) is of lower order than g(x) | (b) | g(x) is of lower order than f(x) |
| | (b) | f(x), g(x) are of same order | (d) | None of the above |
| | Ans | ( c)f(x), g(x) are of same order | | |
| *iii.* | Given a list of seat numbers of students who qualified in certain competitive examination, to check whether a student with seat number x has qualified ,we can use | | | |
| | (a) | Searching algorithm | (b) | Counting algorithm |
| | (c) | Sorting algorithm | (d) | None of these |
| | Ans | (a)Searching algorithm | | |
| *iv.* | The number of moves required to solve the Tower of Hanoi with three disks is | | | |
| | (a) | 6 | (b) | 7 |

| | | | | |
|---|---|---|---|---|
| | (c) | 5 | (d) | None of these |
| | Ans | (b) 7 | | |
| *v.* | The number of edges in a complete graph $K_5$ is | | | |
| | (a) | 9 | (b) | 10 |
| | (c) | 11 | (d) | 12 |
| | Ans | **(b) 10** | | |
| *vi.* | The degree of an isolated vertex in any graph is | | | |
| | (a) | 0 | (b) | 1 |
| | (c) | 2 | (d) | 3 |
| | Ans | **(a) 0** | | |
| *vii.* | The sum of degrees of all the vertices in a graph $G$ is 20. Therefore, the number of edges of $G$ is | | | |
| | (a) | 20 | (b) | 10 |
| | (c) | 40 | (d) | None of these |
| | Ans | **(b) 10** | | |
| *viii.* | If a tree T has 25 vertices then the number of edges are …. | | | |
| | (a) | 24 | (b) | 25 |
| | (c) | 26 | (d) | 12 |
| | Ans | (a)24 | | |
| *ix.* | If a full binary tree has 50 internal vertices then the total number of vertices is | | | |
| | (a) | 100 | (b) | 101 |
| | (c) | 99 | (d) | 51 |
| | Ans | 101 | | |

3

x. Degree of the root vertex in a full binary tree is

| | | | |
|---|---|---|---|
| (a) | 1 | (b) | 3 |
| (c) | 2 | (d) | Atmost2 |

Ans 2

**Q2.** Attempt any **ONE** question from the following: (08)

*a)* i. Design an algorithm to find the maximum element of a finite set of n integers.

Trace your algorithm for n=5 and the set {7,2,10,8,12}.

Also state the number of comparisons needed for finding maximum element in the above set of 5 integers.

Ans Procedure (n, $a_1$ ,$a_2$ ……. $a_n$ )

1.{ max := $a_1$

2. for i =2 to n

3. {if ($a_i$> max) then {max := $a_i$}}

4.print max

5} (4M)

TRACE:

| Step no | i= 2 to 5 | ($a_i$> max)? | $a_i$ | max |
|---|---|---|---|---|
| 1 | | | | 7 |
| 2,3 | 2 | false | 2 | |
| | 3 | true | 10 | 10 |
| | 4 | false | 8 | |

(4)

| | | | | |
|---|---|---|---|---|
| | 5 | true | 12 | 12 |
| | 6 | | | |

(2M)

Max =12

Number of comparisons =5 (2M)

ii. Given an integer x and a list of n distinct integers, write Binary Search algorithm for searching x in the list. Also take the trace of the algorithm for the following data:

x=9, n=6, list:2 ,6 , 8 ,9 , 12,14

Ans Procedure Binary Search( x:integer,n: positive integer, $a_1, a_2 \ldots a_n$: integers

```
{i:=1
 j:=n
 while (i < j)
{ m:= int((i+j)/2)
  If ( x > a_m) then { i = m + 1}
           else  {j = m}
}
If ( x= a_i) then{ location := i}else {location := 0}
}
```

(2M) (3M)

TRACE: x=9 n=6

| | i | j | (i<j)? | m | $a_m$ | ( $x > a_m$)? |
|---|---|---|---|---|---|---|
| initial | 1 | 6 | true | | | |
| | 4 | | | 3 | 8 | true |
| | | 5 | true | 5 | 12 | false |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 4 | true | 4 | 9 | false |
| | | | false | | | |

( x= $a_i$) is true (3M)

Location=4

**Q.2** Attempt any **TWO** questions from the following: (12)

*b)* i. Define recursive algorithm . Write a recursive algorithm for finding the first n terms of Fibonacci sequence 0,1,1,2,3,…..

Ans Definition: An algorithm is called recursive if it solves a problem by reducing it to an instance of the same problem with smaller input. (1M)

```
Procedure(n: nonnegative integer)
{if(n=0) then {fibonacci(0) := 0}                                  (1M)
elseif (n=1) then {Fibonacci(1) := 1}                              (1M)
else {Fibonacci(n) := Fibonacci(n-1) + Fibonacci(n-2)}             (3M)
}
```

ii. Describe the Euclid's algorithm to find the GCD of two nonzero integers a, b. Also trace your algorithm for a=15, b=51.

Ans

```
Procedure gcd (a, b : nonnegative integers)
{x := a
 y := b
 If (x=0) then print "gcd :=" y                                    (1M)
 elseif (y=0) then print "gcd :=" x
 else
      { while y ≠ 0
         { r := x mod y
           x := y
           y := r
         }
```

6

(2M)

}
(1M)

Print " gcd =" x

}
(2M)

TRACE:

| | $(y \neq 0)$? | X | Y | R |
|---|---|---|---|---|
| INITIAL | | 15 | 51 | |
| 1st pass | True | | | 15 |
| | | 51 | | |
| | | | 15 | |
| 2nd pass | True | | | 6 |
| | | 15 | | |
| | | | 6 | |
| 3rd pass | True | | | 3 |
| | | 6 | | |
| | | | 3 | |
| 4th pass | True | | | 0 |
| | | 3 | | |
| | | | 0 | |
| 5th pass | False | | | |

Gcd= 3

iii. Give Selection Sort algorithm to sort the given list of n integers in an ascending order.

Ans Procedure selection sort (n: positive integer, $a_1, a_2 \ldots a_n$: integers)

```
{for i = 1 to n
 x := a_i
 k := I                                   (2M)
 for j = i+1 to n
 {If ( a_j < x ) then { k := j
                 x := a_j }
                                          (2M)
 }
 a_k := a_i
 a_i := x
                                          (2M)
}
```

**iv.** Design an algorithm to find sum of first n terms of sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots\ldots$ .Trace your algorithm for n=3

**Ans**

```
Procedure  summation (n: positive integer)      (1M)
{sum = 0                                         (1M)
 For i = 1 to n                                  (2M)
 {sum := sum + 1/2^i  }
}
 print "sum =" sum
}                                                (2M)
```

TRACE:

| I= | i | Sum |
|---|---|---|
| initial | | 0 |
| $1^{st}$ pass | 1 | 0+1/2=0.5 |
| $2^{nd}$ pass | 2 | 0.5+1/4=0.75 |
| 3rd pass | 3 | 0.75+1/8=0.875 |

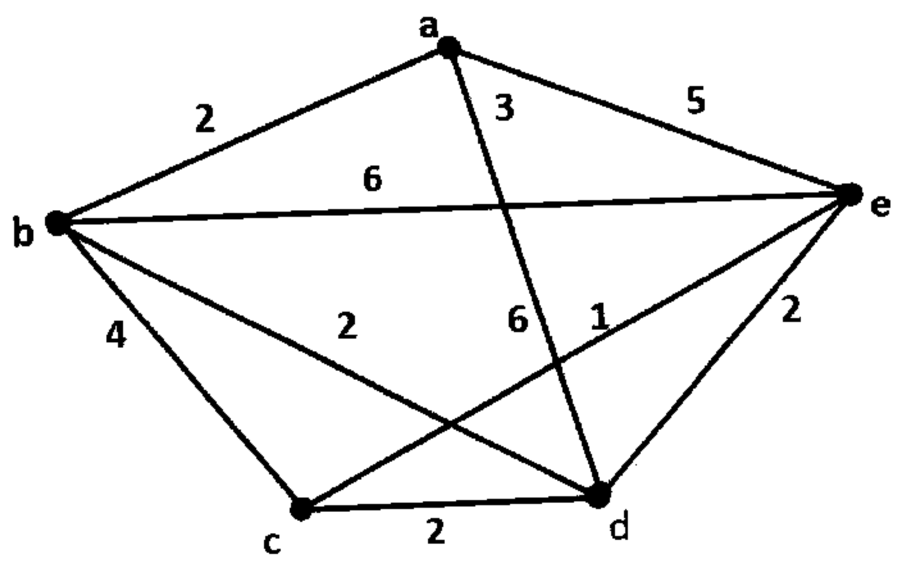
(8)

| | 4 | |
|---|---|---|

Sum=0.875

Q3. Attempt any **ONE** question from the following: (08)

*a)* i. Find shortest path from $b$ to $e$, for the following graph, using Dijkstra's algorithm.




Ans Initially all vertices are labeled ∞. As we have to find shortest path from vertex b to vertex e, so vertex b is permanently labeled 0. 2

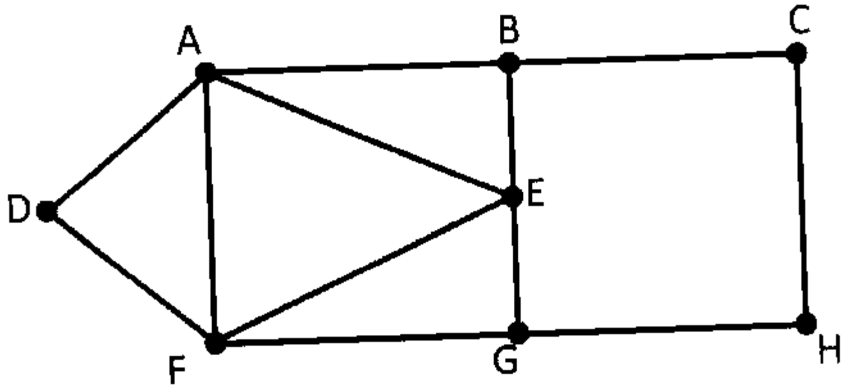
Applying Dijkstra's algorithm, we have:

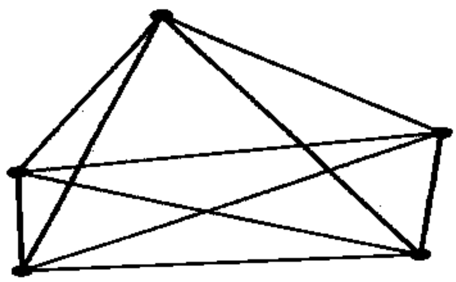
| **Vertex** | **a** | **b** | **c** | **d** | **e** |
|---|---|---|---|---|---|
| **b** | [$2_b$] | 0 | $4_b$ | $2_b$ | $6_b$ |
| **a** | - | - | $4_b$ | [$2_b$] | $6_b$ |
| **d** | - | - | [$4_b$] | - | $4_d$ |
| **c** | - | - | - | - | [$4_d$] |

2

2

Shortest path from vertex $b$ to vertex $e$ is: $\boldsymbol{b - d - e}$.

| | | | |
|---|---|---|---|
| | | Weight of the shortest path is $2 + 2 = 4$. | 2 |
| | ii. | Use Fleury's algorithm to construct Euler path for the following graph.<br><br> | |
| | Ans | If in a connected graph, exactly two vertices are of odd degree, Euler path exists. Fleury's algorithm for Euler path, we first choose an arbitrary vertex of odd degree of connected multigraph, and then forming Euler path by choosing edges successively. Once an edge is chosen, it is removed. Edges are chosen successively so that each edge begins where the last edge ends, and so that this edge is not a cut edge unless there is no alternative.<br>In above graph, Euler path exists as exactly two vertices are of odd degree. Let we start from the vertex B (odd degree), then applying conditions of Fleury's algorithm we get the path as:<br>$\boldsymbol{B - C - H - G - F - D - A - F - E - B - A - E}$. | 2<br>2<br>2<br>2 |
| | | | |
| Q3. | Attempt any **TWO** questions from the following: | | (12) |
| *b*) | i. | Explain with example:<br>Dirac's theorem and Ore's theorem. | |

| | | | |
|---|---|---|---|
| | Ans | **Dirac's Theorem:** If G is a simple graph with $n$ vertices $n \geq 3$ such that the degree of every vertex in $G$ is atleast $\frac{n}{2}$, then G has a Hamilton circuit.<br><br>**Ore's Theorem:** If G is a simple graph with n vertices with $n \geq 3$ such that $\deg(u) + \deg(v) \geq n$, for every pair of non-adjacent vertices $u$ and $v$ in $G$, then $G$ has a Hamilton circuit.<br><br>Example: In following graph, number of vertices $n = 4 \therefore \frac{n}{2} = 2$<br><br>Degree of each vertex = 3<br><br>Clearly, Dirac's condition for Hamilton circuit are satisfied.<br><br>Therefore, Hamilton circuit exists.<br><br>Also sum of degrees of any two non-adjacent vertices is greater than 4. Hence Ore's condition is satisfied, therefore, above graph has Hamilton circuit. | 2<br><br>2<br><br>2 |
| | ii. | Explain adjacency matrix of a simple graph and incidence matrix of an undirected graph with an example in each case. | |
| | Ans | Adjacency matrix: Suppose $G = (V, E)$ is a simple graph where $\|V\| = n$. Suppose that the vertex of $G$ are listed arbitrarily as $v_1, v_2, \ldots. v_n$. The adjacency matrix $A$ of $G$ with respect to this listing of the vertices, is $n \times n$ zero one matrix with 1 as its $(i, j)$ $th$ entry when $v_i$ and $v_j$ are adjacent, and 0 as its $(i, j)$ th entry when they are not adjacent. In other words, if its adjacency matrix is $A = [a_{ij}]$ then,<br><br>$a_{ij} = 1$, if $[v_i, v_j]$ is an edge | 1 |

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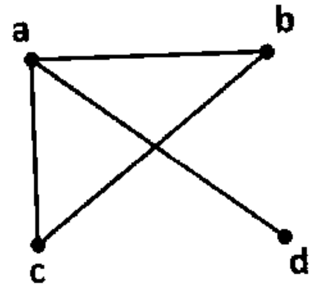
= 0, otherwise.

Example:




*G*

Adjacency matrix of graph G is

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

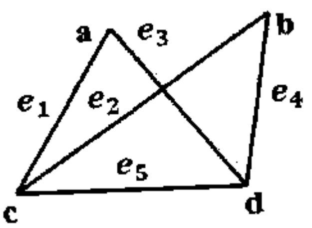
2

Incidence matrix: Let $G = (V, E)$ be an undirected graph. Suppose that $v_1, v_2, \dots v_n$ are the vertices and $e_1, e_2, \dots, e_n$ are the edges of $G$. Then the incidence matrix with respect to this ordering of $V$ and $E$ is the $n \times m$ matrix $M = [m_{ij}]$ where

$m_{ij} = 1$ when edge $e_j$ is incident with $v_j$

= 0 otherwise.

**Example:**



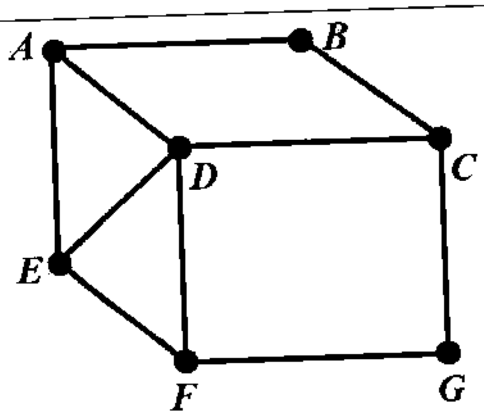

*H*

Incidence matrix for graph H is 1

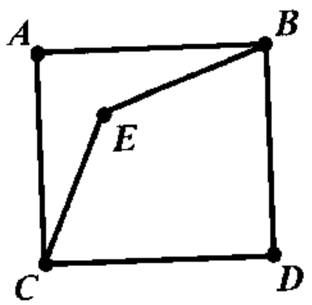
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

2

iii. What do we mean by Hamilton circuit and Hamilton Path? Check whether the following graph has a Hamilton Circuit. State any sufficient condition for any graph to have a Hamilton circuit.




| | | | |
|---|---|---|---|
| Ans | Solution:<br><br>A simple path in a graph $G$ that passes through every vertex exactly once is called a Hamilton path.<br><br>A simple circuit in a graph $G$ that passes through every vertex exactly once is called Hamilton circuit.<br><br>The given graph has a Hamilton circuit<br><br>One such Hamilton circuit is **D-A-B-C-G-F-E-D.**<br><br>For the sufficient condition, the student may write either Dirac's theorem (If $G$ is a simple graph with $n$ vertices with $n \geq 3$ such that the degree of each vertex of $G$ is at least $n/2$, then $G$ has a Hamilton circuit.) or Ore's theorem (If $G$ is a simple graph with $n$ vertices with $n \geq 3$ such that $\deg(u) + \deg(v) \geq n$ for every pair of nonadjacent vertices $u$ and $v$ in $G$, then $G$ has Hamilton circuit.) | 2<br><br>2<br><br>2 |
| iv. | In the following graph, how many paths of length 2 exist from vertex $B$ to $C$? Obtain Adjacency matrix of the graph and using the same find the number of paths of length 2 from vertex $B$ to $C$. | |



**Ans** In the given graph, there are three graphs of length 2 from B to C. (2)

These are as follows: B-E-C, B-A-C, B-D-C

Assuming A, B, C, D, E as 1, 2, 3, 4, 5, we get the adjacency matrix $A$ as below: (1)

$$A=\begin{pmatrix}0&1&1&0&0\\1&0&0&1&1\\1&0&0&1&1\\0&1&1&0&0\\0&1&1&0&0\end{pmatrix}$$

To obtain the number of paths of length 2, we will obtain $A^2$.

$$A^2=\begin{pmatrix}0&1&1&0&0\\1&0&0&1&1\\1&0&0&1&1\\0&1&1&0&0\\0&1&1&0&0\end{pmatrix}\begin{pmatrix}0&1&1&0&0\\1&0&0&1&1\\1&0&0&1&1\\0&1&1&0&0\\0&1&1&0&0\end{pmatrix}=\begin{pmatrix}2&0&0&2&2\\0&3&3&0&0\\0&3&3&0&0\\2&0&0&2&2\\2&0&0&2&2\end{pmatrix}$$ (3)

From the matrix $A^2$, we see that the entry $A^2{}_{2,3} = A^2{}_{3,2} = 2$.

**Q4.** Attempt any **ONE** question from the following: (08)

**a) i.** Prove that a graph is a tree if and only if there is unique path between every pair of vertices in the graph.

**Ans** First assume that $T$ is a tree. Then $T$ is a connected graph with no cycle. Let $x$ and $y$ be two vertices of $T$. Because $T$ is connected, there is a simple path between $x$ and $y$. Moreover, this path must be unique, for if there were a second such path, the path formed by combining the first path from $x$ to $y$ followed by the path from $y$ to $x$ obtained by reversing the order of the second path would form a cycle. This implies that there is a cycle in $T$ which is a contradiction. Hence, there is a unique simple path between any two vertices of a tree.

Now assume that there is a unique simple path between any two vertices of a graph $T$. Then $T$ is connected, because there is a path

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| | | |
|---|---|---|
| | | between any two of its vertices. Further, $T$ cannot contain cycle. Suppose, $T$ had a cycle that contained the vertices $x$ and $y$. Then there would be two paths between $x$ and $y$, because the cycle is made up of a path from $x$ to $y$ and a second path from $y$ to $x$. Hence a graph with a unique path between any two vertices is a tree. |
| | ii. | Show that for a full m-ary tree with '$n$' vertices, $l$ leaves and $i$ internal vertices<br>(I) $n = mi + 1$ (II) $l = (m-1)i + 1$ (III) $i = \frac{l-1}{m-1}$. |
| | Ans | Consider a full $m$-ary tree with '$n$' vertices, $l$ leaves and $i$ internal vertices.<br>(I) Every vertex, except the root, is the child of an internal vertex. Because each of the $i$ internal vertices has $m$ children, there are $mi$ vertices in the tree other than the root. Therefore, the tree contains $n = mi + 1$ vertices.<br>(II) Since every vertex is either a leaf or internal vertex, we have $n = i + l$.<br>$\therefore i + l = mi + 1$ (from (I), $n = mi + 1$)<br>$\therefore l = mi + 1 - i$<br>$\therefore l = (m-1)i + 1$, leaves.<br>(III) From (II), $l = (m-1)i + 1$<br>$\therefore (m-1)i = l - 1$<br>$\therefore i = \frac{l-1}{m-1}$, internal vertices. |
| | | |
| Q4. | Attempt any **TWO** questions from the following: (12) | |
| *b)* | i. | If a tree $T$ has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4, find the number of pendant vertices in $T$. |
| | Ans | Let $T(V, E)$ be a tree with two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4.<br>Let $p$ be the number of pendant vertices.<br>Total number of vertices in $T$ is $\|V\| = 2 + 4 + 3 + p = 9 + p$<br>Therefore, total number of edges in $T$ is |

$|E| = |V| - 1 = (9 + p) - 1 = 8 + p$

By Hand-shaking theorem,
$2(2) + 4(3) + 3(4) + p = 2(8 + p)$

$4 + 12 + 12 + p = 16 + 2p$

$p = 12.$

ii. Define rooted tree, m-ary tree and fully m-ary tree. Draw full 3-ary tree on 13 vertices.

Ans **Rooted tree:** A rooted tree is a tree $T$ in which one vertex is distinguished vertex and the distinguished vertex is called the root of the tree.
***m*-ary tree:** A rooted tree is called m-ary tree if every internal vertex has atmost $m$ children.
**Full *m*-ary tree:** A rooted tree is called full $m$-ary tree if every internal vertex has exactly $m$ children.

iii. Prove that the number of vertices in a full Binary tree is always odd.

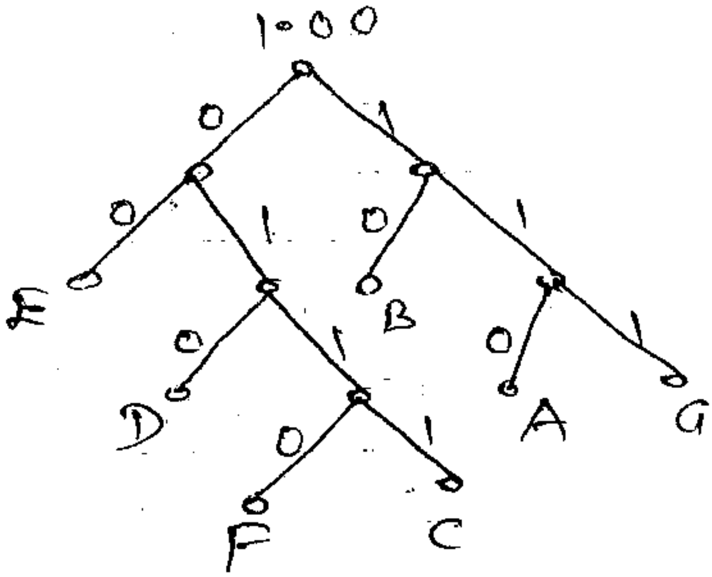
Ans Consider a full binary tree on $n$ vertices. Since, it contains exactly one vertex of degree 2 and other vertices are of degree 1 or 3, it follows that there are $n - 1$ odd degree vertices in the graph.

But, the number of odd vertices in a graph is always even. It follows that $n - 1$ is even. Hence, n is odd.

Thus, the number of vertices in a full Binary tree is always odd.

iv. Use Huffman coding to encode the following symbols with the frequencies listed. A: 0.10, B: 0.25, C: 0.05, D: 0.15, E: 0.30, F: 0.07, G: 0.08. What is the average number of bits used to encode a character?

Ans




The encoding produces encodes

A by 110

B by 10

C by 0111

D by 010

E by 00

F by 0110

G by 111

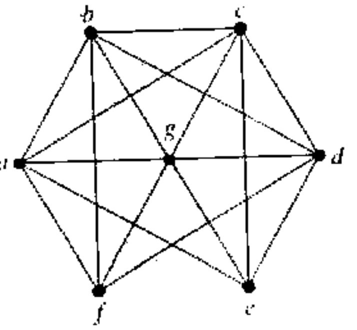
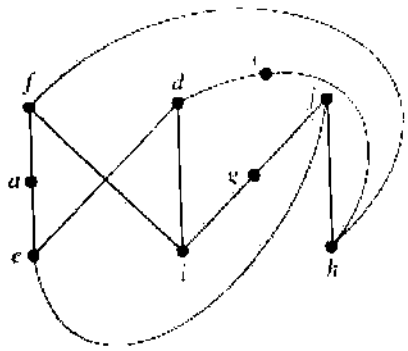
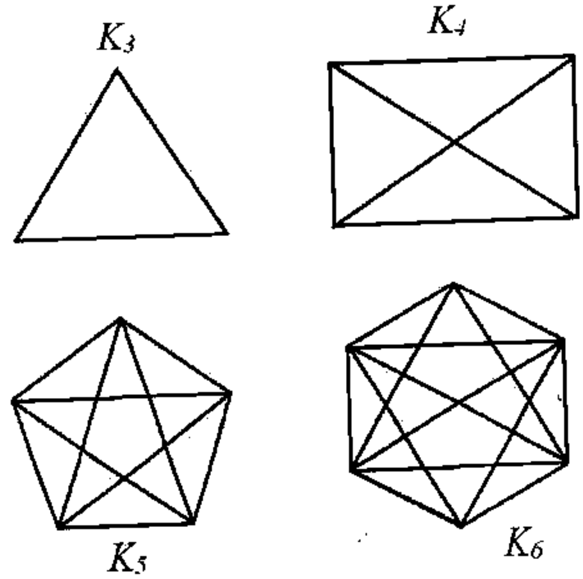
The average number of bits used to encode a symbol using this encoding is

$3(0.10) + 2(0.25) + 4(0.05) + 3(0.15) + 2(0.30) + 4(0.07) + 3(0.08)$

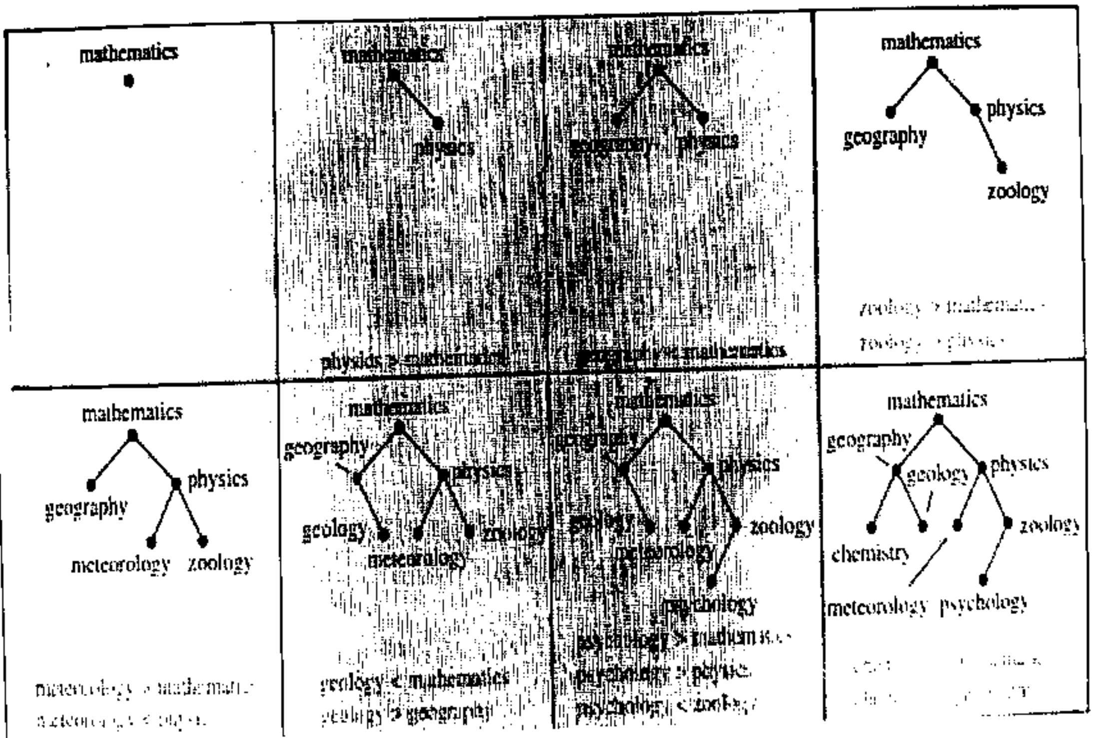
$= 0.30 + 0.50 + 0.20 + 0.45 + 0.60 + 0.28 + 0.24$

$= 2.57$

| | | |
|---|---|---|
| Q5. | Attempt any **FOUR** questions from the following: | (20) |
| *a)* | Design an algorithm to find $a^n$ for a number a and for $n \in N$ | |
| Ans | Procedure product(a, n:natural number)<br>{ prod =1 (1M)<br>For i = 1 to n (1M)<br>{prod := prod * a} (3M)<br>Print prod<br>}<br>OR (recursive algorithm)<br>Procedure power (a,n)<br>{if n=0 then power (a,n) :=1<br>else power (a,n):= a* power (a,n-1)<br>} | |
| *b)* | Design algorithms to exchange the values of a and b,<br>(i) using temporary variable and (ii) without using temporary variable | |
| Ans | (i)procedure exchange (x ,y ,t)<br>{ t :=x<br>x :=y<br>y := t<br>} (2M)<br><br>(ii) procedure exchange (x ,y)<br>{ x:= x+y<br>x := y-x<br>y := y- x<br>} (3M) | |
| *c)* | What is Kuratowski's theorem? Explain with an example. | |
| Ans | Kuratowski's Theorem: A graph G is nonplanar if and only if it contains a subgraph homomorphic to $K_5$ or $K_{3,3}$.<br>Example: In graph $(i)$ we observe that $K_5$ is a subgraph of $(i)$. Hence by Kuratowski's theorem given graph is non-planar. | 2 |

| | | |
|---|---|---|
| | In graph $(ii)$ we observe that $K_{3,3}$ is a subgraph of $(ii)$. Hence by Kuratowski's theorem given graph is non-planar.<br><br>$(i)$<br><br>$(ii)$ | 2<br>1 |
| $d)$ | Define Complete Graph of $n$ vertices. Draw $K_3$, $K_4$ and $K_5$. | |
| Ans | Solution: A simple graph of $n$ vertices is said to be complete, denoted by $K_n$ if it contains exactly one edge between each pair of distinct vertices.<br> | 1<br>4 |
| $e)$ | Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology and chemistry (using alphabetical order) | |
| Ans | The word mathematics is the key of the root. Because physics comes after mathematics (in alphabetical order), add a right child of the root with key physics. Because geography comes before mathematics, add a left child of the | |

root with key geography. Next, add a right child of the vertex with key physics, and assign it the key zoology, because zoology comes after mathematics and after physics. Similarly, add a left child of the vertex with key physics and assign this new vertex the key meteorology. Add a right child of the vertex with key geography and assign this new vertex the key geology. Add a left child of the vertex with key zoology and assign it the key psychology. Add a left child of the vertex with key geography and assign it the key chemistry.




| | |
|---|---|
| *f*) | Show that there is no tree with degree sequence (1,1,2,2,2,2,3,3,3,3,3,3) |
| Ans | Let $T(V, E)$ be a tree with degree sequence (1,1,2,2,2,2,3,3,3,3,3,3).<br>Then $\|V\| = 12$<br>And $\|E\| = \|V\| - 1 = 12 - 1 = 11$<br>By Hand-shaking theorem,<br>$1 + 1 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 3 = 2\|E\|$<br>$28 = 2\times 11$, a contradiction.<br>Hence, there is no tree with degree sequence (1,1,2,2,2,2,3,3,3,3,3,3). |

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