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S.Y.B.Sc.(SEM-III) Paper I –USST301 QP code 30049
SET-1

ANSWER-KEY

Q.1 **Attempt all sub-questions: Maximum Marks 20**

a. **State TRUE or FALSE and correct if necessary. 2 Marks each**

i. False.
$X \sim$ Poisson (4)

ii. False.
Negative Binomial distribution is positively skewed distribution.

iii. True.

iv. True.

v. True.

b. **Answer the following : 2 Marks each**

i. Lack of Memory property of Geometric distribution
$P(X \geq s+t / X \geq s) = P(X \geq t)$

ii. Mode = 3

iii. Binomial distribution

iv. Exp (- x)

v Definition

Q.2			**Attempt any TWO sub-questions:**	**(20)**
	a.	i.	Definition of M.G.F. ....2M Properties(any 2) ....2M M.G.F. of r. v. Y = 2X+7 is $M_Y(t) = e^{7t} M_X(2t)$ ....(1M)	(05)
		ii.	Definition of Characteristic Function ....(2M) Properties (any 3) ....(3M)	(05)
	b.	i.	Definition of C.G.F. of a discrete random variable X ......1M $K_1 = \mu_1'$, $K_2 = \mu_2$, $K_3 = \mu_3$, $K_4 = \mu_4 - 3\mu_2^2$ ......2M Statement Additive property of cumulants 2M	(05)
		ii.	$f(x) = e^{-x}$ $x > 0$ $= 0$ o.w. Y = 2X+4. x= (y-4)/2 $\left\| \frac{dx}{dy} \right\| = 1/2$ ....(2M) p.d.f. of Y = [f(x)] x in terms of y $\left\| \frac{dx}{dy} \right\|$ ....(1M)	(05)

			$= \frac{1}{2} \exp[-((y-4)/2]$ $y > 4$ ....(2M)	
	c.	i.	X ~ Bin(n,p) M.G.F.= Mx(t)= $E(e^{tx}) = (q+pe^t)^n$ ....(2M) X and Y are two independent random variables with X ~ Bin (n_1,p) and Y ~ Bin (n_2,p) Distribution of X+Y $M_{X+Y}(t) = M_X(t)\, M_Y(t)$ $= [q+pe^t]^{n_1}\ [q+pe^t]^{n_2}$ $=[q+pe^t]^{n_1+n_2}$ X+Y ~ Binomial (n_1+n_2 , p) 3M	(05)
		ii.	$P(X=x) = \frac{1}{n}$ x = 1,2.....n = 0 o. w. M.G.F. = $\frac{1}{n}[e^t+e^{2t}+e^{3t}+....e^{nt}]$ ....3M E(X) = (n+1)/2 ....2M	(05)
Q.3			**Attempt any TWO sub-questions:**	**(20)**
	a.		X~ NB(k,p) P.M.F. of Negative Binomial(k,p) ....2M $E(X) = \sum xp(x) = \frac{kq}{p}$ ....3M $E(X^2) = \sum x^2p(x) \sum x^2p(x) = k (k+1)q^2/p^2+kq/p$ ....3M $V(X) = kq/p^2$ 2M	(10)
	b.		X ~ Geometric (p) M.G.F. = $p / (1-qe^t)$ ....2M C.G.F. = $\log_e[p / (1-qe^t)]$ ....2M $E(X) = \frac{q}{p}$ ....3M $V(X) = q/p^2$ 3M	(10)
	c.		X ~ Poisson (λ)	(10)

			M.G.F. = $M_x(t) = \exp[\lambda(e^t - 1)]$ ....2M C.G.F. = $\lambda(e^t - 1)$ $= \lambda[t + t^2/2! + t^3/3! + ... + t^r/r! + ...]$ ....2M $\mu_1' = K_1 = \lambda$, $\mu_2 = K_2 = \lambda$, $\mu_3 = K_3 = \lambda$, Measures of skewness ....3M $\beta_1 = 1/\lambda$ $\gamma_1 = 1/\sqrt{\lambda} > 0$ ....2M Poisson distribution is positively skewed................1	
Q.4			**Attempt any TWO sub-questions:**	**(20)**
	a.		Definition of marginal distribution of X --discrete and continuous case 2M Definition of marginal distribution of Y --discrete and continuous case 2M $p(x) = e^{-\lambda} \lambda^x / x!$ .... 3M $p(y) = e^{-\lambda p} (\lambda p)^y / y!$ 3M	(10)
	b.		Definition of conditional distribution of X given y --discrete and continuous case 2M Definition of conditional distribution of Y given x --discrete and continuous case 2M $f(x/y) = (3x^2 y + 3xy^2) / y(1 + 3y/2)$ .... 2M $f(y/x) = (3x^2 y + 3xy^2) / x(1 + 3x/2)$ 2M	(10)
	c.		(i) k = 1/36 ....2M (ii) X : 1 2 3 4 P(x) : 10/36 9/36 8/36 9/36 ....2M Y : 1 2 3 4 P(y) : 11/36 9/36 7/36 9/36 .... 2M E (X) = 88/36 ....2M E (Y) = 86/36 ....2M	(10)
Q.5			**Attempt any TWO sub-questions:**	**(20)**
	a.	i.	Distribution of sum of k independent identically distributed Bernoulli random variables with parameter p X ~ Bernoulli(p) M.G.F. = $(q + pe^t)$ 2M	(05)

			$M_{X_1+X_2+\ldots X_n}(t) = M_{X_1}(t)\, M_{X_2}(t) \ldots\ldots M_{X_n}(t)$ $= (M_X(t))^n$ $= [q+pe^t]^n$ ....2M $X_1+X_2+\ldots X_n \sim$ Binomial (n , p) 1M	
		ii.	X ~ Binomial(n, p) $\mu_{r+1} = pq.\,[\,n\, r\, \mu_{r-1} + \frac{d}{dp}\mu_r\,]$ Derivation 5M	(05)
	b.	i.	X ~ Hypergeometric (N, A, n) P.M.F. ....1M $E(X) = \frac{An}{N} = nP$ where $P = \frac{A}{N}$ 4M	(05)
		ii.	Definition of truncated distribution ....2M p.m.f. of truncated Binomial distribution, truncated at X=0 ....2M Mean = $np/(1-q^n)$ 1M	(05)
	c.		E (X) = 5/6, E (Y) = 17 / 6, E (XY) = 1 / 12, Cov (XY) = - 1/ 36	(10)