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| SEM-II | F. Y. B. Sc<br>PHYSICS – I | 100 MARKS 3 HRS |
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Answer Key – F.Y. B.Sc., SEM-II, Paper-I, April-2019

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|---|---|---|---|
| Q.1 | (A) Select the correct option | | 12 |
| | i) b ii) a iii) d iv) c v) b vi) a | | |
| | (B) Answer in one sentence : | | 03 |
| | i) | A unit vector of the given vector is a vector of unit magnitude and has a direction as same as that of the given vector. | |
| | ii) | An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables is said to be differential equation. | |
| | iii) | The transverse wave is that wave in which the particles of the medium vibrate about their mean position in a direction at right angle to the direction of propagation of the wave. | |
| | (C) Fill in the blanks | | 05 |
| | i) wavelength ii) time period iii) L/R iv) Nabla v) Non dispersive | | |
| Q.2 | (A) Attempt any one | | 08 |
| | i) | The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of magnitudes of the two vectors and cosine of angle between of them. Thus $\vec{A}\cdot\vec{B} = \lvert\vec{A}\rvert\,\lvert\vec{B}\rvert \cos\theta$ the product is scaler, the product may be +ve or –Ve depending upon the angle between them.<br>1. DOT product is commutative ($\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$)<br>2. DOT product is distributive $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$.<br>3. Dot product of a vector with itself gives square of magnitude.<br>4. The DOT product of two perpendicular vectors is zero.<br>5. The DOT product of two vectors may be negative, if angle $\theta > 90°$.<br>6. $\vec{A}$ is parallel to $\vec{B}$ then, $\vec{A}\cdot\vec{B} = AB\cos 0 = AB$.<br>7. If $\vec{A}$ is antiparallel to B then $\vec{A}\cdot\vec{B} = AB\cos 180 = -AB$. And so on | |
| | ii) | **(a)** $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{\imath} + \frac{\partial\phi}{\partial y}\hat{\jmath} + \frac{\partial\phi}{\partial z}\hat{k} = 2x\hat{\imath} + (-2y+2z)\hat{\jmath} + (2y\,4z)\hat{k};$<br>$(\nabla\phi)_{(1,-2,1)} = 2\hat{\imath} + 6\hat{\jmath} + 0\hat{k} = 2\hat{\imath} + 6\hat{\jmath}.$<br>**(b)** $\nabla\cdot\vec{r} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right); \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) = 1+1+1 = 3$ | |
| | (B) Attempt any one | | 08 |
| | i) | $let\ \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $\lvert\vec{r}\rvert = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{\frac{1}{2}}$<br>$\ln\lvert\vec{r}\rvert = \frac{1}{2}\ln(x^2+y^2+z^2);$<br>$\vec{\nabla}\phi = \frac{1}{2}\vec{\nabla}\ln(x^2+y^2+z^2)$<br>Ans: $\frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{x^2+y^2+z^2}$<br>$\vec{\nabla}\phi = \nabla\left(\frac{1}{r}\right) = \nabla\left\{(x^2+y^2+z^2)^{-\frac{1}{2}}\right\};$<br>$= \hat{\imath}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-\frac{1}{2}} + \hat{\jmath}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{-\frac{1}{2}}\ \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-\frac{1}{2}}$ | |

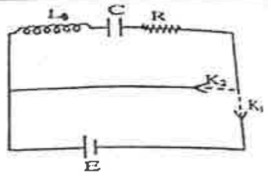
| | | | |
|---|---|---|---|
| | | $= \frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{\frac{3}{2}}} = -\frac{\vec{r}}{r^3}$ In general $\vec{\nabla} r^n = n r^{n-2}\,\vec{r}$ | |
| | ii) | **(a)** Using Dot product, $\vec{A}\cdot\vec{B} = \lvert\vec{A}\rvert \lvert\vec{B}\rvert \cos\theta$; $\therefore \cos\theta = \frac{\vec{A}\cdot\vec{B}}{\lvert\vec{A}\rvert\lvert\vec{B}\rvert} = \frac{(2\hat{i}+2\hat{j}-\hat{k})\cdot(6\hat{i}-3\hat{j}+2\hat{k})}{(\sqrt{2^2+2^2+1^2})\sqrt{6^2+3^2+2^2}} = \frac{4}{21}$; $\theta = \cos^{-1}\left(\frac{4}{21}\right) = 79^{\circ}$<br>**(b)** Let $\hat{a}$ be the unit vector in the direction of $\vec{A}$ then, projection of $\vec{B}+\vec{C}$ on $\vec{A}$ = proj. of $\vec{B}$ on $\vec{A}$ + proj. of $\vec{C}$ on $\vec{A}$ ; $(\vec{B}+\vec{C})\hat{a} = \vec{B}\,\hat{a} + \vec{C}\,\hat{a}$, multiplying by $\vec{A}$ $(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{C}\cdot\vec{A}$. By commutative law of for DOT product<br>$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$. | |
| | (C) | Attempt any one | 04 |
| | i) | $\lvert\vec{A}+\vec{B}\rvert = \sqrt{(A^2+B^2+2AB\cos\theta)}$ and $\lvert\vec{A}-\vec{B}\rvert = \sqrt{(A^2+B^2-2AB\cos\theta)}$<br>But $\lvert\vec{A}+\vec{B}\rvert = \lvert\vec{A}-\vec{B}\rvert$; $\sqrt{(A^2+B^2+2AB\cos\theta)} = \sqrt{(A^2+B^2-2AB\cos\theta)}$,<br>$A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$ ; 4AB $\cos\theta = 0$, $\cos\theta = 0$ $\theta = 90^{\circ}$ , (A ⊥ B ) | |
| | ii) | $4x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ | |
| Q.3 | (A) | Attempt any one | 08 |
| | i) | The general equation in standard form is $\frac{d^2y}{dx^2} + P_0\frac{dy}{dx} + q_0 y = 0$<br>where $p_0$ and $q_0$ are constants. Let us write above eq. in the form of $\frac{d}{dx}$ by D<br>$(D^2 + p_0 D + q_0)y = 0$,--(1) The algebraic equation is $D^2 + p_0 D + q_0 = 0$ ----(2) this eqn. Is called auxiliary equation. Its roots are found by the method of factorization.<br>If the roots are real and equal (repeated)<br>Let $m_1 = m_2 = m$, then eqn.(1) is (D-m) (D-m)y = 0 ----(3);<br>let (D-m)y = $y_1$. Then eqn.(3) becomes (D-m) $y_1$= 0;<br>$\therefore y_1 = C_1 e^{mx}$, substituting this in above eqn.<br>(D-m)y = $C_1 e^{mx}$; $\frac{dy}{dx} - my = C_1 e^{mx}$ the standard form of first order inhomogeneous differential eqn. is $\frac{dy}{dx} + P(x)y = Q(x)$, After comparing, we get<br>P(x) = -m and Q(x) = $C_1 e^{mx}$ $\therefore I.F. = e^{\int p(x)\,dx} = e^{\int -m dx} = e^{-mx}$ and the solution is: y x [ I.F.] = $\int Q(x)[I.F.]\,dx + C$; $ye^{-mx} = \int C_1 e^{mx} e^{-mx} dx + C$<br>$ye^{-mx} = C_1\int dx + C$; $ye^{-mx} = C_1x + C$; **Ans: y =** $(C_1x + C)e^{mx}$ | |
| | ii) | circuit Diagram<br>**Growth of current:** when the current is growing, back induced emf $\left(-L\frac{di}{dt}\right)$, then resultant emf is in the circuit $\left(E - L\frac{di}{dt}\right)$, using Kirchoff's voltage law the emf eqn. for the circuit $E - L\frac{di}{dt} = Ri$; $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$; $\frac{di}{dt} = \frac{E}{L} - \frac{R}{L}i$, intergrating– $\ln\left(\frac{E}{R} - i\right) = \frac{R}{L}t + C$<br>at t = 0, i = 0; $\therefore -\ln\left(\frac{E}{R}\right) = C$; $\therefore -\ln\left(\frac{E}{R} - i\right) = \frac{R}{L}t - -\ln\left(\frac{E}{R}\right)$,<br>$i_0 = \frac{E}{R}$ **final steady state**; $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$<br>when the current has reached its final value then the current in the circuit is maximum.<br>**Decay of current:** E = 0, then emf eqn. of circuit $-L\frac{di}{dt}$ $Ri$; $\frac{di}{i} = -\frac{R}{L}dt$, integrating both side $\ln i = -\frac{Rt}{L} + C$, t = 0, i = $i_0$ , $\ln i_0 = C$ ; $\ln i = -\frac{R}{L}t + \ln i_0$<br>$\therefore i = i_0 e^{-\frac{R}{L}t}$ this gives the expression for the decay of current with time in LR series circuit. | |
| | (B) | Attempt any one | 08 |
| | i) | **(a)** standard form of this differential equation is $\frac{dy}{dx} + P(x)y = Q(x)$; then P(x) = 1, $Q(x) = e^{-x}$, $I.F. = e^{\int P(x)dx} = e^{\int 1\,dx} = e^{x}$ then solution is | |

$y \times [I.F.] = \int Q(x)[I.F.]dx + C;\ ye^{x} = x + C;$ ANS. $\therefore y = (x + C)e^{-x}$

**(b)** The given DE can be written as: $x^2dx = e^y dy$, On integrating $\int x^2dx = \int e^y dy$

$e^y = \frac{x^3}{3} - C \quad \therefore y = \ln\left(\frac{x^3}{3} - C\right)$, which is the required solution. C is arbitrary constant

ii)




Consider a circuit having an inductance L, a capacitor C and a resistance R placed in series with a steady emf E through the key K($K_1$ or $K_2$)as shown in Fig. 4.16. Initially when the key $K_1$ and $K_2$ are open the charge on the capacitor is zero. After closing the key $K_1$ capacitor starts getting charged through the inductance L and resistance R. Let q be the charge on the capacitor at any instant t. The charge on the capacitor slowly grows due to which there is current $i = \frac{dq}{dt}$ at any instant.

P.D. across $R = iR = R\frac{dq}{dt}$ P.D. across $C = \frac{q}{C}$

Back emf in the inductor $L = -L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$

Thus, the resultant emf in the circuit $= E - L\frac{d^2q}{dt^2}$

By KVL, the emf equation for the circuit is, $E - L\frac{d^2q}{dt^2} = \frac{q}{C} + R\frac{dq}{dt}$

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} - E = 0 \qquad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \left(\frac{q}{LC} - \frac{E}{L}\right) = 0$

Put $\frac{R}{L} = 2b$ and $\frac{1}{LC} = K^2 \quad \therefore \quad \frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + \left(K^2q - \frac{E}{L}\right) = 0$

$\frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + K^2\left(q - \frac{E}{LK^2}\right) = 0$ We change variable from q to x by putting $x = q - \frac{E}{LK^2}$

$\therefore \quad \frac{dx}{dt} = \frac{dq}{dt}$ and $\frac{d^2x}{dt^2} = \frac{d^2q}{dt^2}$ Substituting in eqn. $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + K^2x = 0$

This is the standard second order DE which may be solved in the form.

$x = e^{\alpha t} \quad \therefore \quad \frac{dx}{dt} = \alpha e^{\alpha t}$ and $\frac{d^2x}{dt^2} = \alpha^2 e^{\alpha t} \qquad \alpha^2e^{\alpha t} + 2b\alpha e^{\alpha t} + K^2e^{\alpha t} = 0$

$\alpha^2 + 2b\alpha + K^2 = 0$ This is a quadratic equation in $\alpha$. Its roots are

$\alpha = \frac{-2b \pm \sqrt{4b^2 - 4K^2}}{2} = -b \pm \sqrt{b^2 - K^2}$

As there are two values of $\alpha$, the most general solution of differential equation

$x = Ae^{\left(-b+\sqrt{b^2-K^2}\right)t} + Be^{\left(-b-\sqrt{b^2-K^2}\right)t} \qquad \left(\because \frac{1}{LC} = K^2\right)$

where A and B are arbitrary constants. $x = q - \frac{E}{LK^2} = q - EC = q - q_0$

where $q_0$ is the final steady value of charge on the capacitor

$\therefore \quad q - q_0 = Ae^{\left(-b+\sqrt{b^2-K^2}\right)t} + Be^{\left(-b-\sqrt{b^2-K^2}\right)t}$

$q = Ae^{\left(-b+\sqrt{b^2-K^2}\right)t} + Be^{\left(-b-\sqrt{b^2-K^2}\right)t} + q_0$

(i) At $t = 0,\ q = 0$, eqn. becomes $0 = A + B + q_0 \quad \therefore A + B = -q_0$

(ii) At $t = 0,\ i = \frac{dq}{dt} = 0$, eqn becomes

$i = \frac{dq}{dt} = \left(-b + \sqrt{b^2 - K^2}\right)Ae^{\left(-b+\sqrt{b^2-K^2}\right)t} + \left(-b - \sqrt{b^2 - K^2}\right)Be^{\left(-b-\sqrt{b^2-K^2}\right)t}$

$\therefore 0 = \left(-b + \sqrt{b^2 - K^2}\right)A + \left(-b - \sqrt{b^2 - K^2}\right)B \qquad -b(A + B) + \sqrt{b^2 - K^2}(A - B) = 0$

$A = -\frac{q_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - K^2}}\right) \qquad B = -\frac{q_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - K^2}}\right)$

$q = -\frac{q_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - K^2}}\right)e^{\left(-b+\sqrt{b^2-K^2}\right)t} - \frac{q_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - K^2}}\right)e^{\left(-b-\sqrt{b^2-K^2}\right)t} + q_0$

$q = q_0\left[1 - \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - K^2}}\right)e^{\left(-b+\sqrt{b^2-K^2}\right)t} - \frac{1}{2}\left(1 - \frac{b}{\sqrt{b^2 - K^2}}\right)e^{\left(-b-\sqrt{b^2-K^2}\right)t}\right]$

(C) Attempt any one — 04

i) $\frac{dy}{dx} = kxy, \quad \frac{dy}{y} = kx\,dx;$ integrate $\int \frac{dy}{y} = k\,x dx$, $\ln y = k\frac{x^2}{2} + C$

$y = e^{C'}e^{k\frac{x^2}{2}}$, Ans: $\therefore y = e^{k\frac{x^2}{2}}$

ii) $E - L\frac{di}{dt} = iR, \qquad \frac{di}{dt} = \frac{1}{L}(E - iR); \qquad = \frac{1}{10}(100 - 5 \times 10); \qquad \frac{di}{dt} = \frac{50}{10} = 5\ A/s$

Q.4 (A) Attempt any one — 08

i) $x = A\sin(\omega t + \alpha)$ and $y = B\sin(\omega t + \beta)$,

$\frac{x}{A}\sin\beta = \sin\omega t\ \cos\alpha\ \sin\beta + \cos\omega t\ \sin\alpha\ \sin\beta;$

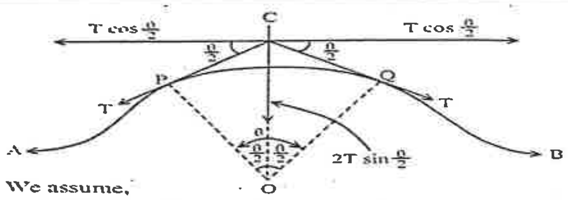
$\frac{y}{B}\sin\alpha = \sin\omega t\,\cos\alpha\beta\,\sin\alpha + \cos\omega t\,\sin\beta\,\sin\alpha$ *substracting*

$\left(\frac{x}{A}\sin\beta - \frac{y}{B}\sin\alpha\right) = \sin\omega t\,(\cos\alpha\,\sin\beta - \cos\beta\,\sin\alpha) = \sin\omega t\,\sin(\beta-\alpha)$

$\left(\frac{x}{A}\cos\beta - \frac{y}{B}\cos\alpha\right) = \cos\omega t\,(\sin\alpha\,\cos\beta - \sin\beta\,\cos\alpha) = -\cos\omega t\,\sin(\beta-\alpha)$

$\frac{x^2}{A^2}+\frac{y^2}{B^2}-2\frac{xy}{AB}\cos(\beta-\alpha) = \sin^2(\beta-\alpha)$, Ans: $\frac{x^2}{A^2}+\frac{y^2}{B^2}-2\frac{xy}{AB}\cos\delta = \sin^2\delta,$

ii)




We assume,

1. The string has a uniform linear density, i.e., mass per unit length.
2. The transverse vibration are confined to a single plane.
3. The tension T remains the same when the string is deformed from its equilibrium position.
4. The effect of gravity is negligible, i.e. The weight of the string is negligible since the tensio acting tangentially at any point is large enough.

Note $\angle POC = \angle QOC = \frac{\theta}{2}$, where $\angle POQ = \theta$.

Total tension along CO $= 2T\sin\frac{\theta}{2}$ $\qquad 2T\sin\frac{\theta}{2} = m\delta x\frac{v^2}{R}$

For small angle of θ, $\sin\frac{\theta}{2} \approx \frac{\theta}{2}$

$\therefore\quad 2T\left(\frac{\theta}{2}\right) = m\delta x\,\frac{v^2}{R} \qquad \therefore\quad T(\theta) = m\delta x\frac{v^2}{R}$ $\qquad$ Substitute $\theta = \frac{\delta x}{R}$, angle $= \frac{\text{Arc}}{\text{Radius}}$

$\frac{T\delta x}{R} = m\delta x\frac{v^2}{R} \qquad v^2 = \frac{T}{m} \qquad v = \sqrt{\frac{T}{m}}$

Thus we can say, velocity of transverse wave along the stretched string depends upon tension (T) and the density (linear) of wire (m). If *l* be the length of string which vibrates in *p* segments, then the length of each segment is equal to the ratio of length (*l*) / segments (*p*). But each segment corresponds to $\frac{\lambda}{2}$.

$\therefore\quad \frac{\lambda}{2} = \frac{l}{p}$ or $\lambda = \frac{2l}{p}$ $\qquad$ Also $\qquad v = n\lambda$ then $v = n\times\frac{2l}{p}$ $\qquad \therefore\quad n\times\frac{2l}{p} = \sqrt{\frac{T}{m}}$

$n = \frac{p}{2l}\times\sqrt{\frac{T}{m}} = \frac{p}{2l}\sqrt{\frac{T}{m}}$

(B) Attempt any one — 08

i) $x_1 = A\sin(\omega t+\alpha)$ and $x_2 = A\sin(\omega t+\beta)$;by using superposition principle $x = x_1 + x_2$ ; $= \sin\omega t(A\cos\alpha + B\cos\beta) + \cos\omega t(A\sin\alpha + B\sin\beta)$

$R\cos\delta = (A\cos\alpha + B\cos\beta)$.......(1) and $R\sin\delta = (A\sin\alpha + B\sin\beta)$..........(2)

after solving the resultant motion is simple harmonic motion $x = R\sin(\omega t+\delta)$. Finding the resultant amplitude R is obtained by squaring and adding eqn.(1) and (2) and solving LHS = $R^2\cos^2\delta + R^2\sin^2\delta = R^2(A\sin\alpha + \cos\beta) = R^2$ ; RHS = $A^2 + B^2\,2AB(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$ then $R^2 = A^2 + B^2 + 2AB\cos(\alpha-\beta)$; $R^2 = A^2 + B^2 + 2AB\cos\delta$; $\tan\delta = \frac{R\sin\delta}{R\cos\delta} = \frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}$

Case 1: $\delta = \alpha - \beta = (2n+1)$; $R = A - B$

Case 2: If A=B then $\alpha$ *and* $\beta$ are different $\delta = \frac{\alpha+\beta}{2} = \frac{1}{2}(\alpha+\beta$

Case 3: If A=B then $\delta = \alpha - \beta = (2n\pi)$; $R_{Max} = A + B = A + A = 2A$

ii)

**GROUP VELOCITY**

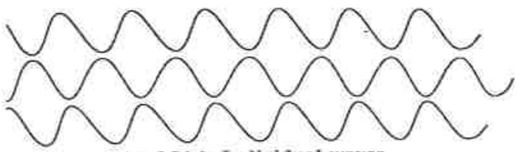


Fig. 6.3(a): Individual waves

Fig. 6.3(b): Superposition of Individual waves (Wave Packet)

The principal behind group velocity is the concept of wave packet. We know that from the expression of wave velocity, the wave or phase velocity is greater than the velocity of light. In order to overcome the above problem, matter (which consists of particles), moves as a group of wave or a wave packets instead of single waves. Thus, wave packets are the concept behind the group velocity.

Consider a group of two or more individual waves with a slight difference in their wavelength, as shown in Fig. 6.3(a). The superposition of two individual waves takes place and hence, the resultant wave is formed as shown in Fig. 6.3(b). The amplitude of the resultant wave consist of a number of components waves with different wavelengths. The resultant wave is known as group of waves or wave packets. The packet travel with a velocity in a medium which is different from the velocity of individual component waves. Therefore, the velocity of the wave packets which travel in a medium is known as a group velocity. The importance of group velocity is the transmission of energy in a wave.

Consider two travelling waves with the same amplitude but slightly different frequencies $\omega_1$ and $\omega_2$ and wave number $k_1$ and $k_2$. Let $y_1$ and $y_2$ be the displacements for the two travelling waves and hence, the equation for the displacement of wave is,

$$y_2 = A\sin(\omega_2 t - k_2 x)$$

$$y = y_1 + y_2$$

$$y = A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$$

$$= A\,[\sin((\omega_1 t - k_2 x) + \sin(\omega_2 t - k_2 x)]$$

We know that $\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$$y = 2A\sin\left[\frac{(\omega_1+\omega_2)}{2}t - \frac{(k_1+k_2)}{2}x\right]\times\cos\left[\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right]$$

Consider, $\omega = \frac{(\omega_1+\omega_2)}{2}$, $k = \frac{(k_1+k_2)}{2}$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$

$$y = 2A\sin[\omega t - kx]\times\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$$

$$y = 2A\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]\times\sin[\omega t - kx]$$

Amplitude of resultant wave $= 2A\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$

$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = 0$ $\quad\therefore\quad$ $V_g = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta k}{2}} = \frac{\Delta\omega}{\Delta k}$ Therefore, the group velocity, when $\frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$ is $V_g = \frac{d\omega}{dk}$.

**PHASE VELOCITY**

The phase velocity of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest) will appear to travel at the phase velocity. The phase velocity is given in terms of the wavelength $\lambda$ and period T as:

$$V_p = \frac{\lambda}{T} \qquad V_p = \frac{\omega}{k}$$

Thus, the velocity of component waves of a wave packet is called the phase velocity the phase $\omega t - kx$ = Constant

$$\therefore \quad \omega\,dt - k\,dx = 0$$

$$\therefore \quad V_p = \frac{dx}{dt} = \frac{\omega}{k}$$

1. In the dispersive medium, the phase velocity is frequency dependent. The dispersion relation expresses relation between angular frequency $\omega$ and the wave number.

   Since, $\omega = kV_p$, $V_g = \frac{d\omega}{dk} = \frac{d}{dk}(kV_p) = V_p + k\frac{dV_p}{dk}$

   $$\therefore \quad V_g = V_p + k\frac{dV_p}{dk} = V_p + \left(\frac{2\pi}{\lambda}\right)\frac{dV_p}{d\left(\frac{2\pi}{\lambda}\right)} = V_p + \frac{1}{\lambda}\frac{dV_p}{d\left(\frac{1}{\lambda}\right)}$$

   But $d\left(\frac{1}{\lambda}\right) = \frac{1}{\lambda^2}d\lambda$

   $$\therefore \quad V_g = V_p - \lambda\frac{dV_p}{d\lambda}$$

2. For non-dispersive medium the different waves with different wavelengths travel with the same phase velocity and hence $\frac{dV_p}{d\lambda} = 0$. Therefore, group velocity is equal to phase velocity $V_g = V_p$.

(C) Attempt any one — 04

i) If $y = f_1(x - ct) + f_2(x + ct)$ is the solution of the wave eqn.

$\frac{\partial y}{\partial t} = -cf_1'(x - ct) + cf_2'(x + ct)$ and $\frac{\partial y}{\partial x} = f_1'(x - ct) + cf_2'(x + ct)$

$\frac{\partial^2 y}{\partial t^2} = c^2 f_1''(x - ct) + c^2 f_2''(x + ct)$ and $\frac{\partial^2 y}{\partial x^2} = f_1''(x - ct) + f_2''(x + ct)$

Hence proved $\quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\left(\frac{\partial^2 y}{\partial t^2}\right)$

ii) Frequency of first tuning fork=250Hz; Time for complete one cycle =T=0.2 S

Difference in frequencies $= \frac{1}{T} = \frac{1}{0.2} = 0.5Hz$; Then possible frequencies for the other fork are 250+5 = 255Hz or 250-5 = 245Hz.

Q.5 Attempt any four — 20

i) $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$; $\quad \vec{B}\times(\vec{C}\times\vec{A}) = (\vec{B}\cdot\vec{A})\vec{B} - (\vec{B}\cdot\vec{C})\vec{A}$

$\vec{C}\times(\vec{A}\times\vec{B}) = (\vec{C}\cdot\vec{B})\vec{A} - (\vec{C}\cdot\vec{A})\vec{B}$, adding all three

$(\vec{C}\cdot\vec{A})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C} + (\vec{A}\cdot\vec{B})\vec{C} - (\vec{B}\cdot\vec{C})\vec{A} + (\vec{B}\cdot\vec{C})\vec{A} - (\vec{C}\cdot\vec{A})\vec{B} = 0$

ii) $$\nabla\times V = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos y & x\sin y & \cos z \end{vmatrix} = 0\,\hat{\imath} + 0\,\hat{\jmath} + (-\sin y + \sin y)\hat{k} = 0;\ \nabla\times V = 0$$

| | | |
|---|---|---|
| | iii) | Consider an eqn. $\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$; M(x, y)dx +N(x, y) dy = 0; let the soln.F(x, y)=C<br>dF = $\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$ ∴ $\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$ ; M(x, y)= $\frac{\partial F}{\partial x}$ , N(x, y) = $\frac{\partial F}{\partial x}$<br>$\frac{\partial M(x,y)}{\partial y} = \frac{\partial}{\partial y}\frac{\partial F}{\partial x} = \frac{\partial^2 F}{\partial x \partial y}$ and $\frac{\partial N(x,y)}{\partial x} = \frac{\partial}{\partial x}\frac{\partial F}{\partial y} = \frac{\partial^2 F}{\partial x \partial y}$ If F has continuous first derivatives, then order of differentiation of F is immaterial and hence<br>$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ is said to be an exact differential equation. |
| | iv) | Time constant $\Gamma = \frac{L}{R} = \frac{200}{20} = 10\,s$; max. current in the circuit $\frac{E}{R} = \frac{5}{20} = 0.25A$ |
| | v) | P.E.=V= $\frac{1}{2}Kx^2$, Force acting on $F = -\frac{dV}{dx}$ , F = $\frac{1}{2}K(2x)$ = -2$x$ , F = -Kx. |
| | vi) | |