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Sem I : Paper II : Solution : Oct, 2018 59173

Q 1] (a)(1) False. 0 & 1, (2) False, $P(A\cap B)=0$, (3) True, (4) True, (5) False. Poisson distn.

(b)(1) For ~~event~~ A, non-occurence of event A is called Complementary event. e.g. $\bar{A}$

Sample points of two or more events taken together constitutes sample space are exhaustive events e.g. $A\cup B=S$

(2) If an expt. can result into any n m.e, exhaustive, equally likely & exhaustive outcomes, out of which m are favourable to the occurrence of event A, then $P(A)=\frac{m}{n}$.

(3) $Cov(X,Y) = E[(X-E(X))(Y-E(Y))] = E(X\cdot Y) - E(X)\cdot E(Y)$.

(4) ~~c.d.f = X is discrete r.v. with p.m.f P(X=x) then F(x)=P(X≤x)~~, R.V. : A real valued fn associated with outcomes of a random expt.

(5) Bernoulli trial: Each trial results in 2 possible exh. & m.e outcomes and prob of any outcome of the trial remain fixed over time.

Q.2] (a) (I) $P(A\cap B)=P(A)\cdot P(B|A)=P(B)\cdot P(A/B)$

Proof: n(S): Sample points in S, n(A): favourable points to A, n(B): favourable to B., $n(A\cap B)$: favourable to both A & B

$\therefore P(A)=\frac{n(A)}{n(S)}$, $P(B)=\frac{n(B)}{n(S)}$, $P(A\cap B)=\frac{n(A\cap B)}{n(S)}$

$P(B|A)=\frac{n(A\cap B)}{n(A)}=\frac{P(A\cap B)}{P(A)}$ $\therefore P(A\cap B)=P(A)P(B|A)$

III^{ly}, $P(A\cap B)=P(B)\cdot P(A/B)$.

For A & B indep. $P(A\cap B)=P(A)\cdot P(B)$

(II) (i) $P(\text{no girl})=\frac{^4C_3}{^7C_3}$

(ii) $P(\text{At least one girl})=1-P(\text{No girl})=1-\frac{^4C_3}{^7C_3}$,

(iii) $P(\text{A particular girl})=\frac{^4C_2}{^7C_3}+\frac{^4C_1\times {^3C_1}}{^7C_3}=$

(b) (I) Bayes' thm: $P(A_i/B)=\frac{P(A_i)\cdot P(B|A_i)}{\Sigma P(A_i)\cdot P(B|A_i)}$, $\forall i=1,2,\dots n$.

Proof: $A_1, A_2 \dots A_n$ are m.e & exh. events

$\therefore A_i\cap A_j=\phi$ & $A_1\cup A_2\cup\dots\cup A_n=S$

$B=S\cap B=(A_1\cup\dots\cup A_n)\cap B=(A_1\cap B)\cup(A_2\cap B)\cup\dots\cup(A_n\cap B)$

$\therefore P(B)=\Sigma P(A_i\cap B)=\Sigma P(A_i)\cdot P(B|A_i)$ & $P(A_i|B)=\frac{P(A_i\cap B)}{P(B)}$. proved.

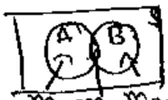
(II) P(Selected marble is white) $=\left(\frac{3}{10}\times\frac{5}{11}\right)+\left(\frac{7}{10}\times\frac{4}{11}\right)=\frac{15+28}{110}$

$=\frac{43}{110}$

(2)

Q.2] (c) (I) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

Proof: $n(S) = n$, $n(A) = m_1$, $n(B) = m_2$ & $n(A\cap B) = m$.



$P(A) = \frac{m_1}{n}$, $P(B) = \frac{m_2}{n}$, $P(A\cap B) = \frac{m}{n}$,

$m_1 - m$, m, $m_2 - m$

$n(A\cup B) = m_1 + m_2 - m$ $\therefore P(A\cup B) = \frac{n(A\cup B)}{n(S)} = \frac{m_1 + m_2 - m}{n}$ $= P(A) + P(B) - P(A\cap B)$.

(i) A & B m.e., $P(A\cap B) = 0$. $\therefore P(A\cup B) = P(A) + P(B)$

(ii) A & B indep., $P(A\cap B) = P(A)\cdot P(B)$, $\therefore P(A\cup B) = P(A) + P(B) - P(A)\cdot P(B)$.

(iii) A is subset of B, $A\subset B$. $\therefore P(A\cap B) = P(A)$. $\therefore P(A\cup B) = P(B)$.

(II) $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

(d) (I) (i) S.S: Set of all possible outcomes, (ii) Event: Any subset of S.S,

(iii) Impossible event: Event corresponding to null set. (ϕ),

(iv) Certain event: Event corresponding to S.S.,

(v) Conditional prob: Prob. of occurrence of one event given that other event has already occurred of an expt.

(II) (i) $P(\bar{A}) = 1 - P(A) = 0.6$, (ii) $P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B) = 0.2$, (iii) $P(A/\bar{B}) = \frac{P(A\cap \bar{B})}{P(\bar{B})}$

Q.3]

$$= \frac{P(A) - P(A\cap B)}{1 - P(B)} = \frac{0.4 - 0.2}{1 - 0.6} = \frac{0.2}{0.4} = 0.5$$

$P(A\cap B) = 0.2 \neq P(A)\cdot P(B) = 0.4\times 0.6 = 0.24$ $\therefore$ A & B are not indep

Q.3] (a)(I) $E(X\cdot Y) = E(X)\cdot E(Y)$ for X & Y indep.

$P(x,y) = P_1(x)\cdot P_2(y)$ $\therefore E(XY) = \sum_x\sum_y p(x,y)\, x\cdot y = \sum x P_1(x) \sum y\cdot P_2(y)$ $= E(X)\cdot E(Y)$.

For n events, $E(X_1 \cdots X_n) = E(X_1)\cdots E(X_n)$

(II) $E(X^r) = \sum_x x^r P(x)$, $E[(X-\mu)^r] = \sum_x (x-\mu)^r P(x)$

It is affected by change in scale & is multiplied/divided by the same power of the change.

(b) (I) A r.v. which takes finite/countably infinite values occurring at intervals is discrete r.v.

p.m.f., $p(x) = P(X=x)$, $0 \le p(x) \le 1$ & $\sum_x p(x) = 1$, c.d.f: $P(X\le x) = F(x)$

c.d.f properties $0 \le F(x) \le 1$, $\forall x$, $F(-\infty) = 0$ & $F(\infty) = 1$,

$F(x)$ is monotonically non-decreasing fn of x i.e. if $a \le b$, $F(a) \le F(b)$

$P(a < X \le b) = F(b) - F(a)$

(c) $V(aX+bY) = E[aX + bY - E(aX+bY)]^2 = E[a^2(X - E(X))^2 + b^2(Y - E(Y))^2 + 2ab(X - E(X))(Y - E(Y))]$

$= a^2V(X) + b^2V(Y) + 2ab\, Cov(X,Y)$.

(i) $a = -1$, $b = 1$. $V(-X+Y) = V(X) + V(Y) - 2\,Cov(X,Y)$.

(ii) X & Y - indep. $V(aX+bY) = a^2V(X) + b^2V(Y)$, $Cov(X,Y) = 0$.

(iii) $Corr^n = 0.6 = \frac{Cov(X,Y)}{\sqrt{V(X)}\sqrt{V(Y)}}$ $\Rightarrow Cov(X,Y) = 0.6\sqrt{V(X)}\sqrt{V(Y)}$

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Q.3](d).

x \ y	1	2	3	$P_1(x)$
1	1/36	2/36	3/36	6/36
2	2/36	4/36	6/36	12/36
3	3/36	6/36	9/36	18/36
$P_2(y)$	6/36	12/36	18/36	1

$E(X) = \Sigma x P_1(x) = \frac{6}{36} + \frac{24}{36} + \frac{54}{36} = \frac{84}{36} = E(Y).$

$Cov(X,Y) = E(X \cdot Y) - E(X) \cdot E(Y)$

$= \frac{194}{36} - \left(\frac{84}{36}\right)\left(\frac{84}{36}\right) = 5.388 - 5.444$

$\therefore$ X & Y are indep. $\doteq 0$

Q.4 (a) Bin. distⁿ $P(x) = {}^nC_x p^x q^{n-x}$; $x = 0, 1, 2 .. n$, $q = 1-p$, $0 < p < 1$

Mean = np, variance = npq. ∀ mean > variance.

(b) Poisson distⁿ $P(x) = \frac{e^{-m} m^x}{x!}$; $x = 0, 1, 2 \ldots$, $m > 0$. Illustration.

mean = Variance = m.

(c) Hypergeometric distⁿ: $P(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$;

Mean = $\frac{Mn}{N}$ (derive), Variance = $\frac{(N-n)}{(N-1)} m\left(1 - \frac{M}{N}\right)$.

Diff. Bin. - WR, Hyper - WoR.

(d) $P(x) = \frac{1}{N+1}$; $x = 0, 1, 2 .. N$. Example.

mean = $\frac{N}{2}$, variance = $\frac{N(N+2)}{12}$, $E(x^2) = \frac{N(2N+1)}{6}$.

Q.5) (a)(i) $P(B/A) = P(B/\bar{A}) \Rightarrow 1 - P(B/A) = 1 - P(B/\bar{A}) \Rightarrow P(\bar{B}/A) = P(\bar{B}/\bar{A})$.

$\therefore$ $\bar{B}$ & A are indep.

(ii) $\bar{A}$ & $\bar{B}$. $P(A|\bar{B}) = P(A|B) \Rightarrow 1 - P(A|\bar{B}) = 1 - P(A|B) \Rightarrow P(\bar{A}|\bar{B}) = P(\bar{A}|B)$.

(iii) $\bar{A}$ & B. $P(A|B) = P(A|\bar{B}) \Rightarrow 1 - P(A|B) = 1 - P(A|\bar{B}) \Rightarrow P(\bar{A}|B) = P(\bar{A}|\bar{B})$.

(b) Addition thm on expectation. $E(X+Y) = E(X) + E(Y)$

(c) $\mu_1 = 0$, $\mu_2 = \mu_2' - \mu_1'^2$ & $\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$.

(d) Expt. - two outcomes, success/failure, p: Prob.(success) same for each trial. WR. n trials indep. x: no. of successes.

$P(X=x) = {}^nC_x p^x q^{n-x}$; $x = 0, 1, 2 .. n$, $0 < p < 1$, $q = 1-p$.

$= 0$; o.w.

(e) $P(x+1) = \frac{m}{x+1} \cdot P(x)$; $x = 0, 1, 2, \ldots$

(f) $E(X) = 12$, $E(Y) = 15$, $V(X) = 9$, $V(Y) = 16$

(i) $E(5X+3Y) = 105$, (ii) $E(5X-3Y) = 15$, (iii) $V(5X+3Y) = 369$, (iv) $V(5X-3Y) = 369$.

(e) A: Item from A, B: from Y, C: from Z.

$P(A) = 0.4$, $P(B) = 0.25$, $P(C) = 0.35$.

D: Defective. $P(D/A) = 0.03$, $P(D/B) = 0.01$, $P(D/C) = 0.02$

$$P(A/D) = \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)} = \frac{0.4 \times 0.03}{(0.4 \times 0.03) + (0.25 \times 0.01) + (0.35 \times 0.02)}$$

$$= \frac{0.012}{0.012 + 0.0025 + 0.007} = \frac{0.012}{0.0215} = 0.558$$