

UNIVERSITY OF MUMBAI

Syllabus

for

T.Y.B.A./B.Sc. (CBCS)

Program: B.A/B.Sc.

Course: Mathematics

with effect from the academic year 2018-2019

T.Y.B.A./T.Y.B.Sc. (CBCS) Semester V

Discrete Mathematics				
Course Code	Unit	Topics	Credits	L/W
USMT501,UAMT501	Unit I	Preliminary Counting	2.5	3
	Unit II	Advanced Counting		
	Unit III	Permutations		
Algebra V				
USMT502, UAMT502	Unit I	Quotient Vector spaces and Orthogonal Transformations	2.5	3
	Unit II	Eigenvalues, Eigenvectors		
	Unit III	Diagonalisation		
Topology of Metric Spaces				
USMT503, UAMT503	Unit I	Metric Spaces	2.5	3
	Unit II	Sequences, Closed subsets, Limit Points		
	Unit III	Continuity		
Numerical Analysis-I (Elective A)				
USMT5A4,UAMT5A4	Unit I	Error Analysis	2.5	3
	Unit II	Transcendental & Polynomial Equations		
	Unit III	Linear System of Equations		
Number Theory and its Applications-I (Elective B)				
USMT5B4,UAMT5B4	Unit I	Congruences and Factorisations	2.5	3
	Unit II	Diphantine Equations and their solutions		
	Unit III	primitive Roots and Cryptography		
Graph Theory (Elective C)				
USMT5C4,UAMT5C4	Unit I	Basics of Graphs	2.5	3
	Unit II	Trees		
	Unit III	Eulerian and Hamiltonian Graphs		
Basics Concepts of probability and Random Variables (Elective D)				
USMT5D4,UAMT5D4	Unit I	Basics Concepts of probability and Random Variables	2.5	3
	Unit II	Properties of Distribution Function and Joint Density Function		
	Unit III	Weak Law of Large Numbers		
Practicals				
USMTP05, UAMTP05		Practicals based on USMT501/UAMT501, USMT502/UAMT502 and USMT503/UAMT503	3	6
USMTPJ5,UAMTPJ5		Project	3	6

T.Y.B.A./T.Y.B.Sc. (CBCS) Semester VI

Real and Complex Analysis				
Course Code	Unit	Topics	Credits	L/Week
USMT601,UAMT601	Unit I	Sequences and series of functions	2.5	3
	Unit II	Introduction to Complex Analysis		
	Unit III	Complex Power series		
Algebra V				
USMT602, UAMT602	Unit I	Normal Subgroups	2.5	3
	Unit II	Ring Theory		
	Unit III	Factorisation		
Metric Topology				
USMT603, UAMT603	Unit I	Complete Metric Spaces	2.5	3
	Unit II	compact Metric spaces		
	Unit III	Connected sets		
Numerical Analysis-II (Elective A)				
USMT6A4,UAMT6A4	Unit I	Interpolation	2.5	3
	Unit II	Polynomial Approximations and Numerical differentiation		
	Unit III	Numerical Integration		
Number Theory and its Applications-II (Elective B)				
USMT6B4,UAMT6B4	Unit I	Quadratic Reciprocity	2.5	3
	Unit II	Continued Fractions		
	Unit III	Pell's equation, Arithmetic Functions and Special Numbers		
Graph Theory and Combinatorics (Elective C)				
USMT6C4,UAMT56C4	Unit I	Colorings of Graphs	2.5	3
	Unit II	Planar Graphs		
	Unit III	Combinatorics		
Operations Research (Elective D)				
USMT6D4,UAMT6D4	Unit I	Linear Programming I	2.5	3
	Unit II	Linear Programming II		
	Unit III	Queing Systems		
Practicals				
USMTP06, UAMTP06		Practicals based on USMT601/UAMT601,USMT602/UAMT602 and USMT603/UAMT603	3	6
USMTPJ6,UAMTPJ6		Project	3	6

Note:

1. USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.
2. Candidate has to opt one Elective Course from USMT5A4/UAMT5A4, USMT5B4/UAMT5B4, USMT5C4/UAMT5C4 and USMT5D4/UAMT5D4 for Semester V.
3. USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.
4. Candidate has to opt one Elective Course from USMT6A4/UAMT6A4, USMT6B4/UAMT6B4, USMT6C4/UAMT6C4 and USMT6D4/UAMT6D4 for Semester VI.
5. Passing in theory and practical shall be separate in the compulsory courses.
6. Candidate has to do a project in the courses USMT5PR/UAMT5PR of Semester V and USMT6PR/UAMT6PR of Semester VI.

Teaching Pattern for SY B.A./B.Sc:

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).
3. Each project for the courses USMT5PR/UAMT5PR in Semester V and USMT6PR/UAMT6PR in Semester VI shall have at most 08 (eight) students and the workload for each project is 1L/W. However a teacher guiding more than one project gets 1L/W workload, irrespective of the number of projects he/she guides.

Syllabus for Semester V & VI

SEMESTER V

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT501/UAMT501 Discrete Mathematics

Unit I: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, Countable and uncountable sets, examples such as \mathbb{N} , $\mathbb{N} \times \mathbb{N}$, \mathbb{Q} , \mathbb{R} and open interval $(0, 1)$.
2. Addition and multiplication principle, Counting sets of pairs, two ways counting.
3. Stirling numbers of second kind, Simple recursion formulae satisfied by $S(n, k)$

and direct formulae for $S(n, k)$ for $k = 0, 1, \dots, n - 1$.

4. Pigeon hole principle and its strong form, its applications to geometry, monotonic sequences.

Reference for para 1 of unit I: Sections 2.1 and 2.4 of **Discrete Mathematics & Its Applications** by KENNETH ROSSEN, Tata McGraw Hill.

Reference for para 2 of unit I: Sections 10.1 and 10.2 of **Discrete Mathematics** by NORMAN L. BIGGS, (Second Edition) Oxford University press

Unit II: Advanced Counting (15 Lectures)

Binomial and Multinomial Theorem, Pascal identity, examples of standard identities

such as the following with emphasis on combinatorial proofs: $\sum_{i=0}^n \binom{n}{i} = 2^n$ and

$$\sum_{k=0}^r \binom{m}{k} \binom{m}{r-k} = \binom{m+n}{r}, \sum_{k=r}^n \binom{k}{r} = \binom{n+1}{r+1}, \sum_{i=0}^k \binom{k}{i}^2 = \binom{n+1}{r+1}.$$

Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems. Non-negative and positive integral solutions of equation $x_1 + x_2 + \dots + x_k = n$.

Principle of Inclusion and Exclusion and its applications, derangements, explicit formula for d_n , various identities involving d_n .

Unit III: Permutations (15 Lectures)

1. Permutation of objects, composition of permutations, results such as every permutation is product of disjoint cycles, every cycle is product of transpositions, even and odd permutations, S_n, A_n , rank and signature of permutation, results such as

$$\epsilon(\sigma \circ \eta) = \epsilon(\sigma)\epsilon(\eta) \quad (\sigma, \eta \in S_n), \quad \epsilon(\sigma) = \prod_{1 \leq i < j \leq n} \frac{\sigma(i) - \sigma(j)}{i - j}.$$

2. Partially ordered sets, Mobius Inversion Formula with application to deriving the formula for Eulers phi-function $\varphi(n)$.

3. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non-homogeneous) recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Reference for para 1 of unit III: **A First Course in Abstract Algebra** by JOHN. B. FRALEIGH, third edition, Narosa Publishing House.

Recommended Text Book:

RICHARD BRUALDI: *Introductory Combinatorics*, Pearson (Fourth Edition).

(Sections 2.1, 2.2, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 5.1, 5.2, 5.3, 6.1, 6.2, 6.3, 6.6, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 8.2.)

Additional Reference Books:

1. NORMAN BIGGS, *Discrete Mathematics*, Oxford University Press.

2. V. KRISHNAMURTHY, *Combinatorics Theory and Applications*.
3. A. TUCKER, *Applied Combinatorics*, John Wiley and Sons.

USMT502/UAMT502 Algebra IV

Unit I: Quotient Spaces and Orthogonal Linear Transformations (15 Lectures)

Review of vector spaces over \mathbb{R} , subspaces and linear transformations.

Quotient Spaces: For a real vector space V and a subspace W , the cosets $v + W$ and the quotient space V/W . First Isomorphism theorem for real vector spaces (Fundamental theorem of homomorphism of vector spaces), dimension and basis of the quotient space V/W when V is finite dimensional.

Inner product spaces: Examples of inner product including the inner product $\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$ on $\mathcal{C}[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$. Orthogonal sets and orthonormal sets in an inner product space. Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process and simple examples in $\mathbb{R}^3, \mathbb{R}^4$.

Real Orthogonal transformations and isometries of \mathbb{R}^n . Translations and Reflections with respect to a hyperplane. Orthogonal matrices over \mathbb{R} .

Equivalence of orthogonal transformations and isometries of \mathbb{R}^n fixing the origin. Characterization of isometries as composites of orthogonal transformations and translations.

Orthogonal transformation of \mathbb{R}^2 . Any orthogonal transformation in \mathbb{R}^2 is a reflection or a rotation.

Unit II: Eigenvalues, Eigenvectors (15 Lectures)

Eigenvalues and eigenvectors of a linear transformation $T : V \rightarrow V$ where V is a finite dimensional real vector space and examples, eigenvalues and eigenvectors of $n \times n$ - real matrices, linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation / Matrix.

Characteristic polynomial of an $n \times n$ - real matrix. Result: A real number λ is an eigenvalue of an $n \times n$ matrix A if and only if λ is a root of the characteristic polynomial of A . Cayley-Hamilton Theorem (statement only), Characteristic roots. Similar matrices and relation with a change of basis. Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices.

Reference for Unit II:

Sections 1, 2, 3 of Chapter VIII of Introduction to Linear Algebra (Second Edition) by SERGE LANG.

Recommended Text Books:

1. SERGE LANG: *Introduction to Linear Algebra*, Springer Verlag.
2. S. KUMARESAN: *Linear Algebra A geometric approach*, Prentice Hall of India Private Limited.

Unit III: Diagonalisation (15 Lectures)

Diagonalizability of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself. Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation. Examples of non-diagonalisable matrices over \mathbb{R} .

An $n \times n$ real matrix A is diagonalisable if and only if \mathbb{R}^n has a basis of eigenvectors of A if and only if the sum of dimension of eigen spaces of A is n if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide.

Diagonalisation of real Symmetric matrices and applications to real quadratic forms, rank and signature of a real quadratic form, classification of conics in \mathbb{R}^2 and quadric surfaces in \mathbb{R}^3 .

Recommended Text Books for Unit I & unit II:

1. S. KUMARESEN, *Linear Algebra: A Geometric Approach*, PHI.
2. M. ARTIN, *Algebra*, Pearson India.
3. L. SMITH, *Linear Algebra*, Springer.
4. T. BANCHOFF AND J. WERMER, *Linear Algebra through geometry*, Springer.

Additional Reference books:

1. N.S. GOPALAKRISHNAN, *University Algebra*, Wiley Eastern Limited.
2. M. ARTIN, *Algebra*, Prentice Hall of India, New Delhi.
3. P.B. BHATTACHARYA, S.K. JAIN, AND S.R. NAGPAUL, *Abstract Algebra*, Second edition, Foundation Books, New Delhi, 1995.
4. T.W. HUNGERFORD, *Algebra*, Springer.
5. D. DUMMIT & R. FOOTE, *Abstract Algebra*, John Wiley & Sons, Inc.

USMT503/UAMT503 Topology of Metric Spaces

Note: In this course, definitions of *closed set* in a metric space, *limit point* and *closure* of a subset of metric space shall be used as indicated below in Unit II.

Unit I: Metric spaces (15 Lectures)

Definition of metric space, Euclidean space \mathbb{R}^n with its Euclidean norm function and the distance metric induced by it, sup metric and sum metric on \mathbb{R}^n and \mathbb{C} (complex numbers). Discrete metric spaces and examples such as \mathbb{Z} .

Normed linear spaces: Definition, the distance (metric) induced by the norm, translation invariance of the metric induced by the norm. Examples of normed linear spaces including

1. \mathbb{R}^n with sum norm $\|\cdot\|_1$, the Euclidean norm $\|\cdot\|_2$, and the sup norm $\|\cdot\|_\infty$.
2. $\mathcal{C}[a, b]$, the space of continuous real valued functions on $[a, b]$ with norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ where $\|f\|_1 = \int_a^b |f(t)| dt$, $\|f\|_2 = \left(\int_a^b |f(t)|^2 dt\right)^{1/2}$, $\|f\|_\infty = \sup\{|f(t)| : t \in [a, b]\}$.

Open balls, open subsets of a metric space. Verification of the result: any open ball of a metric space is an open subset of the metric space. Examples of open sets in various metric spaces, structure of an open set in \mathbb{R} .

Properties of open subsets of a metric space: the intersection of finitely many open subsets of a metric space is an open subset of the metric space, the union of arbitrary collection of open subsets of a metric space is an open subset of the metric space. Interior of a subset of a Metric space. Hausdorff property of a metric space. Subspaces of a Metric space. Product of two metric spaces. Equivalent metrics.

Distance of a point from a set, distance between two sets, diameter of a set in a metric space.

Unit II: Sequences, closed sets, limit Points (15 Lectures)

Sequences in a metric space, convergent sequences and Cauchy sequences in a metric space, subsequence of a sequence, examples.

Closed set in a metric space (as complement of an open set), limit point of a set (If A is subset of a metric space X , $x \in X$ is a limit point of A if each open ball of X with center at x contains a point of A other than x), isolated point. A closed set contains all its limit points. Closed balls. Closure of a subset of a metric space (closure \bar{E} of a subset E of a metric space is $E \cup E'$ where E' denotes the set of all limit points of E in X) and properties: If X is a metric space and $E \subset X$. Then

1. \bar{E} is closed in X .
2. $E = \bar{E}$ if and only if E is closed in X .
3. $\bar{E} \subset F$ for every closed subset F of X such that $E \subset F$.
4. \bar{E} equals the intersection of all the closed supersets of E in X .

Boundary of a set in a metric space. Complete metric spaces.

Unit III: Continuity (15 Lectures)

ϵ - δ definition of continuity at a point for a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets.

Continuity of a function on a metric space. Characterization of continuity of a function in terms of inverse image of open sets and closed sets. Algebra of continuous real valued functions. Uniform continuity of a function defined on a metric space: definition and examples (emphasis on \mathbb{R}).

Recommended Text Books:

1. W. RUDIN, *Principles of Mathematical Analysis*, Tata McGraw- Hill Education in 2013.

2. G.F. SIMMONS, *Introduction to Topology and Modern Analysis*, McGraw Hill Education (India) Edition.
3. IRVIN KAPLANSKY, *Set Theory and Metric spaces*, Allyn and Bacon Inc, Boston.
4. S. KUMARESAN, *Topology of Metric spaces*, Narosa.

Additional Reference Books:

1. MÍCHEÁL Ó SEARCÓID, *Metric spaces*, Springer Undergraduate Mathematics Series, 2007.
2. T. APOSTOL, *Mathematical Analysis*, Narosa.
3. R. R. GOLDBERG, *Methods of Real Analysis*.
4. P. K. JAIN, K. AHMED, *Metric Spaces*, Narosa, New Delhi, 1996.
5. D. SOMASUNDARAM, B. CHOUDHARY, *A first Course in Mathematical Analysis*.
6. E. T. COPSON, *Metric Spaces*, Universal Book Stall, New Delhi, 1996.

USMT5A4/UAMT5A4 Numerical Analysis I (Elective A)

Note: Derivations and geometrical interpretation of all numerical methods have to be covered.

Unit I: Errors Analysis, Transcendental and Polynomial Equations (15 Lectures):

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Taylors series example. Significant digits and numerical stability. Concept of simple and multiple roots. Iterative methods, error tolerance, use of intermediate value theorem. Iteration methods based on first degree equation: Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method. Condition of convergence and Rate of convergence of all above methods

Unit II: Transcendental and Polynomial Equations (15 Lectures)

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method. Iterative methods for polynomial equations; Descarts rule of signs, Birge-Vieta method, Bairstrow method. Methods for multiple roots. Newton-Raphson method. System of non-linear equations by Newton-Raphson method. Methods for complex roots. Condition of convergence and Rate of convergence of all above methods.

Unit III: Linear System of Equations (15 Lectures)

Matrix representation of linear system of equations. Direct methods: Gauss elimination method. Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle and Crouts method, Choleskys method. Error analysis of direct methods. Iteration methods: Jacobi iteration method, Gauss-Siedal method. Convergence analysis of iterative method. Eigen value problem, Jacobis method for symmetric matrices Power method to determine largest eigenvalue and eigenvector.

Recommended Text Books:

1. E. KENDALL AND ATKINSON, *An Introduction to Numerical Analysis*, Wiley.
2. M. K. JAIN, S. R. K. IYENGAR AND R. K. JAIN, *Numerical Methods for Scientific and Engineering Computation*, New Age International Publications.
3. S.D. COMTE AND CARL DE BOOR, *Elementary Numerical Analysis, An Algorithmic Approach*, McGraw Hill International Book Company.
4. S. SASTRY, *Introductory methods of Numerical Analysis*, PHI Learning.
5. F.B. HILDEBRAND, *Introduction to Numerical Analysis*, Dover Publication, New York.
6. S.B. JAMES, *Numerical Mathematical Analysis*, Oxford University Press, New Delhi.

USMT5B4/UAMT5B4 Number Theory and its applications I (Elective B)

Unit I. Congruences and Factorization (15 Lectures)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.

Congruences : Definition and elementary properties, Complete residue system modulo m , Reduced residue system modulo m , Euler's function ϕ and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of higher degree, The Fermat-Kraitchik Factorization Method.

Reference for Unit I: Sections 2.1, 2.2, 2.3, 2.4, 2.5 of Niven, H. Zuckerman and H. Montgomery, *An Introduction to the Theory of Numbers*, John Wiley & Sons. Inc. and section 5.4 of David M. Burton, *An Introduction to the Theory of Numbers*. Tata McGraw Hill Edition.

Unit II: Diophantine equations and their solutions (15 Lectures)

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$ where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, the equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions $(x; y; z)$ with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic form $x_1^2 + x_2^2 + x_3^2 + x_4^2$.

Reference for Unit II: Sections 5.1, 5.2, 5.3, 5.4, 5.5 of Niven, H. Zuckerman and H. Montgomery, *An Introduction to the Theory of Numbers*, John Wiley & Sons. Inc.

Unit III: Primitive Roots and Cryptography (15 Lectures)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Crypto-systems, symmetric key cryptography, simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Crypto-system; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference for Unit III: Elementary number theory, David M. Burton, Chapter 8 sections 8.1, 8.2 and 8.3, Chapter 10, sections 10.1, 10.2 and 10.3.

Additional Reference Books:

1. G. H. HARDY AND E.M. WRIGHT, *An Introduction to the Theory of Numbers*, Oxford University Press.
2. NEVILLE ROBINS, *Beginning Number Theory*, Narosa Publications.
3. S.D. ADHIKARI, *An introduction to Commutative Algebra and Number Theory*, Alpha Science International.
4. N. KOBLITZ, *A course in Number theory and Cryptography*, Springer.
5. M. ARTIN, *Algebra*, Prentice Hall.
6. K. IRELAND, M. ROSEN, *A classical introduction to Modern Number Theory*, Springer.
7. W. STALLING, *Cryptology and network security*, Prentice Hall.

USMT5C4/UAMT5C4 Graph Theory (Elective C)

Unit I: Basics of Graphs (15 Lectures)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems.

Unit II: Trees (15 Lectures)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of K_n , Binary and m -ary tree, Weighted graphs and minimal spanning trees.

Unit III. Eulerian and Hamiltonian graphs (15 Lectures)

Eulerian graph and its characterization Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G - S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

Recommended Text Book:

J.A. BONDY AND U.S.R. MURTY, *Graph Theory with Applications*, Elsevier.

Additional Reference books:

1. R. BALAKRISHNAN AND K. RANGANATHAN, *A Textbook of Graph Theory*, Springer.
2. BEHZAD AND CHARTLAND, *Graph Theory*.
3. CHOUDAM S.A., *Introduction to Graph Theory*.
4. WEST D.G., *Graph Theory*. Allyn and Bacon.

USMT5D4/UAMT5D4

Basic Concepts of Probability and Random Variables (Elective D)

Unit I: Basic Concepts of Probability and Random Variables

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field \mathcal{F} , Probability measure P on \mathcal{F} , Probability Space as a triple (Ω, \mathcal{F}, P) , Properties of P including Sub-additivity.

Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on (Ω, \mathcal{F}, P) definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of \mathbb{R} , Absolutely continuous random variable. Function of a random variable; Result on a random variable R with distribution function F to be absolutely continuous, Assume F is continuous everywhere and has a continuous derivative at all points except possibly at finite number of points, Result on density function f_2 of R_2 where $R_2 = g(R_1)$, h_j is inverse of g over a suitable subinterval

$$f_2(y) = \sum_{i=1}^n f_1(h_j(y)) |h'_j(y)| \text{ under suitable conditions.}$$

Reference for Unit 1, sections 1.1-1.6, 2.1-2.5 of *Basic Probability theory* by ROBERT ASH, Dover Publication, 2008.

Unit II: Properties of Distribution function, Joint Density function (15 lectures)

Properties of distribution function F , F is non-decreasing, $\lim_{x \rightarrow \infty} F(x) = 1$, $\lim_{x \rightarrow -\infty} F(x) = 0$, Right continuity of F , $\lim_{x \rightarrow x_0} F(x) = P(\{R < x_0\})$, $P(\{R = x_0\}) = F(x_0) - F(x_0^-)$.

Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and k -th moments of a random variable with properties.

Reference for Unit II: Sections 2.5-2.7, 2.9, 3.2-3.3, 3.6 of *Basic Probability theory* by ROBERT ASH, Dover Publication, 2008.

Unit III: Weak Law of Large Numbers (15 lectures)

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient ρ , Result on ρ as a measure of linear dependence, $\text{Var}(\sum_{i=1}^n R_i) = \sum_{i=1}^n \text{var}(R_i) + 2 \sum_{1 \leq i < j \leq n} \text{Cov}(R_i, R_j)$, Method of Indicators to find expectation of a random variable, Chebyshev's Inequality, Weak law of Large numbers.

Reference for Unit III, Sections 3.4, 3.5, 3.7, 4.1-4.4 of *Basic Probability theory* by ROBERT ASH, Dover Publication, 2008

Additional Reference Books:

M. CAPINSKI, *Probability through Problems*, Springer.

USMTP05, UAMTP05

Practicals for USMT501/UAMT501, USMT502/UAMT502 & USMT503/UAMT503

A. Practical for USMT501/UAMT501:

1. Problems based on counting principles, Two way counting.
2. Stirling numbers of second kind, Pigeon hole principle.
3. Multinomial theorem, identities, permutation and combination of multi-set.
4. Inclusion-Exclusion principle, Euler phi function.
5. Derangement and rank signature of permutation.
6. Recurrence relation.
7. Miscellaneous Theoretical Questions based on full paper.

B. Practical for USMT502/UAMT502:

1. Quotient spaces.
2. Orthogonal transformations, Isometries.
3. Eigenvalues, eigenvectors of $n \times n$ matrices over \mathbb{R}, \mathbb{C} ($n = 2, 3$).
4. Diagonalization.
5. Orthogonal diagonalization.
6. Miscellaneous Theoretical Questions based on full paper.

C. Practical for USMT503/UAMT503:

1. Metric spaces and normed linear spaces, Examples.
2. Open balls, open sets in metric spaces, subspaces and normed linear spaces.
3. Limit points and closure points, closed balls, closed sets, closure of a set, boundary of a set, distance between two sets.
4. Cauchy Sequences, completeness.
5. Continuity.
6. Uniform continuity in a metric space.
7. Miscellaneous Theoretical Questions based on full paper.

USMTPJ5, UAMTPJ5: Projects

A student can submit a project which shall have 20-30 typed pages, on one of the following topics:

1. Computer implementation of rational numbers in python or C++:
R.G. Dromey, *How to Solve it by Compute*, Pearson Education.
2. Various Sorting Algorithms like merge sort, insertion sort, quick sort, heap sort, bucket sort, radix sort:
R.G. Dromey, *How to Solve it by Computer*, Pearson Education.
3. Algorithms: Integer knapsack problem, fractional knapsack problem, back-tracking algorithm for the n-queens problem:
-R.G. Dromey, *How to Solve it by Computer*, Pearson Education.
4. Normalization in databases:
Jeffrey D. Ullman, *Principles of Database and Knowledge-base Systems*, Volume 1.

5. Vector Fields, Integral curves, Phase flows in the plane:
V.I. Arnold, *Ordinary Differential Equations*, PHI.
6. Eigenvalues, Eigenfunctions of the vibrating string and Applications to the Heat Equation, Dirichlet problem for the circle:
G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.
7. Bessel Functions and the vibrating membrane:
G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.
8. Sturm-Liouville Bounday value problems, Eigenvalues, Eigenfunctions:
G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

9. Continued Fractions and applications to irrational numbers:
H.S. Zuckerman and I. Niven, *An Introduction to the Theory of Numbers*, Wiley Eastern Ltd.
10. Distribution of primes:
H.S. Zuckerman and I. Niven, *An Introduction to the Theory of Numbers*, Wiley Eastern Ltd.
11. Prime Number theorem, Zeta function.
D.M. Burton, *Elementary Number Theory*, Tata McGraw-Hill.
12. Transcendental and algebraic numbers, Transcendence of e , Irrationality of π and e :
I.N. Herstein, *Topics in Algebra*, Wiley India Pvt. Limited.
13. The real numbers-a survey of constructions:
<https://arxiv.org/pdf/1506.03467>

14. Fourier series of circular functions and applications to Series of real numbers:
R.R Goldberg, *Methods of Real Analysis*, Oxford IBM Publications.
15. Fourier series, Orthogonal Functions, Dirichlet's problem:
G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.
16. Pointwise convergence of Fourier series, the Gibbs Phenomenon:
R. Bhatia, *Fourier series*, Hindustan Book Agency.
17. Cesaro summability and Fejer's theorem:
R. Bhatia, *Fourier series*, Hindustan Book Agency.
18. Construction of everywhere continuous but no-where differentiable functions:
R.R Goldberg, *Methods of Real Analysis*, Oxford IBM Publications.
19. Study of uniform convergence of various sequences of real valued continuous functions and plotting the functions of the sequences.
R.R Goldberg, *Methods of Real Analysis*, Oxford IBM Publications.
20. Henstock Kurzweil integration:
R.G. Bartle and D. Sherbert, *Introduction to Real Analysis*, Wiley India Pvt. Ltd.

21. Symmetric matrices, Spectral theorem, quadratic forms in n -variables:
M. Artin, *Algebra*, Prentice Hall of India.
22. Classification of Isometries of \mathbb{R}^2 :
M. Artin, *Algebra*, Prentice Hall of India.
23. Discrete subgroups of isometries of the plane:
M. Artin, *Algebra*, Prentice Hall of India.

24. Surface integrals, Line integrals, Theorem on Curl, Divergence theorem of Gauss:
T.M. Apostol, *Calculus*, Volume II, Wiley India Pvt. Limited.
25. Parametrised regular surfaces in \mathbb{R}^3 , tangent spaces, Orientable surfaces:
T.M. Apostol, *Calculus*, Volume II, Wiley India Pvt. Limited.
26. Applications of WX-Maxima plot graphs of surfaces, tangent vectors, level sets of real valued functions $f(x, y, z)$.

27. Circular Permutations, Study of Sterling numbers of First Kind:
K.H. Rosen, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi.
28. Generating Functions and its applications (Counting, Solving Differential Equations): K.H. Rosen, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi.
29. Recurrence relations and applications:
K.H. Rosen, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi.

30. Forbidden position problems:
K.H. Rose, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi.
31. Applications of Pigeon Hole Principle:
K.H. Rosen, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi.
32. Basic Logic, Poset and Lattices:
 - a) K. H. Rosen, *Discrete Mathematics and its Applications* (Sixth edition), Tata McGraw Hill Publishing Company, New Delhi(Chapter 1).
 - b) V. K. Khanna, *Lattices and Boolean Algebras- First Concepts*, Vikas Publishing House Pvt Ltd (Chapter 2).
33. Boolean algebra (Lattices and Algebraic Systems):
C.U. Liu and D.P. Mahapatra, *Discrete mathematics*, McGraw Hill.
34. Algorithms in Cryptography:
 - a) Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Edition, McGraw Hill, 2012.
 - b) Douglas R. Stinson, *Cryptography Theory and Practice*, 3rd Edition, 2005.
35. Berge Vieta and Bairstow Method, proofs and programming implementation:
S.S. Sastry, *Numerical Methods: For Scientific and Engineering Computation*, New Age International Publishers. See also M.K.Jain,S.R.K.Iyengar & R.K.Jain, *Numerical Methods*.
36. Jordan Rational Form, Algorithmic proofs and computations:
D. S. Dummit and R.M. Foote, *Abstract Algebra*, Wiley India Pvt. Limited.
37. Homogeneous coordinates, transformations and computer geometry:
Computer Graphics (Special Indian Edition) (Schaum's Outline Series) 2nd Edition.
38. Bezier curves, B-splines implementation and definition:
S.S. Sastry, *Introductory Methods of Numerical Analysis*, Prentice hall India.
39. Number systems in various bases:
- H. M. Antia, *Numerical Methods for Scientists and Engineers*, American Mathematical Society, 2012.
40. Financial Mathematics (Theory of interest rates and Discounted cash flow):
 - a) Mc Cutch eon and Scot Heinemann, *An introduction to the Mathematics of Finance*, Professional publishing.
 - b) Sheldon M.Ross, *An Introduction to Mathematical Finance*, Cambridge University Press.
41. Financial Mathematics (Valuation of securities, Cumulative Sinking Funds):
 - a) Mc Cutch eon and Scot Heinemann, *An introduction to the Mathematics of Finance*, Professional publishing.

- b) Sheldon M. Ross, *An Introduction to Mathematical Finance*, Cambridge University Press.
42. Mathematical Economics (Demand and Supply Analysis, Cost and Revenue Functions, Theory of Consumer Behaviour):
H.L. Ahuja, *Principles of Micro Economics*, 15th Revised Edition, S. Chand.
43. Basic Statistics (Correlation and Regression):
- G. Gupta and D. Gupta, *Fundamentals of Statistics*, Vol. 1, The World Press Pvt. Ltd., Kolkata.
 - Gupta and Kapoor, *Fundamentals of Mathematical Statistics*, Sultan Chand and Sons, New Delhi.
 - Hogg, R. V. and Craig R. G., *Introduction to Mathematical Statistics*, fourth Edition, MacMillan Publishing Co., New York.
44. Implementing Statistical methods using R:
W. N. Venables, D.M. Smith and the R Development Core Team, *An Introduction to R, Notes on R: A Programming Environment for Data Analysis and Graphics*, Version 3.0.1 (2013-05-16), (URL: <https://cran.r-project.org/doc/manuals/r-release/R-intro.pdf>).
45. Social Network Analysis:
R. A. Hanneman, M. Riddle, *Introduction to Social Network Methods*, University of California, 2005 (Published in digital form and available at <http://faculty.ucr.edu/hanneman/nettext/index.html>).
46. Basics of R programming:
W. N. Venables, D.M. Smith and the R Development Core Team, *Notes on R: A Programming Environment for Data Analysis and Graphics*, Version 3.0.1 (2013-05-16), (URL: <https://cran.r-project.org/doc/manuals/r-release/R-intro.pdf>).
47. Topics in Data Sciences:
C. O'Neil, R. Schutt and O'Reilly, *Doing Data Science, Straight Talk From The Frontline*, 2014.

SEMESTER VI

USMT601/UAMT601 Real and Complex Analysis

Unit I: Sequence and series of functions (15 Lectures)

Sequence of real valued functions, pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true.

Series of functions, convergence of a series of functions, Weierstrass M-test. Examples.

Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence

of functions converge to the integral and derivative of uniform limit on a closed and bounded interval, examples. Consequences of these properties for series of functions, term by term differentiation and integration.

$\liminf_{n \rightarrow \infty} x_n$ & $\limsup_{n \rightarrow \infty} x_n$ for a bounded sequence $(x_n)_{n \in \mathbb{N}}$ of \mathbb{R} .

Properties of $\limsup_{n \rightarrow \infty} x_n =: x^*$:

1. \exists a subsequence $(x_{n_k})_{k \in \mathbb{N}}$ of the sequence (x_n) such that $x_{n_k} \rightarrow x^*$.
2. If $x > x^*$, then $\exists n_0 \in \mathbb{N}$ such that $x_n \leq x \forall n \geq n_0$.

Power series in \mathbb{R} centered at origin and at some point x_0 in \mathbb{R} , radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Reference for Unit I:

1. R.R. GOLDBERG, *Methods of Real Analysis*, Oxford and International Book House (IBH) Publishers, New Delhi.
2. W. RUDIN, *Principles of mathematical Analysis*, Tata McGraw- Hill Education in 2013.
3. AJIT KUMAR, S. KUMARESAN, *Introduction to Real Analysis*, CRC Press.

Unit II: Introduction to Complex Analysis (15 Lectures)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivres formula, \mathbb{C} as a metric space, bounded and unbounded subsets of \mathbb{C} , point at infinity and the extended complex plane, sketching of set in complex plane. (No question be asked).

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions $f : \mathbb{C} \rightarrow \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions.

Derivative of $f : \mathbb{C} \rightarrow \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic functions. If f, g are complex analytic then $f + g, f - g, fg$ and f/g are analytic.

Theorem: If $f'(z) = 0$ everywhere in a domain D , then $f(z)$ must be constant throughout D . Harmonic functions and harmonic conjugate.

Reference for Unit II:

Sections 5,6, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 of J. W. BROWN AND R. V. CHURCHILL, *Complex variables and applications*, McGraw-Hill International, sixth edition.

Unit III: Complex power series (15 Lectures)

Contour integral $\int_C f(z)dz$ over a contour C , the contour integral $\int_C f(z)dz$ where C is the circle $|z - z_0| = r$ in \mathbb{C} .

Cauchy-Goursat theorem (statement only).

Principle of deformation of paths (statement only): Let C_1, C_2 denote positively

oriented circles where C_2 is interior to C_1 . If a function f is analytic in the closed region consisting of those contours and all points between them, then $\int_{C_1} f dz = \int_{C_2} f dz$.

Cauchy integral formula (with proof): If $f : \mathbb{C} \rightarrow \mathbb{C}$ is an analytic function, then $f(w) = \frac{1}{2\pi i} \int_C \frac{f(z) dz}{z - w}$ ($w \in B(z_0, r)$) where C is the circle $|z - z_0| = r$ taken in the positive sense.

Taylor's theorem (with proof) for an analytic function.

Mobius transformations, examples.

Exponential function and its properties (without proof), trigonometric functions, hyperbolic functions.

Power series of complex numbers and related results following from Unit I, radius of convergence of a power series, disc of convergence of a power series, uniqueness of series representation, examples.

Definition of Laurent series, definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, Cauchy's residue theorem (statement only), calculation of residue.

Reference for Unit III:

Sections 23, 24, 25, 30, 31, 32, 33, 39, 44, 45, 46, 47, 49, 50, 53, 54, 55, 56 of J. W. BROWN AND R. V. CHURCHILL, *Complex variables and applications*, McGraw-Hill International, sixth edition. Define residue of a function at a pole using Theorem in section 56. Statement of Cauchy's residue theorem in section 54.

Additional Reference Books:

1. T. APOSTOL, *Mathematical Analysis*, Narosa.
2. M. H. PROTTER AND C. B. MORREY JR., *Intermediate Calculus*.
3. T. W. GAMELIN, *Complex analysis*.
4. R. COURANT AND F. JOHN, *Introduction to Calculus and Analysis*, Vol.2.
5. W. FLEMING, *Functions of Several Variables*.
6. D. V. WIDDER, *Advanced Calculus*, Dover Pub., New York.
7. S. R. GHORPADE AND B. LIMAYE, *A course in Multivariable Calculus and Analysis*.
8. G.B. THOMAS AND R.L FINNEY, *Calculus and Analytic Geometry*.
9. R.E. GREENE AND S.G. KRANTZ, *Function theory of one complex variable*.

USMT602/UAMT602 Algebra VI

Unit I: Normal Subgroups (15 Lectures)

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite

groups, Cyclic groups, The Center $Z(G)$ of a group G , Cosets, Lagranges theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms.

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n , Cycles. List of all normal subgroups of A_4, S_3 .

First Isomorphism theorem (Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem.

Cayleys theorem (statement only). External direct product of a group, properties of external direct products, order of an element in a direct product, criterion for direct product to be cyclic. The classification of groups of order upto 7.

References for unit I:

1. N. HERSTEIN, *Topics in Algebra*, Wiley india Pvt. Ltd, 2015.
2. M. ARTIN, *Algebra*, Pearson India, Fifth Edition, 2017.

Unit II: Ring Theory (15 Lectures)

Definition of a ring (the definition should include the existence of a unity element). Properties and examples of rings including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Q}[X], \mathbb{R}[X], \mathbb{C}[X], \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{5}], \mathbb{Z}_n$.

Commutative rings. Units in a ring. The multiplicative group of units of a ring.

Characteristic of a ring.

Ring homomorphisms. First Isomorphism theorem of rings.

Ideals in a ring, sum and product of ideals in a commutative ring.

Quotient rings. Integral domains and fields. Definition and examples. A finite integral domain is a field. Characteristic of an integral domain, and of a finite field. Construction of quotient field of an integral domain (emphasis on \mathbb{Z}, \mathbb{Q}). A field contains a subfield isomorphic to \mathbb{Z}_p or \mathbb{Q} .

References for Unit II:

1. M. ARTIN, *Algebra*, Pearson India, Fifth Edition, 2017.
2. N.S. GOPALKRISHNAN, *University Algebra*, New Age International, third edition, 2015.

Unit III: Factorisation (15 Lectures)

Prime ideals and maximal ideals. Definition and examples. Characterization in terms of quotient rings.

Polynomial rings. Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field (statement only). Divisibility in an integral domain, irreducible and prime elements, ideals generated by prime and irreducible elements.

Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique

Factor-ization Domain (UFD). Examples of ED including \mathbb{Z} , $F[X]$ where F is a field, and $\mathbb{Z}[i]$. An ED is a PID, a PID is a UFD.

Prime (irreducible) elements in $\mathbb{R}[X]$, $\mathbb{Q}[X]$, $\mathbb{Z}_p[X]$. Prime and maximal ideals in polynomial rings. $\mathbb{Z}[X]$ is not a PID. $\mathbb{Z}[X]$ is a UFD (Statement only).

Reference for Unit III:

1. M. ARTIN, *Algebra*, Pearson India, Fifth Edition, 2017.
2. N.S. GOPALKRISHNAN, *University Algebra*, New Age International, third edition, 2015.

Additional Reference Books:

1. P. B. BHATTACHARYA, S. K. JAIN, AND S. R. NAGPAUL, *Abstract Algebra*, Cambridge University Press, 1995.
2. J. B. FRALEIGH, *A First course in Abstract Algebra*, Narosa.
3. D. DUMMIT AND R. FOOTE *Abstract Algebra*, John Wiley & Sons, Inc.

USMT603/UAMT603 Metric Topology

All concepts have to be taught with plenty of examples and worked out in special case of Euclidean space, Complex plane and other metric spaces.

Unit I. Complete metric spaces (15 Lectures)

Convergent sequences, Cauchy's principle of convergence, convergent Cauchy sequences, Complete metric spaces. Completeness property in subspaces of a complete metric space: Any closed subset of a complete metric space is complete.

Cantor's intersection theorem. Examples of Complete metric spaces: \mathbb{R} , \mathbb{R}^n , $\mathcal{C}[a, b]$. If X, Y are complete metric spaces with metrics d_1, d_2 respectively, then $X \times Y$ is complete with metric $d((x_1, y_1), (x_2, y_2)) = \sqrt{d_1(x_1, x_2)^2 + d_2(y_1, y_2)^2}$.

Reference for unit I:

S. KUMARESAN, *Topology of Metric spaces*, Narosa.

Unit II: Compact metric spaces:

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover), examples. Properties such as: i) Continuous image of a compact set is compact, ii) Compact subsets of a metric space are closed and bounded, iii) A continuous function on a compact set is uniformly continuous.

Compactness and finite intersection property: A metric space X is compact if and only if for every infinite family $\{F_\alpha : \alpha \in S\}$ of closed subsets of X with finite intersection property, $\bigcap_{\alpha \in S} F_\alpha$ is not empty. Every infinite, bounded subset of a compact metric space has an accumulation point (cluster point). A compact metric space is complete.

Characterization of compact sets in \mathbb{R}^n :

The following are equivalent statements for a subset of \mathbb{R}^n to compact:

1. Heine-Borel property.

2. Closed and boundedness property.
3. Bolzano-Weierstrass property.
4. Sequentially compactness property.

Reference for Unit II:

1. S. KUMARESAN, *Topology of Metric spaces*, Narosa.
2. W. RUDIN, *Principles of Mathematical Analysis*, Tata McGraw- Hill Education in 2013..

Unit III. Connected sets (15 lectures)

Connected metric spaces (a metric space which can not be represented as the union two disjoint non-empty open subsets). Characterization of a connected space, namely a metric space X is connected if and only if every continuous function from X to the discrete metric space $\{1, 1\}$ is a constant function. Connected subsets of a metric space, connected subsets of \mathbb{R} are intervals. A continuous image of a connected set is connected, applications such as : i) $GL(2, \mathbb{R}), O(n, \mathbb{R})$ are not connected, ii) graph of a real valued continuous function defined on an interval is a connected subset of \mathbb{R}^2 .

For A, B be two connected subsets of a metric space X , i) $A \cap B \neq \emptyset$ implies $A \cup B$ is connected , ii) $A \subset B \subset \bar{A}$ implies B is connected. Circle S^1 is a connected subset of \mathbb{R}^2 .

Definition of a path connected metric space, examples including $\mathbb{R}^n, S^n (n \in \mathbb{N})$. A path connected metric space is connected and applications including connectedness of $\mathbb{R}^n, \mathbb{C}^n$. An example of a connected subset of \mathbb{R}^2 which is not path connected (proof not required). An open subset of \mathbb{R}^n is connected if and only if it is path-connected ((proof not required)).

Reference for Unit III:

1. S. KUMARESAN, *Topology of Metric spaces*, Narosa.
2. G.F. SIMMONS, *Introduction to Topology and Modern Analysis*, McGraw-Hill Education (India), 2004.

Recommended Text Books:

1. S. KUMARESAN, *Topology of Metric spaces*, Narosa.
2. G.F. SIMMONS, *Introduction to Topology and Modern Analysis*, McGraw-Hill Education (India), 2004.
3. IRVIN KAPLANSKY, *Set Theory and Metric spaces*, Allyn and Bacon Inc, Boston.

Additional Reference Books:

1. MÍCHEÁL Ó SEARCÓID, *Metric spaces*, Springer Undergraduate Mathematics Series, 2007.
2. R.G. GOLDBERG, *Methods of Real Analysis*, Oxford and IBH Publishing House, New Delhi.

USMT6A4/UAMT6A4 Numerical Analysis II (Elective A)

N.B. Derivations and geometrical interpretation of all numerical methods with the-
orem mentioned have to be covered.

Unit I: Interpolation (15 Lectures)

Interpolating polynomials, uniqueness of interpolating polynomials. Linear, Quadratic and higher order interpolation. Lagranges Interpolation.

Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Interpolating polynomials using finite differences: Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirlings Interpolation. Results on interpolation error.

Unit II: Polynomial Approximations and Numerical Differentiation (15 Lectures)

Piecewise Interpolation: Linear, Quadratic and Cubic. Bivariate Interpolation: Lagranges Bivariate Interpolation, Newtons Bivariate Interpolation.

Numerical differentiation: Numerical differentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III: Numerical Integration (15 Lectures)

Numerical Integration based on Interpolation: Newton-Cotes Methods, Trapezoidal rule, Simpsons 1/3-rd rule, Simpsons 3/8-th rule. Determination of error term for all above methods. Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods: Trapezoidal rule, Simpsons rule.

Recommended Text Books:

1. E. KENDALL AND ATKINSON, *An Introduction to Numerical Analysis*, Wiley.
2. M. K. JAIN, S. R. K. IYENGAR AND R. K. JAIN, *Numerical Methods for Scientific and Engineering Computation*, New Age International Publications.
3. S.D. COMTE AND CARL DE BOOR, *Elementary Numerical Analysis, an Algorithmic Approach*, McGraw Hill International Book Company.
4. S. SASTRY, *Introductory methods of Numerical Analysis*, PHI Learning.
5. F.B. HILDEBRAND, *Introduction to Numerical Analysis*, Dover Publication, NY.
6. J.B. SCARBOROUGH, *Numerical Mathematical Analysis*, Oxford University Press, New Delhi.

USMT6B4/UAMT6B4

Number Theory and its applications II (Elective B)

Unit I: Quadratic Reciprocity (15 Lectures)

Quadratic residues and Legendre symbol, Gauss Lemma, Theorem on Legendre symbol $\left(\frac{2}{p}\right)$, the result: If p is an odd prime and a is an odd integer, then

$\left(\frac{a}{p}\right) = (-1)^t$, where $t = \sum_{k=1}^{(p-1)/2} \left[\frac{ka}{p}\right]$, Quadratic Reciprocity law. Theorem on

Legendre Symbol $\left(\frac{3}{p}\right)$. The Jacobi symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II: Continued Fractions (15 Lectures)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III. Pell's equation, Arithmetic function and Special numbers (15 Lectures)

Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted).

Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$), $\sigma(n)$, $\sigma_k(n)$, $\omega(n)$ and their properties, $\mu(n)$ and the Mbius inversion formula.

Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Recommended Text Books:

1. N.H. ZUCKERMAN AND H. MONTGOMERY, *An Introduction to the Theory of Numbers*, John Wiley & Sons.
2. D.M. BURTON, *An Introduction to the Theory of Numbers*, Tata McGraw Hill.

Additional Reference Books:

1. G. H. HARDY AND E.M. WRIGHT, *An Introduction to the Theory of Numbers*.
2. NEVILLE ROBINS, *Beginning Number Theory*, Narosa Publications.
3. S. D. ADHIKARI, *An introduction to Commutative Algebra and Number Theory*.
4. N. KOBLITZ, *A course in Number theory and Cryptography*, Springer.
5. M. ARTIN, *Algebra*, Prentice Hall.
6. K.IRELAND AND M. ROSEN, *A classical introduction to Modern Number Theory*.
7. W. STALLING, *Cryptology and network security*.

USMT6C4/UAMT6C4

Graph Theory and Combinatorics (Elective C)

Unit I. Colorings of graph (15 Lectures)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem.

Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and Edge cuts vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II. Planar graphs (15 Lectures)

Definition of a planar graph. Euler formula and its consequences. Non planarity of K_5 , $K(3;3)$. Dual of a graph. Polyhedrons in \mathbb{R}^3 and existence of exactly five regular polyhedra- (Platonic solids).

Colorability of planar graphs-5 color theorem for planar graphs, statement of 4 color theorem.

Networks and flow and cut in a network- value of a flow and the capacity of cut in a network, relation between flow and cut. Maximal flow and minimal cut in a network and Ford- Fulkerson theorem.

Unit III: Combinatorics (15 Lectures)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems.

Introduction to partial fractions and using Newtons binomial theorem for real power find series expansion of some standard functions.

Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR.

Introduction to matching, M alternating and M augmenting path, Berge theorem. Bipartite graphs.

Recommended Text Books:

1. J. A. BONDY AND U.S.R. MURTY, *Graph Theory with Applications*, Springer, 2008.
2. R. BALKRISHNAN AND K. RANGANATHAN, *Graph theory and applications*, North Holland, 1982.
3. D.G. WEST, *Introduction to Graph theory*, Pearson Modern Classics.

4. R. BRUALDI, *Introduction to Combinatorics*, Pearson Education.

Additional Reference Books:

1. M. BEHZAD AND G. CHARTRAND, *Introduction to the theory of Graphs*, Allyn and Bacon.
2. S.A. CHOUDAM, *A First course in Graph Theory*, Macmillam India Ltd.

USMT6D4/UAMT6D4 Operations Research (Elective D)

Unit I: Linear Programming-I (15 Lectures)

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.

Formation of LPP, Graphical Method. Theory of the Simplex Method- Standard form of LPP, Feasible solution to basic feasible solution, Improving BFS, Optimality Condition, Unbounded solution, Alternative optima, Correspondence between BFS and extreme points. Simplex Method Simplex Algorithm, Simplex Tableau.

Reference for Unit-I:

G. HADLEY, *Linear Programming*, Narosa Publishing.

Unit II: Linear programming-II (15 Lectures)

Simplex Method Case of Degeneracy, Big-M Method, Infeasible solution, Alternate solution, Solution of LPP for unrestricted variable. Transportation Problem: Formation of TP, Concepts of solution, feasible solution, Finding Initial Basic Feasible Solution by North West Corner Method, Matrix Minima Method, Vogels Approximation Method. Optimal Solution by MODI method, Unbalanced and maximization type of TP.

Reference for Unit-II:

1. G. HADLEY, *Linear Programming*, Narosa Publishing.
2. J. K. SHARMA, *Operations Research, Theory and Applications*.

Unit III: Queuing Systems (15 Lectures)

Elements of Queuing Model, Role of Exponential Distribution. Pure Birth and Death Models; Generalized Poisson Queuing Mode. Specialized Poisson Queues: Steady-state Measures of Performance, Single Server Models, Multiple Server Models, Self-service Model, Machine-servicing Model.

Reference for Unit III:

1. J. K. SHARMA, *Operations Research, Theory and Applications*.
2. H. A. TAHA, *Operations Research*, Prentice Hall of India.

Additional Reference Books:

1. HILLIER AND LIEBERMAN, *Introduction to Operations Research*.
2. R. BROSON, *Schaum Series Book in Operations Research*, Tata McGraw Hill Publishing Company Ltd.

USMT607, UAMT607

Practicals for USMT601/UAMT601, USMT602/UAMT602 & USMT603/UAMT603

A. Practical for USMT601/UAMT601:

1. Pointwise and uniform convergence of sequence functions, properties.
2. Point wise and uniform convergence of series of functions and properties.
3. Analytic function, finding harmonic conjugate, Mobius transformations.
4. Cauchy integral formula, Taylor series, power series.
5. Limit continuity and derivatives of functions of complex variables.
6. Finding isolated singularities- removable, pole and essential, Laurent series, calculation of residue.
7. Miscellaneous theory questions based on full paper (3 theory questions from each unit).

B. Practicals for USMT602/UAMT602:

1. Normal Subgroups and quotient groups.
2. Cayleys Theorem and external direct product of groups.
3. Rings, Ring Homomorphism and Isomorphism.
4. Ideals, Prime Ideals and Maximal Ideals.
5. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
6. Fields.
7. Miscellaneous theory questions based on full paper.

C. Practicals for USMT603/UAMT603:

1. Completeness of \mathbb{R}, \mathbb{R}^n .
2. A metric space X is complete if and only if every closed ball of X is complete.
3. Compact sets in a metric space, Compactness in \mathbb{R}^n (emphasis on \mathbb{R}, \mathbb{R}^2), properties.
4. Continuous image of a compact set.
5. Example of a closed and bounded subset of a metric space which is not compact.
6. Connectedness, Path connectedness.
7. Continuous image of a connected set.
8. Miscellaneous Theoretical Questions based on full paper.

USMTPJ6, UAMTPJ6: Projects

A student can submit a project which shall have 20-30 typed pages, on one of the following topics:

1. Apps for small devices using Python:
Chapter 7 of *Head First Python* by Paul Barry, O'Reilly Media, second edition.
2. Apps for small devices using Java:
Java How to Program (early objects) by Paul Deitel and Harvey Deitel, Pearson 9th edition (2012).
3. Elliptic Curves and their uses in Cryptography, Pollard's Algorithm:
D. Hankerson, A.J. Menezes, S. Vanstone, *Guide to Elliptic Curve Cryptography*, Springer.
4. Runge-Kutta methods, principle and proofs of second and fourth order computer programs:
S.S. Sastry, *Introductory Methods of Numerical Analysis*, Prentice hall India.

5. The matrix exponential and applications to system of Differential equations $X' = AX$:
M. Artin, *Algebra*, Pearson India Education.
6. Iterated solutions of Picard's theorem and solutions of second order linear ODE:
M. W. Hirsch, S. Smale and R.L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Academic Press.
7. The Qualitative properties of the solutions of $y'' + P(x)y' + Q(x)y = 0$, Sturm Separation theorem:
G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.
8. Bessel Functions, The Gamma Function and the general solution of bessel's equation: G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

9. Algebraic numbers, algebraic integers:
H.S. Zuckerman and I. Niven, *An Introduction to the Theory of Numbers*, Wiley Eastern Ltd.
10. Quadratic fields ,units, primes, UFD:
H.S. Zuckerman and I. Niven, *An Introduction to the Theory of Numbers*, Wiley Eastern Ltd.

11. Structure of finite Abelian groups:
S. Lang, *Algebra*, Springer.

12. The Class Equation, Application to p -groups:
M. Artin, *Algebra*, Prentice Hall of India.
13. The Class Equation of Icosahedral group.:
M. Artin, *Algebra*, Prentice Hall of India.
14. The Class Equation, classification of groups of order 12:
M. Artin, *Algebra*, Prentice Hall of India.
15. Construction of numbers by Ruler & Compass:
D. S. Dummit and R.M. Foote, *Abstract Algebra*, Wiley India Pvt. Limited.
16. Field Extensions, Cubic equations, Cardano's method:
D. S. Dummit and R.M. Foote, *Abstract Algebra*, Wiley India Pvt. Limited.
17. Character groups of small Order:
D. S. Dummit and R.M. Foote, *Abstract Algebra*, Wiley India Pvt. Limited.
18. Finite Division ring is a feild and sum of two squares:
I.N. Herstein, *Topics in Algebra*, Wiley India Pvt. Limited.

19. Study of Polya theory of Counting:
K.H. Rosen, *Discrete Mathematics and its Applications*, Tata McGraw Hill Publishing Company, New Delhi,(Sixth edition).
20. Hall's Marriage Theorem, Graph theory & Applications:
K.H. Rosen, *Discrete Mathematics and its Applications*, Tata McGraw Hill Publishing Company, New Delhi,(Sixth edition).
21. Ramsey numbers:
K.H. Rosen, *Discrete Mathematics and its Applications*, Tata McGraw Hill Publishing Company, New Delhi,(Sixth edition).
22. Axiom of choice, Zorn's Lemma:
Set theory related topics, Schaum series. See also S. Lang, Analysis II.
23. Introduction to Cryptography:
 - a) Kenneth H. Rosen, Discrete Mathematics and Its Applications, 7th Edition, McGraw Hill, 2012.
 - b) Cryptography Theory and Practice, 3rd Edition, Douglas R. Stinson, 2005.

24. Separable metric spaces, study of completions of $\mathcal{C}[a, b]$ under norms such as sup-norm, L^1 -norm, L^2 -norm:
R.R Goldberg, *Methods of Real Analysis*, Oxford IBM Publications.
25. Study of Baire spaces and application to limit of a sequence of real valued continuous functions defined on \mathbb{R} :
J. R. Munkres, *Topology*, Pearson Education India.
26. Completion of Metric spaces:
J. R. Munkres, *Topology*, Pearson Education India.

27. Maximum principle for analytic functions and applications:
Lars Ahlfors, *Complex Analysis*, McGraw Hill Education (India) Private Limited, 2013.

28. Exponential function e^z , Epimorphism Theorem, $e^{ix} = c(x) + is(x)$, study of circular functions $c(x), s(x)$ and identification with Trigonometric functions:
R. Remmert, *Classical Topics in Complex Function Theory*, Springer.
29. Plotting regions under Mobius Transformations:
S. Lang, *Complex Analysis*, Springer.
30. Study of Mobius Transformations, cross-ratio, Applications:
J.B. Conway, *Functions of One Complex Variable I*, Mcgraw Hill.
31. Radius of Convergence of power series, Abel's Limit theorem and applications:
T.M Apostol, *Mathematical Analysis*, Narosa.
32. Dixon's proof of Cauchy's theorem (Homology version) and deduction of simply connected version:
S. Lang, *Complex analysis*, Springer. see also: Lars Ahlfors, *Complex Analysis*, McGraw Hill Education (India) Private Limited.

33. Classification of Isometries of \mathbb{R}^3 :
J.T. Smith, *Methods of geometry*.
34. Discrete subgroups of isometries of the plane:
M. Artin, *Algebra*, Prentice Hall of India.
35. Tiling and Crystallographic groups:
M. Berger, *Geometry I*, Springer.
36. Group of symmetries of regular polyhedra:
R. Hartshorne, *Geometry: Euclid and Beyond*, Springer.
37. Jordan blocks of matrices and application to solving a system of linear ODE:
M. W. Hirsch, S. Smale and R.L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Academic Press.

38. Alternating k -tensors on a finite dimensional real vector space, orientation and volume elements and theory of differential forms:
M. Spivak, *Calculus on manifolds*, W.A. Benzamin Inc.
39. Differential forms, Stokes' theorem and applications to Green's theorem, Gauss' Divergence theorem:
M. Spivak, *Calculus on manifolds*, W.A. Benzamin Inc.

40. Locally compact metric spaces, One-Point Compactification, One-Point compactification of \mathbb{C} is homeomorphic to the unit sphere $S^2 \subset \mathbb{R}^3$:
J. R. Munkres, *Topology*, Pearson Education India.
41. First Fundamental Groups of a metric space, computation of $\pi_1(S^1, 1)$:
J. R. Munkres, *Topology*, Pearson Education India.
42. Covering spaces of a Metric space, examples:
J. R. Munkres, *Topology*, Pearson Education India.
43. Path-homotopy relation, Simply Connected metric Spaces, Examples:
J. R. Munkres, *Topology*, Pearson Education India.

44. Simplicial complexes in $\mathbb{R}^2, \mathbb{R}^3$ singular chains, Homology groups H_0, H_1 .
J. R. Munkres, *Elements of Algebraic Topology*, Addison-Wesley Publishing.
45. Homology groups:
John B. Fraleigh, *A First Course in Abstract Algebra*, Pearson Education, India.
46. A non-computational proof of Cayley-Hamilton theorem, canonical isomorphism with double dual Tensor products:
S. Lang, *Introduction to Linear Algebra*, Springer Verlag.
47. Topics in Projective Geometry:
R. Artzy, *Linear geometry*, Addison-Wesley.
48. Topics in Non-Euclidean Geometries:
R. Artzy, *Linear Geometry*, Dover Publications.
49. Special relativity:
W. H. Greub, *Linear Algebra*, Springer.
50. LU factorization using Gaussian elimination:
S.S. Sastry, *Numerical Methods: For Scientific and Engineering Computation*, New Age International Publishers.
51. Error estimates of proofs and implementation of Trapezoidal rule, simpson rule, Romberg method adaptive integration:
S.S Sastry, *Numerical methods: For Scientific and Engineering Computation*, New Age International Publishers.
52. Discrete Fourier transform, Fast Fourier transform:
S.S Sastry, *Numerical methods: For Scientific and Engineering Computation*, New Age International Publishers.
53. Topics in Automata Theory:
K.L.P. Mishra and N. Chandrasekaran, *Theory of Computer Science, Automata, Languages and Computation* (Third Edition), Prentice- Hall of India Pvt. Ltd.
54. Operations Research (Game Theory and Quality Control):
a) Kanti Swarup, P. K. Gupta and Man Mohan, *Operation Research*, Sultan Chand and Sons.
b) Taha, *Operations Research: introduction*, Pearson.
55. Operations Research (Integer L.P.P. and Inventory models):
a) P. K. Gupta, Man Mohan and Kanti Swarup, *Operation Research*, Sultan Chand and Sons.
b) Taha, *Operations Research: introduction*, Pearson.
56. Operations Research and Markov Chains:
a) Kanti Swarup, P. K. Gupta and Man Mohan, *Operation Research*, Sultan Chand and Sons.

- b) Taha, *Operations Research: introduction*, Pearson.
57. Discrete and Continuous probability distributions:
W.G. Cochran, *Sampling techniques*, third Edition, Wiley Eastern Ltd., New Delhi.

Scheme of Examination

I. Semester End Theory Examinations:

There shall be a Semester-end external Theory examination of 100 marks for all the courses of Semester V and VI- except for the two project courses USMTPJ5/UAMTPJ5, USMTPJ6/UAMTPJ6- to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.
2. Theory Question Paper Pattern:
 - a) There shall be FIVE questions. All the questions shall be compulsory.
The first question Q1 shall be of objective type for 20 marks based on the entire syllabus.
The next four questions Q2, Q3, Q4, Q5 shall be of 20 marks each.
The questions Q2, Q3, Q4 shall be based on the units I, II , III respectively.
The question Q5 shall be based on the entire syllabus.
 - b) The questions Q2,Q3,Q4,Q5 shall have internal choices within each question. Including the choices, the marks for each question shall be 30-32.
 - c) The questions Q2,Q3,Q4,Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
 - d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. Semester End Examinations Practicals:

There shall be a Semester-end practical examinations of three hours duration and 150 marks for each of the courses USMTP05/UAMTP05 of Semester V and USMTP06/UAMTP06 of semester VI.

In semester V, the Practical examinations for USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are conducted together by the college.

Similarly in semester VI, the Practical examinations for USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are conducted together by the college.

Question Paper pattern: The question paper shall have three parts A,B, C. Every part shall have three questions of 20 marks each. Students to attempt any two questions from each part.

For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT601/UAMT601, USMT602/UAMT602, and USMT603/UAMT603 marks for

journal and viva are as follows:

Journals | : 5 marks
 Viva | : 5 marks

Each Practical of every course of Semester V & VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal.

Practical Course	Part A	Part B	Part C	Marks out of	duration
USMTP05	Questions from USMT501 UAMT501	Questions from USMT502 UAMT502	Questions from USMT503 UAMT503	120	3 hours
USMTP06	Questions from USMT601 UAMT601	Questions from USMT602 UAMT602	Questions from USMT603 UAMT603	120	3 hours

III. Evaluation of Project work

(courses: USMTPJ5/UAMTPJ5 & USMTPJ6/UAMTPJ6):

The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the respective college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee.

The Marks for the project are detailed below:

Contents of the project | : 40 marks
 Presentation of the project | : 30 marks
 Viva of the project | : 30 marks.

Total Marks= 100 per project per student.
