

Institute of Distance and Open Learning
MA/MSc(Mathematics) Part II
Assignment 2018-19

Instructions:

- All questions to be written and submitted in the assignment sheet provided by IDOL
- Answers to all five papers' assignments to be submitted separately
- The last date of submission is Saturday 30th March 2019 before 5pm in Room No. 112, first floor, IDOL building, Kalina campus, Santacruz (E), Mumbai 400098

Paper I: Algebra and Field Theory

1. Define a solvable group. Show that subgroup of a solvable group is solvable.
2. Find the rational canonical form of the following matrices over \mathbb{Q} and determine whether they are similar: $A = \begin{pmatrix} 2 & -2 & 14 \\ 0 & 3 & -7 \\ 0 & 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & -4 & 85 \\ 1 & 4 & -30 \\ 0 & 0 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$
3. Show that if K is an algebraic extension of E and E is an algebraic extension of F then K is an algebraic extension of F .
4. Prove that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic.

Paper II: Advanced Analysis and Fourier Analysis

1. Let A be closed rectangle in \mathbb{R}^n . Prove that the function $\chi_C : A \rightarrow \mathbb{R}$ is integrable if and only if the boundary of C has measure zero.
2. State and prove Bounded Convergence Theorem
3. State and prove the Dirichlet's theorem.
4. Prove that $L^2[-\pi, \pi]$ is separable.

Paper III: Differential Geometry and Functional Analysis

1. For any $x, y \in V$, where V is an inner product space, show that $\|x - y\|^2 = \|x\|^2 + \|y\|^2$ if and only if x is orthogonal to y .
2. Prove or disprove: The unit sphere is a regular surface.
3. State and prove the lemma of Riesz.
4. Prove that the dual space of l^1 is l^∞ .

Paper IV: Numerical Analysis

1. Perform two iterations of the Newton-Raphson method to solve the following system of non-linear equations: $4x^2 + 2xy + y^2 = 30$ and $2x^2 + 3xy + y^2 = 3$. Use initial approximation $x_0 = -3$ and $y_0 = 2$.
2. Use Newton's divided difference formula to find the fourth degree curve passing through the points $(-4, 1245)$, $(-1, 33)$, $(0, 5)$, $(2, 9)$ and $(5, 1335)$.
3. Obtain the least squares quadratic approximation to the function $y(x) = \sin x$ on $[0, \pi/2]$ with respect to the weight function $W(x) = 1$.
4. Derive a numerical method (Crank-Nicolson's method) to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

Paper V: Graph Theory

1. Prove that the block graph of a connected graph is a tree.
2. Prove that a graph is Eulerian if and only if it is connected and even.
3. Prove that every simple outer planar graph has a vertex of degree less than or equal to 2.
4. Compute the spectrum of the complete bipartite graph with m vertices.