Institute of Distance and Open Learning
MA/MSc(Mathematics) Part I
Assignment 2018-19

Instructions:

• All questions to be written and submitted in the assignment sheet provided by IDOL

• Answers to all five papers' assignments to be submitted separately

• The last date of submission is Saturday 30th March 2019 before 5pm in Room No. 112, first floor,IDOL building, Kalina campus, Santacruz (E), Mumbai 400098

Paper I: Algebra

1. Prove that the $\mathbb{Q} - \{1\}$ with operation * defined as $a * b = a + b - ab$, is an abelian group.
2. Let V be a finite dimensional vector space over a field F. Let B and B' be bases of V. Show that for every linear functional T on V the matrix of T with respect to B' is similar to the matrix of T with respect to B.
3. Prove that a finite integral domain is a field.
4. Show that similar matrices have the same minimal polynomial.

Paper II: Analysis and Topology

1. Define metric space. If (X, d) is a metric space, then prove that in (X, d) :
   (i) Union of open sets is open
   (ii) Intersection of a finite number of open sets is open
2. Define a connected set. If A and B are connected sets the prove or disprove that $A \cap B$, $A \cup B$ are also connected.
3. Define a Hausdroff Topological space. Prove that every metric space is a Hausdroff space.
4. Show that continuous image of a connected space is connected.

Paper III: Complex Analysis

1. Prove that if G is an open connected set and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0 \ \forall z \in G$, then f is constant.
2. Prove that a Möbius Transformation is a composition of translation, rotation, inversion and magnification.
3. State and prove Cauchy's Integral Formula.
4. Find all the possible Laurent Series expansions of $f(z) = \dfrac{1}{z(z+1)(z-2)}$.

Paper IV: Discrete Mathematics and Differential Equations

1. Prove that $a \equiv b \pmod{n}$ if and only if a and b leave the same reminder when divided by n.
2. State and prove Euler's criterion for quadratic residue of p.
3. Show that the Legendre polynomial $P_n(x)$ of degree n is given by $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}(x^2-1)^n$.
4. Solve $(1-x^2)y'' - 2xy' + p(p+1)y = 0$ using power series.

Paper V: Set theory, Logic and Elementary Probability Theory

1. Define finite set. Show that if a set A is finite then there is no bijection of A with a proper subset of itself.
2. By using Zorn's lemma, prove that a nonzero unit ring contains a maximal proper ideal.
3. State and prove continuity property of probability.
4. Show that $E_y E[X/Y = y] = E[X]$